Shewhart control charts

$$(\text{LCL, UCL})(W) = m_W \mp k\, s_W$$

$$k = 3$$

$$(\text{LCL, UCL})(W) = m_W \mp 3 s_W$$

Location

1. $\overline{X}$ charts

$$\boxed{W = \overline{X}}$$

$$m_{\overline{X}} = m \Leftrightarrow \overline{\overline{X}} \qquad\qquad s_{\overline{X}} = \frac{s}{\sqrt{n}}$$

$$(\text{LCL, UCL})(\overline{X}) = \overline{\overline{X}} \mp 3 s_{\overline{X}}$$

a) $\overline{X}$ & s

$$s = \frac{m_s}{c_4} \Leftrightarrow \frac{\overline{s}}{c_4} \qquad\qquad \begin{cases} \overline{s} \Leftrightarrow E(s) = c_4 s \neq s \\ E(s^2) = s^2 \end{cases}$$

$$c_4 = \frac{m_s}{s} = c_4(n) = \ldots$$

$$(\text{LCL, UCL})(\overline{X}) = \overline{\overline{X}} \mp 3 s_{\overline{X}} = \overline{\overline{X}} \mp 3\frac{s}{\sqrt{n}} = \overline{\overline{X}} \mp \frac{3}{\sqrt{n}}\frac{\overline{s}}{c_4}$$

$$A_3 = \frac{3}{c_4\sqrt{n}}$$

$$\boxed{(\text{LCL, UCL})(\overline{X}) = \overline{\overline{X}} \mp A_3 \overline{s}}$$

b) $\overline{X}$ & R

$$s = \frac{m_R}{d_2} \Leftrightarrow \frac{\overline{R}}{d_2}$$

$$d_2 = \frac{m_R}{s} = d_2(n) = \ldots$$

$$(\text{LCL, UCL})(\overline{X}) = \overline{\overline{X}} \mp 3 s_{\overline{X}} = \overline{\overline{X}} \mp 3\frac{s}{\sqrt{n}} = \overline{\overline{X}} \mp \frac{3}{\sqrt{n}}\frac{\overline{R}}{d_2}$$

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

$$\boxed{(\text{LCL, UCL})(\overline{X}) = \overline{\overline{X}} \mp A_2 \overline{R}}$$

Variability

2. s charts

$$\boxed{W = s}$$

$$m_s = c_4 s \qquad\qquad s_s = c_5 s$$

$$c_5 = \sqrt{1 - c_4^2}$$

$$(\text{LCL, UCL})(s) = (c_4 \mp 3c_5)s$$

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a) $\overline{X}$ & s

$$(\text{LCL, UCL})(s) = (c_4 \mp 3c_5)\frac{\overline{s}}{c_4} = \left(1 \mp 3\frac{c_5}{c_4}\right)\overline{s}$$

$$B_{3,4} = 1 \mp 3\frac{c_5}{c_4}$$

$$\boxed{(\text{LCL, UCL})(s) = B_{3,4}\overline{s}}$$

3. R charts

$$\boxed{W = R}$$

$$m_R = d_2 s \qquad\qquad s_R = d_3 s$$

$$(\text{LCL, UCL})(R) = (d_2 \mp 3d_3)s$$

a) $\overline{X}$ & R

$$(\text{LCL, UCL})(R) = (d_2 \mp 3d_3)s = (d_2 \mp 3d_3)\frac{\overline{R}}{d_2}$$

$$d_2 = \frac{m_R}{s} = d_2(n) = \ldots$$

$$d_3 = \frac{s_R}{s} = d_3(n) = \ldots$$

$$(\text{LCL, UCL})(R) = \left(1 \mp 3\frac{d_3}{d_2}\right)\overline{R}$$

$$D_{3,4} = 1 \mp 3\frac{d_3}{d_2}$$

$$\boxed{(\text{LCL, UCL})(R) = D_{3,4}\overline{R}}$$

Usual combinations

1.a) + **2.** $\overline{X}$ and s **(recommended)**
1.b) + **3.** $\overline{X}$ and R **(traditional)**