Parallel Branch and Bound to solve Mixed Integer Linear Programs

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$$maximize \sum_{1 \leq j \leq n} c_j x_j \tag{1}$$

$$subject\ to \sum_{1 \leq j \leq n} a_{ij} x_j \leq b_j \ for\ i = 1, 2, ..., m \tag{2}$$

$$x_j \geq 0 \ for\ j = 1, 2, ..., n \tag{3}$$

General form of a Liner Program.

A simplex is solved using pivot points to adjust the coefficients of the optimization function and constraints.

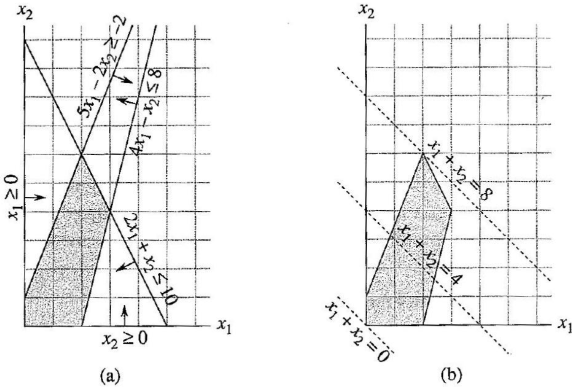


Figure: Geometric view of a simplex.

Branch and Bound

Solving at each node

Consider the following:

maximize:

$$f = 4x_1 + 11x_2$$

subject to:

$$2x_1 - x_2 + x_3 = 4 \quad (4)$$

$$2x_1 + 5x_2 + x_4 = 16 \quad (5)$$

$$-x_1 + 2x_3 + x_5 = 4 \quad (6)$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \quad (7)$$

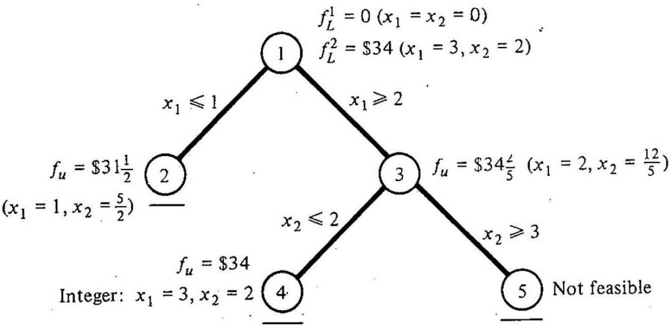
After the first application of the simplex solver to the problem we get:

$$f = 34\frac{2}{3}, x_1 = \frac{4}{3}, x_2 = \frac{8}{3}$$

To bound the search to the integers, we conduct two sub-searches, one where $x_1 \leq 1$, and the other where $x_1 \geq 2$.

These two searches cover the entire valid solution region, but are not necessarily symmetric.

The search would look like like:



In a single threaded program, each branch is searched in sequence:

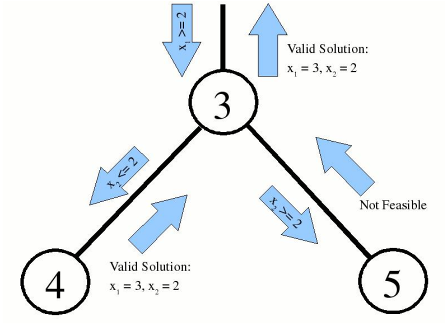


Figure: View of a node in the ILP search space.

Parallel Search
A node in the tree

In a multi threaded program, two branches can be search simultaneously:

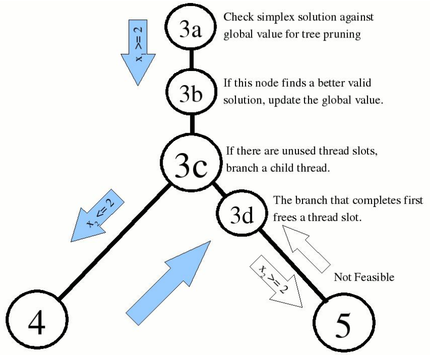
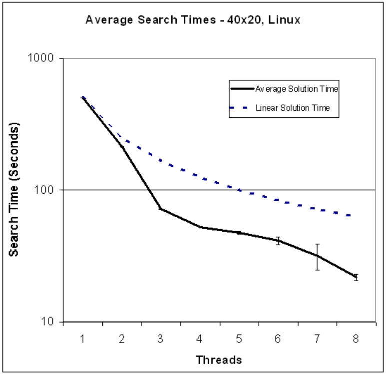


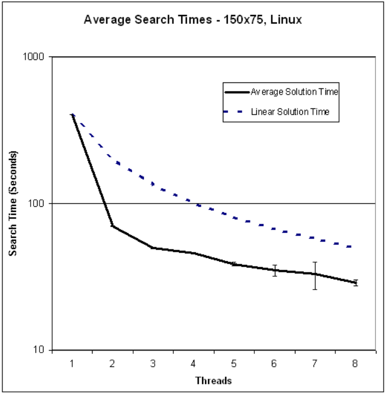
Figure: Branching and joining of parallel searches.

The following times were observed from a problem with 40 variables and 20 constraints, running on an 8-core Linux machine:



Results
150 variable problem

The following times were observed from a problem with 150 variables and 75 constraints, running on an 8-core Linux machine:



References

1. Cormen, Thomas. "Introduction to Algorithms." MIT Press. 2001.
2. Lau, Hang T. "A Java Library of Graph Algorithms and Optimization." Taylor & Francis Group. 2007.
3. Miltersen, Peter. "MILP, ILP, and TSP." Search and Optimization. Version 1.1. Spring 2004.
4. Mumey, Brendan, Tang, Jian, and Hahn, Timothy. "Joint Stream Control and Scheduling in Multihop Wireless Networks with MIMO Links." IEEE ICC 2008(in submission).
5. Press, William H. "Numerical Recipes in C: The Art of Scientific Computing." Cambridge University. 1992.
6. Wu, Nesa and Coppins, Richard. "Linear Programming and Extensions." McGraw Hill. 1981
7. Zionts, Stanley. "Linear and Integer Programming." Prentice-Hall. 1974.