A brief comment on the numerical results in 'Electrostatic potential of a uniformly charged triangle in barycentric coordinates'

Diogo R. Ferreira

Instituto Superior Técnico (IST), Universidade de Lisboa, Portugal

E-mail: diogo.ferreira@tecnico.ulisboa.pt

June 2021

Abstract. In a recent paper (2021 Eur. J. Phys. 42 045205), U-Rae Kim et al carry out an analytical effort to arrive at an expression for the electrostatic potential due to a uniformly charged triangle. This appears to have been attempted by other authors before, with similar, but not exactly equal, numerical results. In addition, the paper leaves some doubts with regard to the comparison of the analytical expressions by different authors and the agreement of their numerical results. What, then, is the true electrostatic potential of a uniformly charged triangle? This comment does not answer this question but provides an independent effort to confirm the numerical results.

1. Discretization of space

The common approach that is adopted in both [1] and its predecessors [2] and [3] is to proceed with the numerical integration of an analytical expression that has been previously derived. Since our goal is to confirm the numerical results rather than the validity of the proposed expressions, we will leave the analytical expressions aside and try to obtain the same numerical results from first principles.

For this purpose, we will discretize the space where the uniformly charged triangle is located and, rather than a continuous charge distribution, we will approximate it as a collection of point charges with the same overall, global shape. Such discretization can be taken to an arbitrary resolution so that, for all practical effects, the collection of point charges becomes indistinguishable from the uniformly charged triangle.

This approach is bound only by the computational resources that are needed to take into account all the point charges that result from the desired level of discretization. For our purposes, what needs to be discretized is the space where the triangle is located. The electrostatic potential can then be computed at any spatial point, either within or outside the triangle, subject to the restriction that such point must not coincide with a point charge, as the potential would become infinite in that case.

Fortunately, the discretization procedure is such that the point charges are naturally misaligned with the points of interest where the potential is to be calculated.

Figure 1 shows the triangle ΔABC with $A = (0, 0)$, $B = (1, 0)$ and $C = (0.25, 1)$ as defined in [1]. In this example, the region where the triangle is located has been discretized by a grid of 10×10 cells across the x and y axes. The amount of charge that is contained in each cell is proportional to the area of the triangle that is inside that cell. An equivalent point charge is placed at the center of each cell. This collection of point charges replaces the uniform charge distribution of the original triangle.

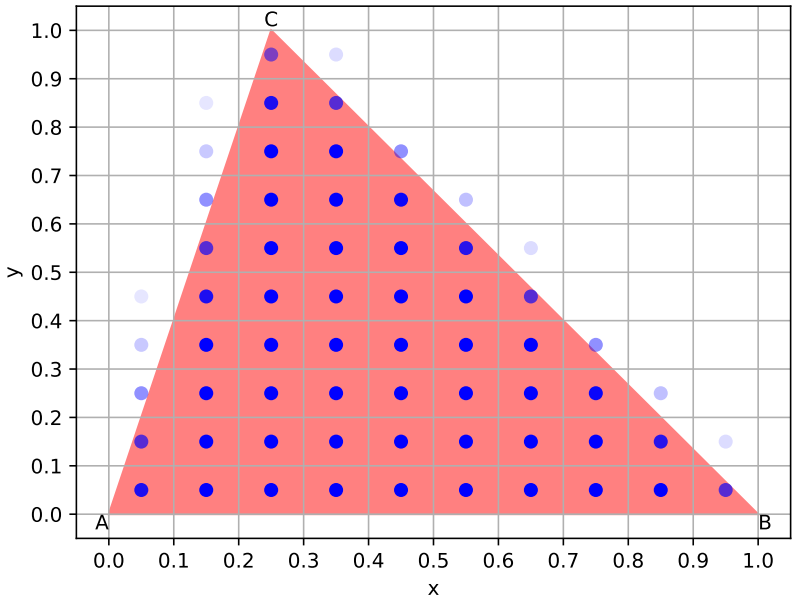

Figure 1. Discretization of a uniformly charged triangle ΔABC via point charges.

Although the approximation in Figure 1 is quite crude, it should be possible to convince the reader that, as the number of cells is increased by several orders of magnitude, the collection of point charges will resemble the original triangle.

2. Calculating the potential

The aim of this space discretization is to enable the computation of the electrostatic potential as a sum of contributions from individual point charges. According to the superposition principle,

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|} \tag{1}$$

In our context, each point charge $q_i = \sigma a_i$ comes as a result of the uniform charge density σ and the area a_i of the triangle that is inside the corresponding cell. In addition, the potential V can be calculated in units of $\sigma/4\pi\epsilon_0$ as in [1]. Finally, given that the

point charges lie in the $z = 0$ plane, the potential at any point $P = (x, y, z)$ that does not coincide with a point charge can be calculated as:

$$V(x, y, z) = \sum_i \frac{a_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + z^2}} \tag{2}$$

Table 1 shows the results for the same set of points that were used in [1, 3] and for different grid sizes. From 100×100 onwards, there are no differences in the results, at least not up to the fifth decimal place, so it becomes unnecessary to increase the grid size further. The results agree with those reported in Table 1 of [1].

Table 1. Numerical results for the electrostatic potential with different grid sizes.

	grid size			
P	10×10	100×100	1000×1000	10000×10000
$(2, 2, 2)$	0.16377	0.16372	0.16372	0.16372
$(2, 1.62, 2)$	0.17451	0.17447	0.17447	0.17447
$(2, 0, 2)$	0.19378	0.19379	0.19379	0.19379
$(2, 2, 0)$	0.21761	0.21748	0.21748	0.21748
$(2, 2, -2)$	0.16377	0.16372	0.16372	0.16372
$(2, 2, 9)$	0.05380	0.05380	0.05380	0.05380
$(9, 2, 9)$	0.03984	0.03984	0.03984	0.03984
$(9, 0, 0)$	0.05822	0.05822	0.05822	0.05822

In conclusion, this suggests that the electrostatic potential of a uniformly charged triangle, as calculated by the expression provided by U-Rae Kim et al [1], appears to be more accurate than the previous results provided by [2, 3].

References

[1] Kim U R, Han W, Jung D W, Lee J and Yu C 2021 *European Journal of Physics* **42** 045205

[2] Rao S, Glisson A, Wilton D and Vidula B 1979 *IEEE Transactions on Antennas and Propagation* **27** 604–608

[3] Okon E E and Harrington R F 1982 *International Journal for Numerical Methods in Engineering* **18** 1401–1411