Quantum Walks with Rydberg atoms

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1. Motivation

- Many optimisation problems can be cast into **graph problems**
- However, these are often difficult to encode due to **connectivity** limitations

Solution → **Rydberg atoms**

- Long-range interactions
- Reconfigurable geometry
- Native multiqubit gates

- **Quantum walks** are the quantum analogue of random walks
- They are a natural **algorithmic** tool to address graph problems
- Although there are implementations for special graphs (e.g., lattices [1]), an efficient general implementation is **not known**

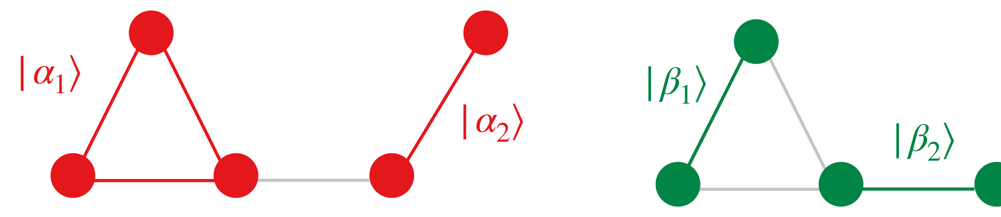
Can Rydberg features enable the implementation of quantum walks on arbitrary graphs?

2. The staggered quantum walk

- To define a staggered quantum walk [2], we need some additional structure in the graph

Tessellation → Partition of the vertices into cliques

Tessellation cover → Set of tessellations that cover all the edges



Example of a tessellation cover with two tessellations

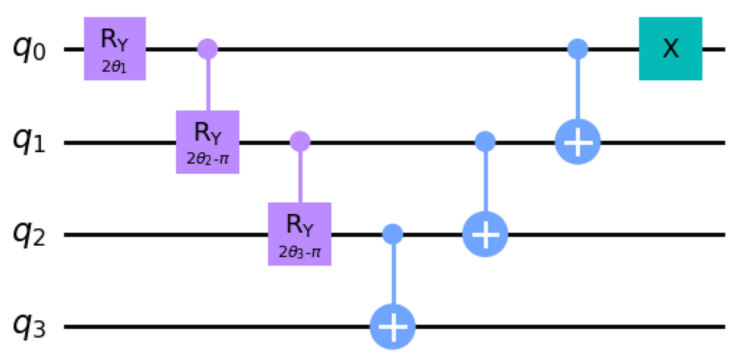
- To each clique associate a superposition of vertices and define:

$$W_\alpha = 1 - 2\sum_k |\alpha_k\rangle\langle\alpha_k|$$
$$W_\beta = 1 - 2\sum_k |\beta_k\rangle\langle\beta_k|$$
$$\longrightarrow \quad W = W_\alpha W_\beta$$

Walk operator

3. Our proposal

- Use N atoms with **excitation encoding** $|i\rangle = |0\ldots1_i\ldots0\rangle$
- **Diagonalize** each walk operator $W_\alpha = \otimes_k U_k(\alpha)\ \mathrm{C^nZ}\ U_k^\dagger(\alpha)$
- U_k, which prepares state $|\alpha_k\rangle$, can be implemented with a **linear** amount of gates



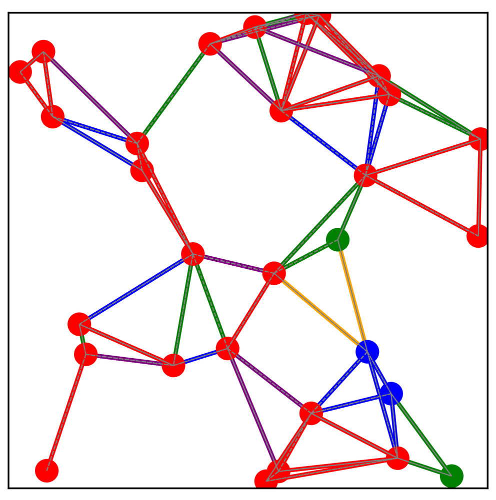
Example of U_k for a clique of size 4

Due to the **locality** of the evolution, our proposal is especially suited for

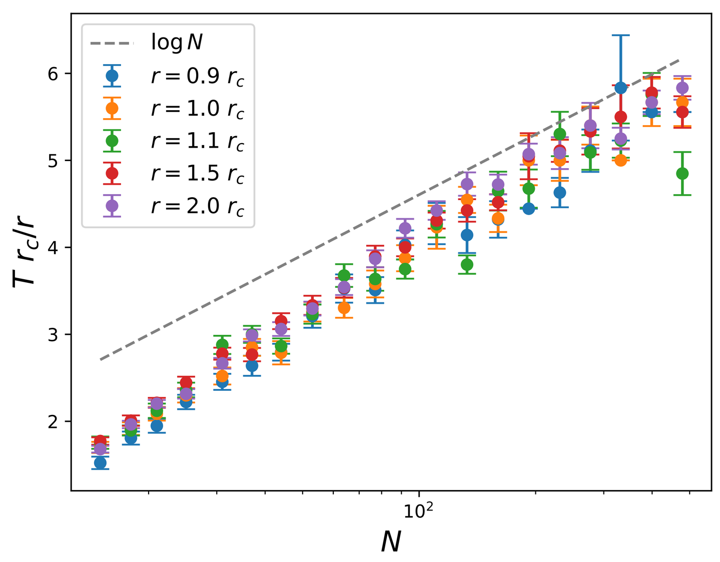
Spatial networks

4. Tessellation cover algorithm

- We developed a **constructive** algorithm to find a tessellation cover in time $O(N\,\mathrm{polylog}\,N)$
- We analysed its performance on **random geometric graphs** $\mathrm{RGG}(N, r)$
- We set the **natural scaling** of r to the critical radius of connectivity $r_c = \sqrt{\log n/(\pi n)}$



Example of random geometric graph with $N = 30$ and $r/r_c = 1.5$

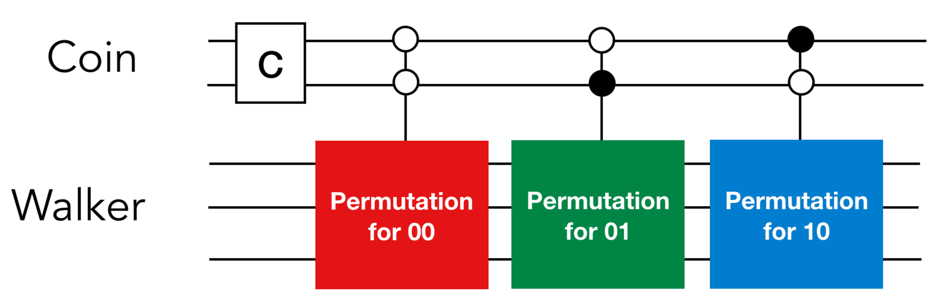


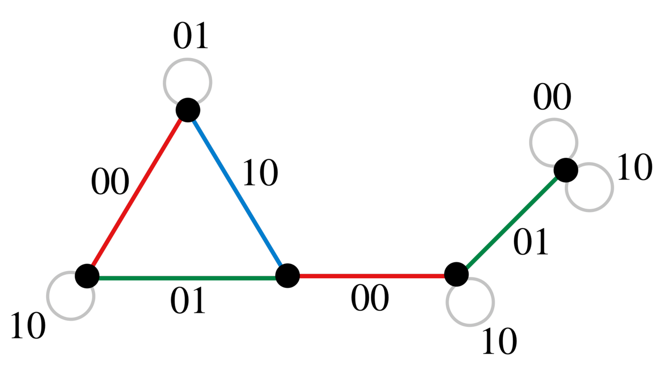
We found that the **number of tessellations** T scales as $T \sim \frac{r}{r_c}\log N$

5. Comparison with coined quantum walks

- A general quantum circuit for **coined** quantum walks has been proposed [3]
- Although it uses only $\log N + \log\Delta$ qubits, we found that it requires $\Delta\cdot\Omega(N\log N)$ gates
- Thus, the **parallelised circuit depth** is exponentially larger than the staggered approach

	Number of qubits	Number of gates	Parallelised depth	Classical costs
Staggered walk	N	$t\cdot T\cdot O(N)$	$t\cdot T$	$O(N\,\mathrm{polylog}\,N)$
Coined walk	$\log N + \log\Delta$	$t\cdot\Delta\cdot\Omega(N\log N)$	$t\cdot\Delta\cdot\Omega(N)$	$O(N\log N)$



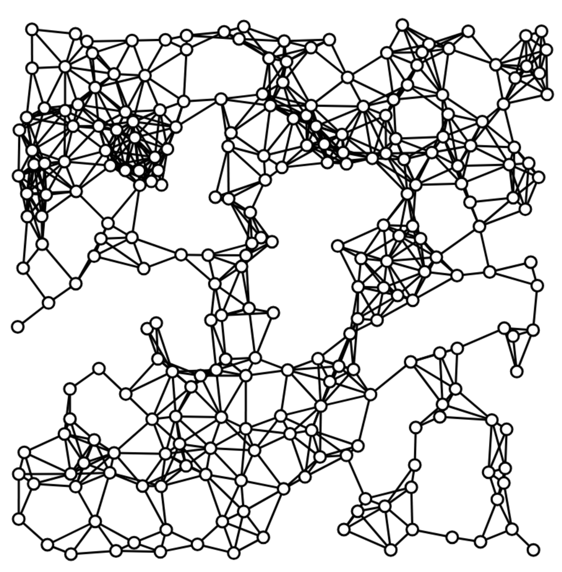


Example of circuit for coined quantum walk

6. Conclusions and outlook

- We developed a novel **Rydberg implementation** of staggered **quantum walks**
- Our proposal is well suited for **spatial networks** due to the locality of the evolution
- When compared to coined quantum walks, our implementation has an exponential advantage in **parallelised depth**, at the cost of using exponentially more **qubits**. Nevertheless, achieving efficient depth is critical for reducing the overall time complexity of algorithms
- **Paper** to be submitted soon

Example of spatial network



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References

[1] Young et al., *Science* 377, 885 (2022)

[2] Portugal et al., *Quantum Inf. Process.* 15, 85 (2016)

[3] Chakrabarti et al., IEEE Comput. Soc. Annu. Symp. VLSI, 2012

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