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AI/ML with High School Math

Hello world

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Understanding AI

Notes from my research

Hands on! From data to decision

Optimization and functions

Classifier Examples

Why base concepts matter?

Cool applications of AI

Computer Vision

Deep Dream: Text to image

Large Language Models (LLMs)

Understanding AI

Notes from my research

Based on the work of Silveira and Lopes in "Intelligence across humans and machines: a joint perspective":

Information Processing Function

$$proc : Ext \times Int \to Act \tag{1}$$

Notes from my research

Action execution function

$$exec : Act \rightarrow Feedback \tag{2}$$

Notes from my research

Adaptation to the environment

$$adapt : Ext \times Int \times Feedback \to Ext \times Int \quad (3)$$

Notes from my research

Function of Intelligence

$$\mathcal{I}(e, i) = \mathcal{I}(adapt(e, i, exec(proc(e, i)) \tag{4}$$

Correct?

- Knowledge
- Cognition
- Reality

Hands on! From data to decision

What is AI?

- **AI as a Function:** $f(x) = y$ - Mapping inputs to outputs
- **Universal approximation:** Approximating any function with a model
- **Challenges:** Multiple features (high-dimensional data), ... what else?
- **Key concept:** All boils down how we model our data (matrices, random variables, graphs, ..)[1]

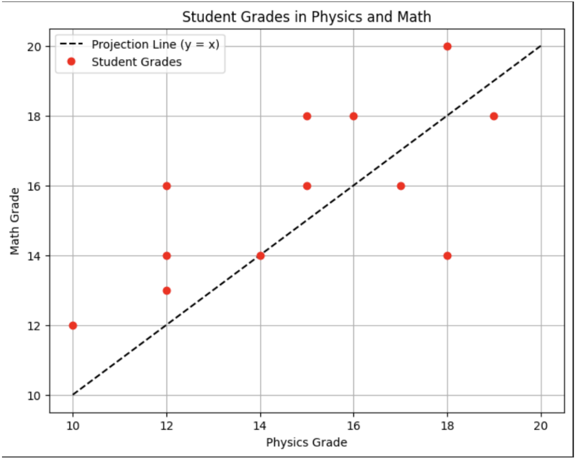
[1] Using matrices in AI enables linear transformations and efficient computation but may lose relational structure (handled better by graphs).

Hands on! From data to decision

Let's consider the grades of students in Math and Physics. We can represent these grades in a matrix format:

$$\mathbf{G} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ \vdots & \vdots \\ g_{n1} & g_{n2} \end{pmatrix}$$

Where g_{ij} is the grade of the ith student in the jth subject.



Can you separate the students into groups/classes based on their grades?

Projecting vectors onto (1,1)

What happens when we project any vector in $\mathbb{R}^2$ onto the vector $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

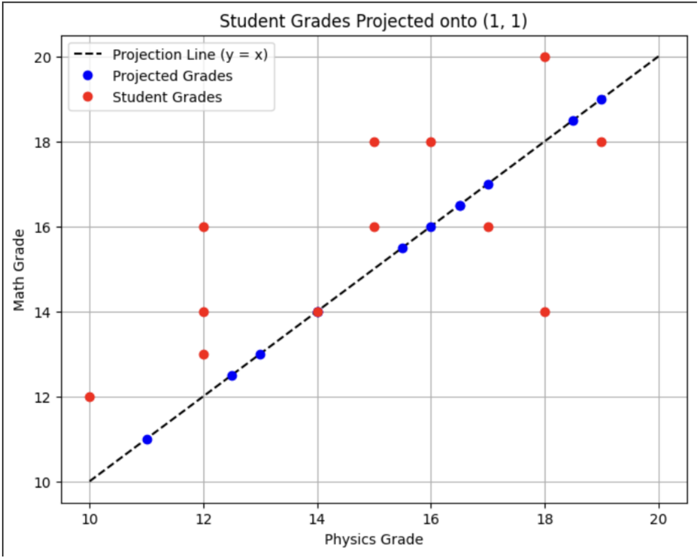
The projection of $\mathbf{v}$ onto $\mathbf{u}$ is given by:

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u}$$

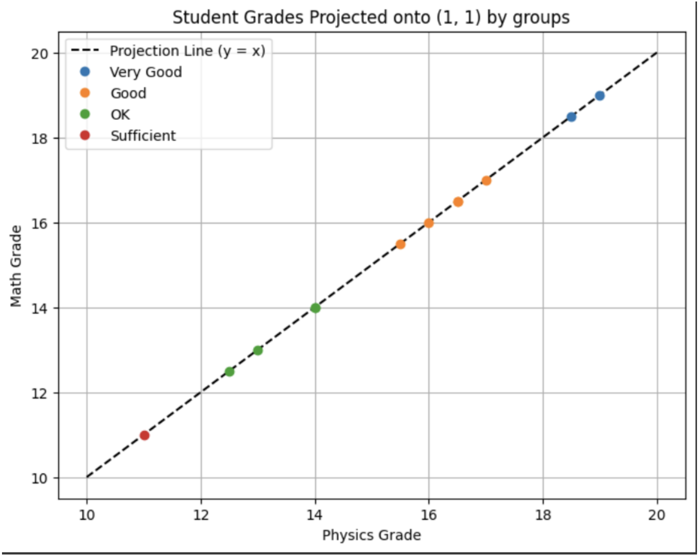
Projecting vectors onto (1,1)

For $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$, we have:

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{v_1 + v_2}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \cdot (v_1 + v_2) \\ \frac{1}{2} \cdot (v_1 + v_2) \end{pmatrix}$$



Student Grades Projected onto (1, 1)
Projection Line (y = x)
Projected Grades
Student Grades
Math Grade
Physics Grade
10
12
14
16
18
20

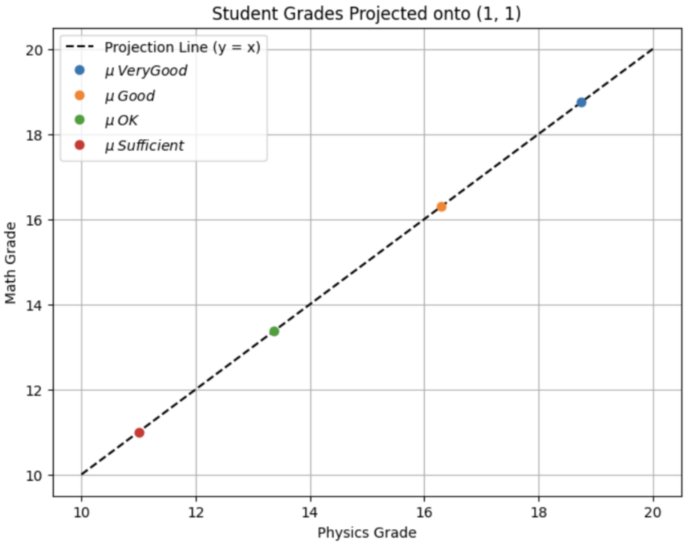

Student Grades Projected onto (1, 1) by groups
Projection Line (y = x)
Very Good
Good
OK
Sufficient
Math Grade
Physics Grade
10
12
14
16
18
20

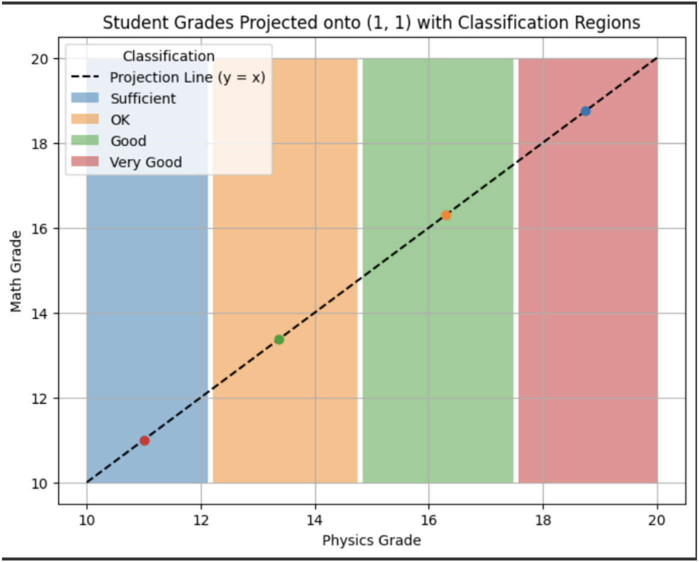
Classifying a new student

If a new student appears with grades, how would you classify them into one of the groups?
For example, if the new student has grades:

$$\mathbf{v}_{\text{new}} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$$

Which group would they fit into?


Student Grades Projected onto (1, 1)
Projection Line (y = x)
μ VeryGood
μ Good
μ OK
μ Sufficient
Math Grade
Physics Grade
10
12
14
16
18
20


Student Grades Projected onto (1, 1) with Classification Regions
Classification
Projection Line (y = x)
Sufficient
OK
Good
Very Good
Math Grade
Physics Grade
10
12
14
16
18
20

Notebook

It should be obvious by now that $\mathbf{v}_{\text{new}}$ would be classified as **OK**.

Feel free to check yourself by running the notebook (click here) and making the required changes.

Optimization and functions

AI as an optimization problem

Optimization minimizes error: It's how AI learns.

- **Minimizing loss:**
  - Loss function: Measures the difference between predicted and actual values.
  - Least squares loss:

$$L = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

AI as an optimization problem

A familiar problem. Anyone? :)

- **Problem:** Find the value that minimizes the sum of squared differences from a given set of numbers.
- **Mathematical formulation:**

$$\min_{x} f(x) = \min_{x} \sum_{i=1}^{N} (x - x_i)^2$$

AI as an optimization problem

Let's play with intuition. How about now? :)

Given points in a circumference, what is the solution?

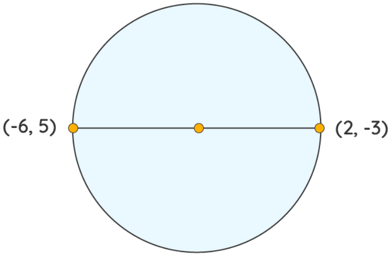


Figure: Circumference

AI as an optimization problem

A familiar problem. Anyone? :)

- **Problem:** Find the value that minimizes the sum of squared differences from a given set of numbers.
- **Mathematical formulation:**

$$\min_x f(x) = \min_x \sum_{i=1}^{N} (x - x_i)^2$$

- **Solution:** The average! [note: f is convex]

AI as an optimization problem

$$\frac{d}{dx}\sum_{i=1}^{N}(x - x_i)^2 = 2\sum_{i=1}^{N}(x - x_i)$$

Setting the derivative to zero:

$$2\sum_{i=1}^{N}(x - x_i) = 0$$

$$\implies x = \frac{1}{N}\sum_{i=1}^{N}x_i$$

Classifier Examples

Simple classifier example

Prior (A priori) probability classifier:

- Assigns the most probable class based on prior probabilities
- Accuracy in the worst case: ˜$P(A)$% (e.g., ˜70% if $P(A) = 0.7$)
- Demonstrates how basic math concepts are used in AI

Example:

$$P(A) = 0.7, P(B) = 0.3 \rightarrow \text{Always classify as } A$$

Simple+ classifier example

Can we do better? Anyone? :)

A posteriori probability classifier

$$P(Y = y|X = x) = \frac{P(Y = y \cap X = x)}{P(X = x)}$$

$$P(Y = y|X = x) = \frac{P(X = x|Y = y) \cdot P(Y = y)}{P(X = x)}$$

For a given input x, classify as the class i that maximizes $P(Y = i|X = x)$. Assigns the most probable class based on posterior probabilities

AI: Vectors and Matrices

- Some AI operations rely on vector and matrix mathematics:
  - Representing data as vectors (e.g., features of a sample: $\mathbf{x} = [x_1, x_2, x_3]^T$).
  - Applying transformations using weights (e.g., $\mathbf{w} = [w_1, w_2, w_3]$).
- Example: Linear combination of features

$$\hat{y} = \mathbf{w} \cdot \mathbf{x} = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = w_1 x_1 + w_2 x_2 + w_3 x_3$$

AI: Vectors and Matrices

- We would like to minimize the distance between the predictions of our model ($\hat{Y}$) and the actual values (Y).
- Distance can be measured using the squared Euclidean distance:

$$\text{Distance: } \|\hat{Y} - Y\|^2$$

AI: Vectors and Matrices

- This leads to an optimization problem:

$$\min_{\hat{Y}} \|\hat{Y} - Y\|^2 = \min_{W} \|W \cdot X - Y\|^2$$

  - The goal is to find the model parameters (e.g., weights W) that minimize this distance.

AI: Vectors and Matrices

$$\min_{\hat{Y}} \|\hat{Y} - Y\|^2 = \min_{W} \|W \cdot X - Y\|^2$$

- Why is this important?
  - Minimizing the distance ensures our model predictions are as close as possible to the actual values.
  - This principle underpins many AI techniques, such as linear regression and neural networks.

Why base concepts matter?

- High school math is the foundation for advanced AI concepts
- Example: Probability, linear algebra, etc.
- Real-world applications make learning easier and more rewarding

Cool applications of AI

What is Computer Vision?

- **Ambition:** Teaching machines to see and interpret images
- **Images as surfaces:** derivatives are so important! (it can measure how quickly image intensity changes... is it useful? how?)
- **Applications:**
  - **Aerial projection:** Map images to coordinates (e.g., Google Maps)
  - **Image classification:** Identifying objects in images

Deep Dream: Reverse image classifier

Instead of classifying images, we can use a powerful image processing technique to generate images based on a given input.

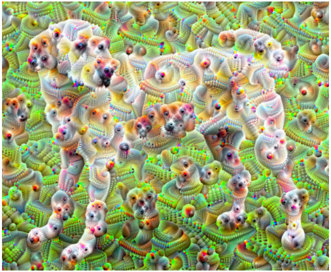


Figure: src: https://www.tensorflow.org/tutorials/generative/deepdream

Deep Dream: Scary times!

deepdreamgenerator.com/processing/18140544?ut=bbMswwkiMutj

Date	Model	Size
Dec 05, 2024	DaVinci2	1152x864

Delete Download

high-school alumni teaches high-school students about AI

LLMs - Exercise for students

Try completing a sentence:

"In **science** school, I love learning about..."

LLMs - Exercise for students

Try completing a sentence:

"In **science** school, I love learning about..."

.... how about "In **art** school, I love learning about..."?

LLMs - How?

- **How it works:** Predict the next word in a sequence
- **Example:** Autocomplete in a text editor

Closing and questions

- **Summary:** From theory to applications, math is the bridge to AI's magic!
- **Takeaway:** Always connect theory with real-world applications!
- Explore further - ask questions, and experiment!!

Thank you!

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