

Session Types and Linear Logic and Lightweight Applications of Session Types in Java

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Introduction

Concurrent Processes

- ▶ Coordination of multiple simultaneously executing agents.
- ▶ Programming Models: Shared-Memory vs Message-Passing
- ▶ Hard to reason about:
 - ▶ Deadlocks
 - ▶ Data races
 - ▶ Concurrent interleavings make behaviour hard to predict
 - ▶ ... and also hard to replicate (e.g. testing).

Introduction

A Concurrency-Theoretic Approach: Session Types

Structuring Communication

- ▶ Communication without structure is hard to reason about.
- ▶ Structure communication around the concept of a **session**.
- ▶ Predetermined sequences of interactions along a (session) channel:
 - ▶ “Input a number, output a string and terminate.”
 - ▶ “Either output or input a number.”

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Session-based Structuring

- ▶ Specify communication behaviour as sessions.
- ▶ Check that programs adhere to specification (session **fidelity**).

Introduction

A Concurrency-Theoretic Approach: Session Types

Session Types [Honda93]

- ▶ Types **are** descriptions of communication behaviour, assigned to channels.
- ▶ A way of guaranteeing communication discipline, **statically**.
- ▶ A **linear** typing discipline for π -calculus processes.
- ▶ Intrinsic notion of duality: Send/Receive, Offer choice/Select

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Syntax

$$P, Q ::= x\langle y \rangle.P \mid x(y).P \mid x.l; P \mid x.\text{case}\{l_i : P_i\}_{i \in I} \mid (\nu x)P \mid \mathbf{0} \mid (P \mid Q)$$
$$S, T ::= !S.T \mid ?S.T \mid \oplus\{l_i : T_i\}_{i \in I} \mid \&\{l_i : T_i\}_{i \in I}$$

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A pair of processes interacting on dual sessions is **deadlock-free!**

Introduction

Session Types and Linear Logic

Linear Logic [Girard87]

- ▶ A marriage of classical dualities and constructivism.
- ▶ A logic of resources and interaction.
- ▶ Resource independence captures non-determinism/concurrency.
- ▶ Connected to concurrency early on [Abramsky93,BellinScott94]

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Session Types and ILL [CairesPfenning01]

- ▶ Its possible to interpret session types as linear logic propositions.
- ▶ Linear logic proofs as process typing derivations.
- ▶ Proof dynamics as process dynamics.

Introduction

Session Types and Linear Logic

Why does it matter?

- ▶ Good metalogical properties map to good program properties.
- ▶ Progress, session fidelity, type preservation “for free”.
- ▶ Reasoning built-in.
- ▶ Much more compositional than traditional approaches (i.e. extensibility).

Introduction

Roadmap

1. Basic interpretation
2. Enriching the type structure (dependent types and polymorphism)
3. Reasoning Techniques: Logical equivalence, type isomorphisms and proof conversions.
4. Multiparty session types.

ILL Interpretation

Key Ideas

- ▶ Session Types as Intuitionistic Linear Propositions.
- ▶ Sequent calculus rules as π -calculus typing rules.
- ▶ Cut reduction as process reduction.

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Propositions

$$A, B ::= A \otimes B \mid A \multimap B \mid \mathbf{1} \mid A \& B \mid A \oplus B \mid !A$$

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Sequents

- ▶ Duality of offering (right rules) and using (left rules) a session.
- ▶ Proof composition (cut) as process composition.
- ▶ Identity as forwarding/renaming.

ILL Interpretation

Judgmental Principles

Typing Judgment

$$\overbrace{u_1:A_1, \dots, u_m:A_m}^{\Gamma}; \overbrace{x_1:A_1, \dots, x_n:A_n}^{\Delta} \Longrightarrow P :: x:A$$

ILL Interpretation

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Process P provides A along x if composed with sessions in Δ and Γ .

ILL Interpretation

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Cut as Composition

$$\frac{\Gamma; \Delta \Longrightarrow P :: x:A \quad \Gamma; \Delta', x:A \Longrightarrow Q :: z:C}{\Gamma; \Delta, \Delta' \Longrightarrow (\nu x)(P \mid Q) :: z:C} \text{ cut}$$

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Parallel composition of P , offering $x:A$ and Q , using $x:A$.

Identity as Renaming

$$\overline{\Gamma; x:A \Longrightarrow [x \leftrightarrow z] :: z:A}$$

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ILL Interpretation

Propositions

Multiplicative Conjunction

$$\frac{\Gamma; \Delta \Longrightarrow P_1 :: y:A \quad \Gamma; \Delta' \Longrightarrow P_2 :: x:B}{\Gamma; \Delta, \Delta' \Longrightarrow (\nu y)x\langle y \rangle.(P_1 \mid P_2) :: x:A \otimes B} \otimes R$$

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$$\frac{\Gamma; \Delta, y : A, x : B \Longrightarrow Q :: z:C}{\Gamma; \Delta, x:A \otimes B \Longrightarrow x(y).Q :: z:C} \otimes L$$

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Proof Reduction

$$\Gamma; \Delta, \Delta' \Longrightarrow (\nu x)((\nu y)x\langle y \rangle.(P_1 \mid P_2) \mid x(y).Q) :: z:C$$

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ILL Interpretation

Propositions

Linear Implication

$$\frac{\Gamma; \Delta, y : A \Longrightarrow P :: x : B}{\Gamma; \Delta \Longrightarrow x(y).P :: x : A \multimap B} \multimap R$$

ILL Interpretation

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ILL Interpretation

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Linear Implication as input. Reduction is the same as for \otimes .

ILL Interpretation

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Multiplicative Unit

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Proof Reduction

$$\Gamma; \Delta \Rightarrow (\nu x)(\mathbf{0} \mid Q) :: z:C$$

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Multiplicative Unit as Termination

ILL Interpretation

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Proof Reduction

$$\begin{aligned} \Gamma; \Delta, \Delta' \Longrightarrow (\nu x)(x.\text{case}(P_1, P_2) \mid x.\text{inl}; Q) :: z:C \\ \longrightarrow \Gamma; \Delta, \Delta' \Longrightarrow (\nu x)(P_1 \mid Q) :: z:C \end{aligned}$$

ILL Interpretation

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Same proof reductions as $\&$.

ILL Interpretation

Judgmental Principles for Exponential

Persistent Cut

$$\frac{\Gamma; \cdot \Longrightarrow P :: x:A \quad \Gamma, u:A; \Delta \Longrightarrow Q :: z:C}{\Gamma; \Delta \Longrightarrow (\nu u)(!u(x).P \mid Q) :: z:C} \text{ cut!}$$

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Parallel composition of P , offering $x:A$ and Q , using $u:A$ persistently.

ILL Interpretation

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Parallel composition of P , offering $x:A$ and Q , using $u:A$ persistently.

Copy

$$\frac{\Gamma, u:A; \Delta, x:A \Longrightarrow P :: z:C}{\Gamma, u:A; \Delta \Longrightarrow (\nu x)u\langle x \rangle.Q :: z:C} \text{ copy}$$

ILL Interpretation

Judgmental Principles for Exponential

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Proof Reduction

$$\begin{aligned} & \Gamma; \Delta \Longrightarrow (\nu u)(!u(x).P \mid (\nu x)u\langle x \rangle.Q) :: z:C \\ \longrightarrow & \Gamma; \Delta \Longrightarrow (\nu u)(!u(x).P \mid (\nu x)(P \mid Q)) :: z:C \end{aligned}$$

ILL Interpretation

Propositions

Exponential

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Proof reduction transforms a cut into a cut[!] (struct. equivalence).

ILL Interpretation

Metatheory

Operational Correspondence and Subject Reduction

If $\Gamma; \Delta \Longrightarrow P :: z:A$ and $P \rightarrow P'$ then $\exists Q$ such that $\Gamma; \Delta \Longrightarrow Q :: z:A$ and $P' \equiv Q$.

ILL Interpretation

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Global Progress

$live(P) \triangleq (\nu \bar{x})(Q \mid R)$ with $Q \equiv \pi.Q'$ or $Q \equiv [x \leftrightarrow y]$

If $\vdash P :: x:1$ and $live(P)$ then $\exists Q$ such that $P \rightarrow Q$.

Much stronger property than in classical session types, “for free”!

ILL Interpretation

Summary

- ▶ Linear Propositions as Session Types.
- ▶ Intuitionistic proofs as session-typed processes.
- ▶ Process reduction maps to proof conversion.

ILL Interpretation

Summary

- ▶ Linear Propositions as Session Types.
- ▶ Intuitionistic proofs as session-typed processes.
- ▶ Process reduction maps to proof conversion.
- ▶ ... but not all proof conversions are process reductions!

Richer Type Theories

Motivation

Session Types

- ▶ Only express simple communication patterns.
- ▶ No interesting properties of exchanged **data**.
- ▶ No sophisticated properties of processes.

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Answers from Logic

- ▶ Enrich the logic/types: Quantifiers, Modalities
- ▶ Dependent Session Types [Toninho et al.11]
- ▶ Polymorphic Session Types [Pérez et al.11, Wadler11]
- ▶ Internalisation in a linear monad [Toninho et al.12]

Richer Type Theories

Where are we?

Dependent Session Types [Toninho et al.11, Pfenning et al.11]

- ▶ Two new types: $\forall x:\tau.A$ and $\exists x:\tau.A$
- ▶ Parametric in the language of types τ .

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Richer Type Theories

Where are we?

Polymorphism and Parametricity [Pérez et al.11, Wadler11]

- ▶ Second-order quantification ($\forall X.A$ and $\exists X.A$).
- ▶ Communication of session types / abstract protocols.
- ▶ Relational parametricity results in the style of System F.

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Monadic Integration [Toninho et al.12]

- ▶ $M : \{\Gamma; \Delta \vdash z:A\}$, type of an open process P (construct $\{P\}$).
- ▶ Process construct $\text{bind}(M, z.Q)$:
 - ▶ Evaluate M to a (suspended) process $\{P\}$.
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 - ▶ Evaluate M to a (suspended) process $\{P\}$.
 - ▶ Run underlying process P in parallel with Q .
- ▶ Higher-Order Sessions, e.g.: $\forall x:\{z:A\}.\forall y:\{z:B\}.A \otimes B$

Proof Conversions and Type Isomorphisms

Introduction

Proof Conversions

- ▶ Process reductions map to principal cut reductions.
- ▶ What about the remaining proof conversions?
- ▶ Can we understand them in concurrency theoretic terms?

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Approach

We decompose proof conversions into three classes:

- ▶ Computational Conversions (i.e. principal cut conversions).
- ▶ Cut Conversions (i.e. permuting two cuts in a proof).
- ▶ Commuting Conversions (i.e. commuting inference rules).

First two correspond to **reductions** and **structural equivalences**.

Proof Conversions

Commuting Conversions

Commuting Conversions induce a congruence \simeq_c on typed processes

$\otimes L / \otimes L$ Commuting Conversion

$$x:A \otimes B, z:C \otimes D \implies x(y).z(w).P \simeq_c z(w).x(y).P :: v:E$$

Commuting (input) prefixes appears, at first, counterintuitive.

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Typed Contextual Equivalence

In any well-typed context, we cannot distinguish the two processes:

$$(\nu x)(\nu z)(x(y).z(w).P \mid R_x \mid S_z) \cong (\nu x)(\nu z)(z(w).x(y).P \mid R_x \mid S_z) :: v:E$$

Actions along x and z are not observable.

Proof Conversions

Typed Contextual Equivalence

Typed Contextual Equivalence

- ▶ How to define this equivalence in a tractable way?
- ▶ Typed Contextual **Bisimilarity**.

Proof Conversions

Typed Contextual Equivalence

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- ▶ How to define this equivalence in a tractable way?
- ▶ Typed Contextual **Bisimilarity**.

Contextual Bisimilarity

- ▶ Contextual: For all typed contexts...
- ▶ Typed bisimilarity on closed processes:
 - ▶ $P \sim Q :: x:A \multimap B$ iff $P \xrightarrow{x(y)} P'$ implies $Q \xrightarrow{x(y)} Q'$ and $\forall R. \cdot \Longrightarrow R :: y:A$ we have $(\nu y)(P \mid R) \sim (\nu y)(Q \mid R) :: x:B$
 - ▶ $P \sim Q :: x:C$ iff $P \xrightarrow{\tau} P'$ implies $Q \Rightarrow Q'$ and $P' \sim Q' :: x:C$.
 - ▶ ...

Proof Conversions

Issue

$\otimes L / \otimes L$ Conversion Revisited

$$(\nu x)(\nu z)(x(y).z(w).P \mid R_x \mid S_z) \sim (\nu x)(\nu z)(z(w).x(y).P \mid R_x \mid S_z) :: v:E?$$

- ▶ Suppose input along x matches an output in R_x on the left proc.

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- ▶ What if S_z never outputs to z ?

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Termination and Bisimilarity

- ▶ Can we develop a uniform solution?
- ▶ Inspiration from functional “world”: Linear Logical Relations!

Proof Conversions

Termination and Bisimilarity

Linear Logical Relations [Pérez et al. 12]

- ▶ Termination: Inductively defined unary predicate.
- ▶ Contextual Bisimulation: Co-inductively defined binary relation.

Proof Conversions

Termination and Bisimilarity

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Logical Predicate

- ▶ Terminating by construction.
- ▶ Inductive on typing derivations: $\mathcal{L}[\Gamma; \Delta \vdash T]$
 - ▶ $P \in \mathcal{L}[\Gamma; y:A, \Delta \vdash T]$ if $\forall R \in \mathcal{L}[y : A]. (\nu y)(R \mid P) \in \mathcal{L}[\Gamma; \Delta \vdash T]$.

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- ▶ Base case is inductive on types:
 - ▶ $P \in \mathcal{L}[z:A \multimap B] \triangleq$ if $P \stackrel{z(y)}{\Rightarrow} P'$ then $\forall Q \in \mathcal{L}[y:A]. (\nu y)(P' \mid Q) \in \mathcal{L}[z:B]$
 - ▶ ...

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 - ▶ ...

Typing implies Termination

If $\Gamma; \Delta \vdash P :: T$ then $P \in \mathcal{L}[\Gamma; \Delta \vdash T]$.

Proof Conversions

Termination and Bisimilarity

Typed Bisimilarity

- ▶ Relational generalization of the predicate.
- ▶ Inductive on typing derivations: $\Gamma; \Delta \vdash PRQ :: T$
 - ▶ If $\Gamma; \Delta, y:A \vdash PRQ :: T$ then $\forall R. \vdash R :: y:A, \Gamma; \Delta \vdash (\nu y)(P|R)\mathcal{R}(\nu y)(Q|R) :: T$.

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- ▶ \approx is the largest such relation.

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- ▶ \approx is the largest such relation.

Soundness of Commuting Conversions

If $\Gamma; \Delta \vdash P \simeq_c Q :: T$ then $\Gamma; \Delta \vdash P \approx Q :: T$

Type Isomorphisms

Definition and Validation

Type Isomorphism ($A \simeq B$)

Types A and B are iso. if there are proofs π_A of $B \vdash A$ and π_B of $A \vdash B$, composing in both direction to identity.

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Session Type Isomorphisms ($A \simeq_S B$)

Session types A and B are iso. if there are processes P and Q :

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- ▶ $x:A \vdash P :: y:B$ and $y:B \vdash Q :: x:A$.
- ▶ $x:A \vdash (\nu y)(P \mid Q) \approx [x \leftrightarrow z] :: z:A$.
- ▶ $y:B \vdash (\nu x)(Q \mid P) \approx [y \leftrightarrow z] :: z:B$.

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Validating Isomorphisms

If $A \simeq B$ then $A \simeq_S B$.

Summary

So far:

- ▶ Explored a logical interpretation of session-based concurrency
- ▶ Explain concurrency theoretic concepts using logic
- ▶ Map logical phenomena to concurrency theory
- ▶ Clean and elegant reasoning through logic.

Not in this talk:

- ▶ Asynchrony
- ▶ Parametricity
- ▶ Inductive, Coinductive and Recursive Session Types.

Multiparty Session Types

Motivation and Basic Concepts

- ▶ Session types specify interaction between exactly two endpoints.
- ▶ Cannot precisely capture protocols with more than two parties.

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- ▶ Generalise to **multiparty** session types [Honda et al.08]:
 - ▶ Global types specify a global choreography.
 - ▶ Local types specify the protocol for each participant.

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- ▶ Generalise to **multiparty** session types [Honda et al.08]:
 - ▶ Global types specify a global choreography.
 - ▶ Local types specify the protocol for each participant.
 - ▶ Local types are generated mechanically from global types.
 - ▶ Duality is generalised to **consistency** to ensure deadlock-freedom.

Multiparty Session Types

Global Types

Global Types

$$\begin{aligned} G &::= p \rightarrow q : (T).G \\ &| p \rightarrow q : \{\ell_j : G_j\}_{j \in J} \\ &| \mu t. G \mid t \\ &| \mathbf{end} \end{aligned}$$

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2-Buyer Protocol

$$\begin{aligned} G_{2B} &\triangleq \mathbf{B1} \rightarrow \mathbf{S} : (\text{str}). \mathbf{S} \rightarrow \mathbf{B1} : (\text{int}). \mathbf{S} \rightarrow \mathbf{B2} : (\text{int}). \\ &\mathbf{B1} \rightarrow \mathbf{B2} : (\text{int}). \mathbf{B2} \rightarrow \mathbf{S} : \{\text{ok} : \mathbf{B2} \rightarrow \mathbf{S} : (\text{str}). \mathbf{end} \\ &\quad \text{quit} : \mathbf{end}\} \end{aligned}$$

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$$\begin{aligned} G & ::= p \rightarrow q : (T).G \\ & \quad | p \rightarrow q : \{\ell_j : G_j\}_{j \in J} \\ & \quad | \mu t. G \mid t \\ & \quad | \mathbf{end} \end{aligned}$$

2-Buyer Protocol

$$\begin{aligned} G_{2B} & \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}). \\ & \quad B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end} \\ & \quad \quad \text{quit} : \mathbf{end}\} \end{aligned}$$

Multiparty Session Types

Global Types

Global Types

$$\begin{aligned} G &::= p \rightarrow q : (T).G \\ &| p \rightarrow q : \{\ell_j : G_j\}_{j \in J} \\ &| \mu t. G \mid t \\ &| \mathbf{end} \end{aligned}$$

2-Buyer Protocol

$$\begin{aligned} G_{2B} &\triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}). \\ &B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end} \\ &\quad \text{quit} : \mathbf{end}\} \end{aligned}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_i: T_i\}_{i \in I} \mid \&p\{l_i: T_i\}_{i \in I} \mid \mathbf{end}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_i: T_i\}_{i \in I} \mid \&p\{l_i: T_i\}_{i \in I} \mid \mathbf{end}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_i: T_i\}_{i \in I} \mid \& p\{l_i: T_i\}_{i \in I} \mid \mathbf{end}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_i: T_i\}_{i \in I} \mid \& p\{l_i: T_i\}_{i \in I} \mid \mathbf{end}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_j:T_j\}_{j \in I} \mid \&p\{l_j:T_j\}_{j \in I} \mid \mathbf{end}$$

Projection

$$\begin{aligned} s \rightarrow r:(T).G' \upharpoonright p &= \begin{cases} r!(T); (G' \upharpoonright p) & \text{if } p = s \\ s?(T); (G' \upharpoonright p) & \text{if } p = r \\ G' \upharpoonright p & \text{otherwise} \end{cases} \\ s \rightarrow r:\{l_j:G_j\}_{j \in J} \upharpoonright p &= \begin{cases} \oplus r\{l_j:G_j \upharpoonright p\}_{j \in J} & \text{if } p = s \\ \&s\{l_j:G_j \upharpoonright p\}_{j \in J} & \text{if } p = r \\ G_1 \upharpoonright p & \forall i, j \in J. G_i \upharpoonright p = G_j \upharpoonright p \end{cases} \\ \mathbf{end} \upharpoonright p &= \mathbf{end} \end{aligned}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_j:T_j\}_{j \in I} \mid \&p\{l_j:T_j\}_{j \in I} \mid \mathbf{end}$$

Projection

$$\begin{aligned} s \rightarrow r:(T).G' \upharpoonright p &= \begin{cases} r!(T); (G' \upharpoonright p) & \text{if } p = s \\ s?(T); (G' \upharpoonright p) & \text{if } p = r \\ G' \upharpoonright p & \text{otherwise} \end{cases} \\ s \rightarrow r:\{l_j:G_j\}_{j \in J} \upharpoonright p &= \begin{cases} \oplus r\{l_j:G_j \upharpoonright p\}_{j \in J} & \text{if } p = s \\ \&s\{l_j:G_j \upharpoonright p\}_{j \in J} & \text{if } p = r \\ G_1 \upharpoonright p & \forall i, j \in J. G_i \upharpoonright p = G_j \upharpoonright p \end{cases} \\ \mathbf{end} \upharpoonright p &= \mathbf{end} \end{aligned}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_j: T_j\}_{j \in I} \mid \&p\{l_j: T_j\}_{j \in I} \mid \mathbf{end}$$

Projection

$$\begin{aligned} s \rightarrow r:(T).G' \upharpoonright p &= \begin{cases} r!(T); (G' \upharpoonright p) & \text{if } p = s \\ s?(T); (G' \upharpoonright p) & \text{if } p = r \\ G' \upharpoonright p & \text{otherwise} \end{cases} \\ \mathbf{s} \rightarrow r:\{l_j: G_j\}_{j \in J} \upharpoonright p &= \begin{cases} \oplus r\{l_j: \mathbf{G}_j \upharpoonright p\}_{j \in J} & \text{if } p = s \\ \&s\{l_j: G_j \upharpoonright p\}_{j \in J} & \text{if } p = r \\ G_1 \upharpoonright p & \forall i, j \in J. G_i \upharpoonright p = G_j \upharpoonright p \end{cases} \\ \mathbf{end} \upharpoonright p &= \mathbf{end} \end{aligned}$$

Multiparty Session Types

Local Types and Projection

Local Types

$$S, T ::= p!(T); S \mid p?(T); S \mid \oplus p\{l_j: T_j\}_{j \in I} \mid \& p\{l_j: T_j\}_{j \in I} \mid \mathbf{end}$$

Projection

$$\begin{aligned} s \rightarrow r:(T).G' \upharpoonright p &= \begin{cases} r!(T); (G' \upharpoonright p) & \text{if } p = s \\ s?(T); (G' \upharpoonright p) & \text{if } p = r \\ G' \upharpoonright p & \text{otherwise} \end{cases} \\ s \rightarrow r:\{l_j: G_j\}_{j \in J} \upharpoonright p &= \begin{cases} \oplus r\{l_j: G_j \upharpoonright p\}_{j \in J} & \text{if } p = s \\ \& s\{l_j: G_j \upharpoonright p\}_{j \in J} & \text{if } p = r \\ G_1 \upharpoonright p & \forall i, j \in J. G_i \upharpoonright p = G_j \upharpoonright p \end{cases} \\ \mathbf{end} \upharpoonright p &= \mathbf{end} \end{aligned}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}).$$
$$B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end}$$
$$\text{quit} : \mathbf{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); B2!(\text{int}); \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); B1?(\text{int}); \oplus S\{\text{ok} : S!(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \&B2\{\text{ok} : B2?(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq \mathbf{B1} \rightarrow \mathbf{S} : (\mathbf{str}).\mathbf{S} \rightarrow \mathbf{B1} : (\mathbf{int}).\mathbf{S} \rightarrow \mathbf{B2} : (\mathbf{int}).$$
$$\mathbf{B1} \rightarrow \mathbf{B2} : (\mathbf{int}).\mathbf{B2} \rightarrow \mathbf{S} : \{\mathbf{ok} : \mathbf{B2} \rightarrow \mathbf{S} : (\mathbf{str}).\mathbf{end}$$
$$\mathbf{quit} : \mathbf{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright \mathbf{B1} = \mathbf{S}!(\mathbf{str}); \mathbf{S}?(int); \mathbf{B2}!(int); \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright \mathbf{B2} = \mathbf{S}?(int); \mathbf{B1}?(int); \oplus \mathbf{S}\{\mathbf{ok} : \mathbf{S}!(\mathbf{str}); \mathbf{end}, \mathbf{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright \mathbf{S} = \mathbf{B1}?(str); \mathbf{B1}!(int); \mathbf{B2}!(int); \& \mathbf{B2}\{\mathbf{ok} : \mathbf{B2}?(str); \mathbf{end}, \mathbf{quit} : \mathbf{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}).$$
$$B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end}$$
$$\text{quit} : \mathbf{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); B2!(\text{int}); \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); B1?(\text{int}); \oplus S\{\text{ok} : S!(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \&B2\{\text{ok} : B2?(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}).$$
$$B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end}$$
$$\text{quit} : \mathbf{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); B2!(\text{int}); \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); B1?(\text{int}); \oplus S\{\text{ok} : S!(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \&B2\{\text{ok} : B2?(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}).$$
$$B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end}$$
$$\text{quit} : \mathbf{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); \mathbf{B2!(int)}; \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); \mathbf{B1?(int)}; \oplus S\{\text{ok} : S!(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \& B2\{\text{ok} : B2?(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}).$$
$$B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end}$$
$$\text{quit} : \mathbf{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); B2!(\text{int}); \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); B1?(\text{int}); \oplus S\{\text{ok} : S!(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \& B2\{\text{ok} : B2?(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$G_{2B} \triangleq B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}).$$
$$B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\text{end}$$
$$\text{quit} : \text{end}\}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); B2!(\text{int}); \text{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); B1?(\text{int}); \oplus S\{\text{ok} : S!(\text{str}); \text{end}, \text{quit} : \text{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \& B2\{\text{ok} : B2?(\text{str}); \text{end}, \text{quit} : \text{end}\}$$

Multiparty Session Types

Projection and Properties

Projecting 2-Buyer Protocol

$$\begin{aligned} G_{2B} \triangleq & B1 \rightarrow S : (\text{str}).S \rightarrow B1 : (\text{int}).S \rightarrow B2 : (\text{int}). \\ & B1 \rightarrow B2 : (\text{int}).B2 \rightarrow S : \{\text{ok} : B2 \rightarrow S : (\text{str}).\mathbf{end} \\ & \quad \text{quit} : \mathbf{end}\} \end{aligned}$$

$$T_{B1} \triangleq G_{2B} \upharpoonright B1 = S!(\text{str}); S?(\text{int}); B2!(\text{int}); \mathbf{end}$$

$$T_{B2} \triangleq G_{2B} \upharpoonright B2 = S?(\text{int}); B1?(\text{int}); \oplus S\{\text{ok} : S!(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

$$T_S \triangleq G_{2B} \upharpoonright S = B1?(\text{str}); B1!(\text{int}); B2!(\text{int}); \& B2\{\text{ok} : B2?(\text{str}); \mathbf{end}, \text{quit} : \mathbf{end}\}$$

Processes implementing a complete multiparty session projected from a global type are **deadlock-free**.

Conclusion

Summary:

- ▶ Explored a logical interpretation of session-based concurrency
- ▶ Explain concurrency theoretic concepts using logic, and vice-versa.
- ▶ Clean and elegant reasoning through logic.

Coming Up:

- ▶ More on Multiparty Session Types.
- ▶ Implementation and practical considerations.

Thank you!
Questions?

Session Types and Linear Logic and Lightweight Applications of Session Types in Java

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