

Simple Linear Regression Example 1:

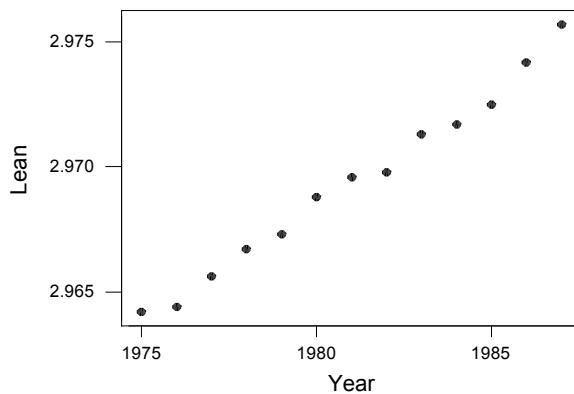
Leaning Tower of Pisa –

The Leaning Tower of Pisa is an architectural wonder. Engineers concerned about the tower's stability have done extensive studies of its increasing tilt. Measurements of the lean of the tower over time provide much useful information. The following table gives measurements for the years 1975 to 1987. The variable "lean" represents the difference between where a point on the tower would be if the tower were straight and where it actually is. The lean is measured in meters.

| | | | | | | | |
|------|--------|--------|--------|--------|--------|--------|--------|
| Year | 1975 | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 |
| Lean | 2.9642 | 2.9644 | 2.9656 | 2.9667 | 2.9673 | 2.9688 | 2.9696 |

| | | | | | | |
|------|--------|--------|--------|--------|--------|--------|
| Year | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 |
| Lean | 2.9698 | 2.9713 | 2.9717 | 2.9725 | 2.9742 | 2.9757 |

Scatterplot of Leaning Tower of Pisa Data



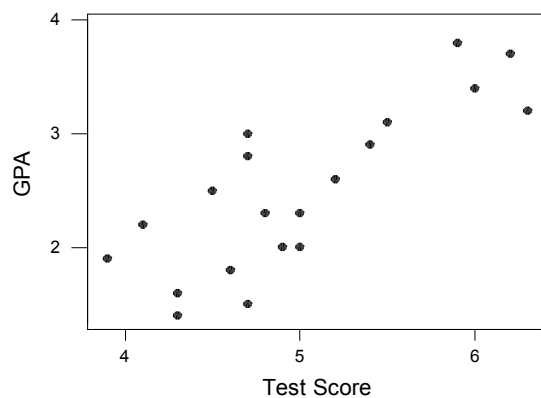
Simple Linear Regression Example 2:

GPA (Tests and Confidence Intervals on the Intercept and Slope Parameters) –

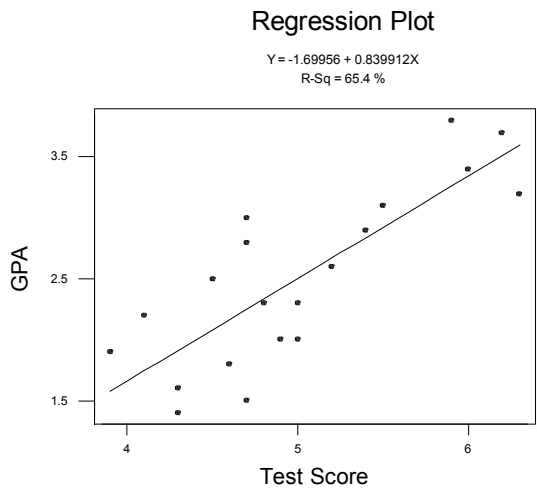
The director of admissions of a small college administered a newly designed entrance test to 20 students selected at random from the new freshman class in a study to determine whether a student's grade point average (GPA) at the end of the freshman year (Y) can be predicted from the entrance test score (X). The results of the study follow.

| Test Score | GPA |
|------------|------|
| 5.50 | 3.10 |
| 4.80 | 2.30 |
| 4.70 | 3.00 |
| 3.90 | 1.90 |
| 4.50 | 2.50 |
| 6.20 | 3.70 |
| 6.00 | 3.40 |
| 5.20 | 2.60 |
| 4.70 | 2.80 |
| 4.30 | 1.60 |
| 4.90 | 2.00 |
| 5.40 | 2.90 |
| 5.00 | 2.30 |
| 6.30 | 3.20 |
| 4.60 | 1.80 |
| 4.30 | 1.40 |
| 5.00 | 2.00 |
| 5.90 | 3.80 |
| 4.10 | 2.20 |
| 4.70 | 1.50 |

Scatterplot of GPA vs. Test Score



The regression equation is
 $GPA = - 1.70 + 0.840 \text{ Test Score}$



Simple Linear Regression Example 2:

GPA (Tests and Confidence Intervals on the Mean Response and New Responses) –

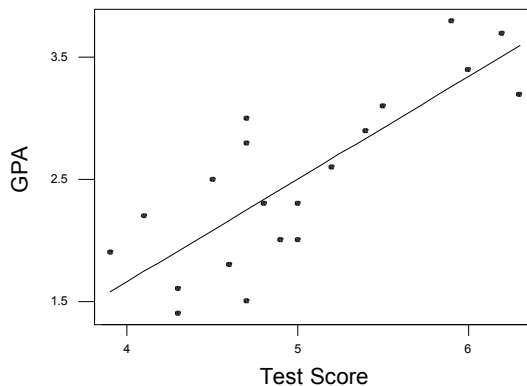
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| 6.20 | 3.70 |
| 6.00 | 3.40 |
| 5.20 | 2.60 |
| 4.70 | 2.80 |
| 4.30 | 1.60 |
| 4.90 | 2.00 |
| 5.40 | 2.90 |
| 5.00 | 2.30 |
| 6.30 | 3.20 |
| 4.60 | 1.80 |
| 4.30 | 1.40 |
| 5.00 | 2.00 |
| 5.90 | 3.80 |
| 4.10 | 2.20 |
| 4.70 | 1.50 |

Regression Plot

$$Y = -1.69956 + 0.839912X$$

R-Sq = 65.4 %



Regression Analysis

The regression equation is
 $GPA = -1.70 + 0.840 \text{ Test Score}$

| Predictor | Coef | StDev | T | P |
|-----------|---------|--------|-------|-------|
| Constant | -1.6996 | 0.7268 | -2.34 | 0.031 |
| Test Sco | 0.8399 | 0.1440 | 5.83 | 0.000 |

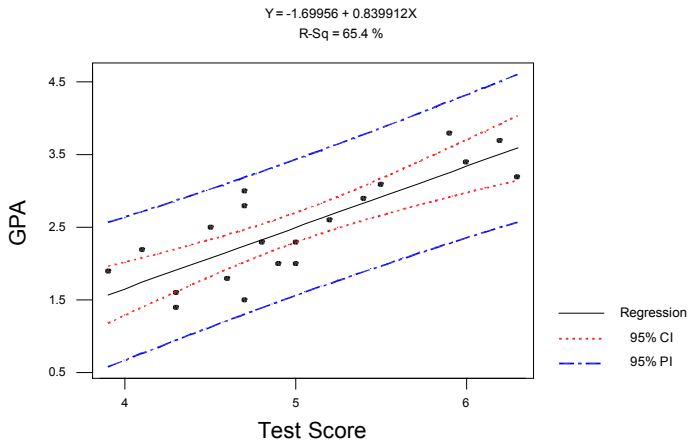
S = 0.4350 R-Sq = 65.4% R-Sq(adj) = 63.5%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 1 | 6.4337 | 6.4337 | 34.00 | 0.000 |
| Residual Error | 18 | 3.4063 | 0.1892 | | |
| Total | 19 | 9.8400 | | | |

| Obs | Test Sco | GPA | Fit | StDev Fit | Residual | St Resid |
|-----|----------|--------|--------|-----------|----------|----------|
| 1 | 5.50 | 3.1000 | 2.9200 | 0.1210 | 0.1800 | 0.43 |
| 2 | 4.80 | 2.3000 | 2.3320 | 0.1014 | -0.0320 | -0.08 |
| 3 | 4.70 | 3.0000 | 2.2480 | 0.1064 | 0.7520 | 1.78 |
| 4 | 3.90 | 1.9000 | 1.5761 | 0.1859 | 0.3239 | 0.82 |
| 5 | 4.50 | 2.5000 | 2.0800 | 0.1210 | 0.4200 | 1.01 |
| 6 | 6.20 | 3.7000 | 3.5079 | 0.1983 | 0.1921 | 0.50 |
| 7 | 6.00 | 3.4000 | 3.3399 | 0.1738 | 0.0601 | 0.15 |
| 8 | 5.20 | 2.6000 | 2.6680 | 0.1014 | -0.0680 | -0.16 |
| 9 | 4.70 | 2.8000 | 2.2480 | 0.1064 | 0.5520 | 1.31 |
| 10 | 4.30 | 1.6000 | 1.9121 | 0.1401 | -0.3121 | -0.76 |
| 11 | 4.90 | 2.0000 | 2.4160 | 0.0983 | -0.4160 | -0.98 |
| 12 | 5.40 | 2.9000 | 2.8360 | 0.1131 | 0.0640 | 0.15 |
| 13 | 5.00 | 2.3000 | 2.5000 | 0.0973 | -0.2000 | -0.47 |
| 14 | 6.30 | 3.2000 | 3.5919 | 0.2110 | -0.3919 | -1.03 |
| 15 | 4.60 | 1.8000 | 2.1640 | 0.1131 | -0.3640 | -0.87 |
| 16 | 4.30 | 1.4000 | 1.9121 | 0.1401 | -0.5121 | -1.24 |
| 17 | 5.00 | 2.0000 | 2.5000 | 0.0973 | -0.5000 | -1.18 |
| 18 | 5.90 | 3.8000 | 3.2559 | 0.1621 | 0.5441 | 1.35 |
| 19 | 4.10 | 2.2000 | 1.7441 | 0.1621 | 0.4559 | 1.13 |
| 20 | 4.70 | 1.5000 | 2.2480 | 0.1064 | -0.7480 | -1.77 |

GPA Regression Plot with CI and PI Bands



Predicted Values

| Fit | StDev Fit | 95.0% CI | 95.0% PI |
|--------|-----------|-------------------|-------------------|
| 2.9200 | 0.1210 | (2.6657, 3.1742) | (1.9713, 3.8686) |
| 2.3320 | 0.1014 | (2.1189, 2.5452) | (1.3936, 3.2705) |
| 2.2480 | 0.1064 | (2.0244, 2.4716) | (1.3071, 3.1889) |
| 1.5761 | 0.1859 | (1.1855, 1.9667) | (0.5822, 2.5700) |
| 2.0800 | 0.1210 | (1.8258, 2.3343) | (1.1314, 3.0287) |
| 3.5079 | 0.1983 | (3.0912, 3.9246) | (2.5034, 4.5123) |
| 3.3399 | 0.1738 | (2.9747, 3.7051) | (2.3557, 4.3241) |
| 2.6680 | 0.1014 | (2.4548, 2.8811) | (1.7295, 3.6064) |
| 2.2480 | 0.1064 | (2.0244, 2.4716) | (1.3071, 3.1889) |
| 1.9121 | 0.1401 | (1.6177, 2.2064) | (0.9519, 2.8722) |
| 2.4160 | 0.0983 | (2.2094, 2.6226) | (1.4790, 3.3530) |
| 2.8360 | 0.1131 | (2.5984, 3.0735) | (1.8917, 3.7803) |
| 2.5000 | 0.0973 | (2.2956, 2.7044) | (1.5635, 3.4365) |
| 3.5919 | 0.2110 | (3.1486, 4.0352) | (2.5761, 4.6077) |
| 2.1640 | 0.1131 | (1.9265, 2.4016) | (1.2197, 3.1083) |
| 1.9121 | 0.1401 | (1.6177, 2.2064) | (0.9519, 2.8722) |
| 2.5000 | 0.0973 | (2.2956, 2.7044) | (1.5635, 3.4365) |
| 3.2559 | 0.1621 | (2.9154, 3.5964) | (2.2806, 4.2312) |
| 1.7441 | 0.1621 | (1.4036, 2.0846) | (0.7688, 2.7194) |
| 2.2480 | 0.1064 | (2.0244, 2.4716) | (1.3071, 3.1889) |