Mean & stdev: particular cases

Sampling WITHOUT replacement

Here, n can be N:

$$\bar{x} = \frac{1+n}{2}$$

Case for $i = 1..N$ ($i.e.$, $i = 1..n$ and $n = N$):

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(i - \frac{n+1}{2}\right)^2 =$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\left[i^2 - i(n+1) + \left(\frac{n+1}{2}\right)^2\right]$$

$$(n-1)s^2 = \sum_{i=1}^{n} i^2 - (n+1)\sum_{i=1}^{n} i + n\left(\frac{n+1}{2}\right)^2$$

$$(n-1)s^2 = n\left(\frac{n^2}{3} + \frac{n}{2} + \frac{1}{6}\right) - (n+1)n\frac{1+n}{2} + n\left(\frac{n+1}{2}\right)^2 =$$

$$= n\left(\frac{n^2}{3} + \frac{n}{2} + \frac{1}{6}\right) + (n+1)^2\left(-\frac{n}{2} + \frac{n}{4}\right) =$$

$$= n\left(\frac{n^2}{3} + \frac{n}{2} + \frac{1}{6}\right) - \frac{n}{4}(n+1)^2$$

$$(n-1)s^2 = n^3\left(\frac{1}{3} - \frac{1}{4}\right) + n^2\left(\frac{1}{2} - \frac{1}{2}\right) + n\left(\frac{1}{6} - \frac{1}{4}\right) =$$

$$= \frac{1}{12}n^3 - \frac{1}{12}n$$

$$s^2 = \frac{1}{12}n(n+1)$$

$$s = \sqrt{\frac{N(N+1)}{12}}$$

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