ZIONTS, Stanley, 2004, **"Linear and integer programming",** Prentice-Hall Internat'l. ISBN: 0-13-536763-8.

[98]). Instead of trying to survey them, we describe what seem to be the four types of structure that appear in these applications. Our structure types are not necessarily unique. They do appear to be collectively exhaustive, because all problems can be decomposed into structures of the four types. The structure types may not be mutually exclusive (some parts of a problem may be categorized as having one or more of them).

The four types of problem structure are as follows:

1. Resource allocation structure.
2. Blending structure.
3. Cutting stock structure.
4. Flow structure.

We shall now consider the four types of structures further by referring to simple problems containing each type.

Resource Allocation Structure

Resource allocation structure is a structure in which it is desirable to determine the most profitable way of allocating available resources (e.g., raw materials and/or processing facilities) to the production of different products. In this formulation the variables are the products and the constraints limit the resources available. The products have per unit profits associated with them. The dog food problem introduced in Chapter 1 is an example of a resource allocation problem. Most simple production planning problems can be viewed as resource allocation problems.

Blending Structure

In blending structure it is desired to determine the least cost blend having certain specified characteristics. Variables are input materials to be used in a specific product. Constraints specify certain maximum, minimum, or constrained characteristics of the blend desired. One of the earliest blending applications was the diet problem in which the lowest cost daily diet was desired, with limitations on vitamins, calories, protein, carbohydrates, and so on. Other applications have been to the blending of gasolines, the burdening of blast furnaces in iron and steel making, and the blending of animal feed-mix, fertilizer, and peanut butter. An example of a problem with blending structure follows.

EXAMPLE 11.1:

A candy shop has available various quantities of nuts, as follows:

100 pounds of peanuts
30 pounds of cashews
50 pounds of hazelnuts

Peanuts sell for $.80 a pound. The Bridge Club mix consists of at least 20 per cent cashews and not more than 50 per cent peanuts, and sells for $1.20 a pound. The Deluxe mix consists of at least 30 per cent cashews and not more than 30 per cent peanuts, and sells for $1.80 a pound. Cashews are sold for $2.20 a pound. The shop likes to have the blended mixes available in advance. Assuming they can sell all that is available, given the nut availability, how should they mix the nuts to maximize sales receipts?

The variables (measured in pounds) are as follows:

w_p peanuts to be sold as peanuts
x_c cashews to be sold as cashews
y_p peanuts to be mixed as Bridge Club mix
y_c cashews to be mixed as Bridge Club mix
y_h hazelnuts to be mixed as Bridge Club mix
z_p peanuts to be mixed as Deluxe mix
z_c cashews to be mixed as Deluxe mix
z_h hazelnuts to be mixed as Deluxe mix

The problem is then to

$$\text{Maximize } z = .8w_p + 1.2y_p + 1.8z_p + 2.2x_c + 1.2y_c + 1.8z_c + 1.2y_h + 1.8z_h$$

subject to:

$$w_p + y_p + z_p \le 100 \quad \text{Peanut availability}$$
$$x_c + y_c + z_c \le 30 \quad \text{Cashew availability}$$
$$y_h + z_h \le 50 \quad \text{Hazel nut availability}$$
$$.5y_p - .5y_c - .5y_h \le 0 \quad \text{upper limit on peanuts in Bridge Mix}$$
$$.2y_p - .8y_c + .2y_h \le 0 \quad \text{lower limit on cashews in Bridge Mix}$$
$$.7z_p - .3z_c - .3z_h \le 0 \quad \text{upper limit on peanuts in Deluxe Mix}$$
$$.3z_p - .7z_c + .3z_h \le 0 \quad \text{lower limit on cashews in Deluxe Mix}$$
$$w_p, x_c, y_p, y_c, y_h, z_p, z_c, z_h \ge 0$$

To illustrate the development of the limit constraints, consider the lower limit on cashews in the Bridge Mix, of which cashews must make up at least 20 per cent. This yields the following constraint, which is equivalent to that given above.

$$y_c \ge .2(y_p + y_c + y_h)$$

Strictly speaking, the last four constraints are blending structure constraints, whereas the first three constraints may be viewed as resource allocation (or, alternatively, flow) constraints.

Cutting Stock Structure

A structure somewhere between resource allocation and blending structure is the cutting stock structure. A problem that contains the cutting stock structure is the cutting stock problem, which occurs in various forms in such industries as paper, glass, steel, and aluminum. The problem has many variations. One example is the combining of orders for rolls of paper ordered in different widths. The rolls are to be cut in an optimal manner from the standard widths available. For this problem the variables are the units of produced widths to be cut in a particular way (e.g., a 20 foot width cut into two 8 foot widths and a 3 foot width, with one foot wasted) and constraints that specify the minimum number of each ordered size which must be produced. Costs include the cost of the paper and the cost of making the cuts. An example of a cutting stock problem follows.

EXAMPLE 11.2:

A lumber yard stocks 2″ by 4″ beams in 3 lengths: 8 feet, 14 feet, and 16 feet. The beams are sold by the foot and no charge is made for cuts. The yard has an order for the following lengths:

80	12 foot lengths
60	10 foot lengths
200	8 foot lengths
100	4 foot lengths

The cost of the 2 by 4's to the lumber yard is \$.30 per 8 foot length, \$.60 per 14 foot length, and \$.70 per 16 foot length. Cutting costs can be assumed to be zero. Assuming that the lumber yard has enough of each of the three lengths in stock, what is the minimum cost method of filling the oruer? Let

x_1 be the number of 8 foot lengths sold uncut
x_2 be the number of 8 foot lengths cut into two 4 foot lengths
y_1 be the number of 14 foot lengths cut into 10 foot and 4 foot lengths
y_2 be the number of 14 foot lengths cut into 12 foot lengths
y_3 be the number of 14 foot lengths cut into 10 foot lengths
w_1 be the number of 16 foot lengths cut into 12 foot and 4 foot lengths

Other possible cutting combinations, such as cutting a 16 foot length into two 8 foot lengths, can be shown to be unprofitable. The linear programming formulation is then

$$\text{Minimize } z = 30x_1 + 30x_2 + 60y_1 + 60y_2 + 60y_3 + 70w_1$$

$$\begin{array}{lrl}
\text{subject to:} & 2x_2 + y_1 + w_1 & \geq 100 \text{ (4 ft. lengths)} \\
& x_1 & \geq 200 \text{ (8 ft. lengths)} \\
& y_1 + y_3 & \geq 60 \text{ (10 ft. lengths)} \\
& y_2 + w_1 & \geq 80 \text{ (12 ft. lengths)} \\
& x_1, x_2, y_1, y_2, y_3, w_1 & \geq 0
\end{array}$$

This problem is, strictly speaking, an integer programming problem because the values of the variables must be integral. In this particular case the solution to the linear programming problem turns out to be integer, fortunately.

Flow Structure

Flow structure appears in most problems, and, of course, in network problems. Flow problems which lead to special methods have been discussed extensively in Chapter 9. These problems generally involve finding the maximal value of flow through a network. Applications of network flows have been made in scheduling transportation systems (e.g., airline operations), as well as flows of goods over time, as in inventory models. Examples of flow structure are given in Chapter 9.

As mentioned above, many problems require more than one type of structure. An example is a multiperiod production and inventory problem, in which each period might consist of a resource allocation or blending problem (possibly both), and the periods would be linked together by forward flows through time.

11.3 FORMULATING THE PROBLEM

In formulating a problem, it must be remembered that the problem solution is primary, and the solution technique secondary. Moreover, *linear programming is not a panacea*. It is a mathematical tool which either does or does not fit or approximate a situation. An example of an ordinary tool is a hammer. A hammer, a versatile tool, can be used to hammer nails. But it can also be