CHAPTER 3

MORE FOR LESS: PURPOSIVE CONTRADICTION, STANDARD UNITS AND MONETARY UNITS

1. Introduction

In Chapter 1 I introduced more for less principles and processes and applications, including applic-ations to potential gains from exchange between individuals. In Chapter 2 I developed general-isations of the Kuhn Tucker conditions for more for less cases in general and for cases in which there are potential gains from exchange between individuals in particular. In Section 2 of this chapter I develop the two person exchange related case and associated roles for precision of measurement and for standard units and in Section 3 I specialize more for less results to Lancaster-like characteristics cases. (See Lancaster 1966, Ryan 1992). In the remaining sections I focus on *processes* of exchange between persons under conditions of uncertainty and of incomplete information. In those contexts I concentrate on a role for standard units in general and for monetary units in particular as providers of precise measures of weight, volume and value for individuals who, due to ignorance and/or for reasons of bargaining advantage, might otherwise disagree on each of these facets of a commodity.

2. Background

Standard approaches to microeconomic theory assume that two or more individuals are each initially endowed with a quantity of one or more commodities and that each has a preference relation defined over any combination of the commodities which may be attainable, either directly, or by processes of exchange between these individuals. It is then shown, for example by means of an Edgeworth Box (Henderson and Quandt, 1958, p. 204, Hirschleifer, 1988, p.387), that, in general, conditions of trade will be Pareto preferable to the status quo. Assumptions leading to conclusions of this kind include those of standard competitive equilibrium analyses stemming from the work of Walras (1954), Arrow and Debreu (1954) and Debreu (1959), (see also Vilks, 1992). These approaches assume optimizing behaviours and emphasise not just gain, but determinacy and self consistency: individuals are assumed to seek to gain in a manner consistent with their prior preferences, even though these are initially defined relative to a relatively abstract nontrading self.

Two crucial observations:

- **Self contradiction.** In such processes of exchange, if an individual chooses to initiate exchange or trade then that individual chooses to initiate a process potentially depriving himself or herself of some or all of their initial endowments of commodities. It follows that, if such initial endowments are "goods" in the sense that less would not be more preferred (and more would not be less preferred) then, with reference to such commodities, such behaviour is apparently not only strictly inconsistent with, but as if purposively strictly contradictory to, that individual's relatively abstract preferences.

- **Incomplete information**. If an individual init-iates a process of exchange, that individual apparently acts as if purposively to reveal a preference for elements of one or more commodities initially relatively more completely known to another - and less completely known to self (in the sense of possessed by another) - in exchange for elements of one or more commodities initially relatively more completely known to self - and less completely known relative to another (because initially possessed by self).

The first observation predicts processes of as if purposive self contradiction for individuals when engaged in processes of exchange since such processes necessarily contradict individuals' initially relatively abstract preference relations by requiring that those individuals (empirically) purposively act as if to offer themselves less relative to self, albeit as if thereby to potentiate more relative to another/others. [Notice interaction in two related senses here - between abstract and empirical and between more and less. This relates directly to relative indeterminacy issues implicit in the second observation.]

The second observation emphasizes the inevitability of self indeterminacy and self incompleteness for contexts of exchange between individuals in the sense that individuals engaged in exchange necessarily act as if purposively less physically determined with reference to their own initial endowments in order, inter alia, to become more physically determined relative to elements of otherwise less completely known endowments of another.

I have pursued these issues from other perspectives in Ryan, 1992. In this chapter I concentrate on implications of exchange related principles and processes of purposive individual and collective contradiction which can provide hitherto unexplored motivations for creating and using standard units in general and standardized monetary units in particular. [A graphic physical example of principles of as if purposive self contradiction is implicit in the well known adage: "The longest journey starts with a single step". Reflection will convince you that that "step" would necessarily be relatively backwards if the resulting motion were to be relatively forwards. Conversely if the initial motion were relatively forwards, the resulting reaction would be relatively backwards. I have argued elsewhere that economists using standard arguments have neglected this apparently simple point with, I think, ultimately profound conse-quences, e.g. for economic implications of arguments employed in tatonnement approaches to proofs of general equilibrium.]

3. Standard units, monetary units and precision

If each agent in an economic system has a different perception of value relative to self and relative to another or others and, if exploitation of that fact to mutual advantage is the purpose of potentially mutually advantageous processes of exchange, then there is a role for standard units in general, and for standard monetary units in particular. This role follows from the recognition that principles and processes of relative indeterminacy and incomplete-ness of measurement may be not only mathematic-ally and physically inevitable, but, in an economic context, where differences of measurement may be the basis of mutually advantageous gains, they may be individually and collectively *desirable*.

Physicists recognize that empirical measurement of physical magnitudes with absolute precision is ultimately inevitably impossible. They recognize a fortiori that to ensure that two different physicists differently located in space-time could measure the same physical magnitude in precisely the same way is ultimately impossible. These reasons alone imply inevitable shortcomings, due to the inevitability of measurement errors, for any kind of empirically based physical research.

But in economic contexts potential differences of measurement can correspond to measures of opportunity for individuals to *gain* from such differences by processes generating gifts, barters or trade. In such cases, not only do individuals inevitably measure differently from each other, they and others may *prefer* to do so.

Here is a role for standardized units, including standardized monetary units, as potentially providing means of agreement on system related values where otherwise, because trades are predicated on *differences* of values between individuals, there could be no agreement on values.

After a review of exchange related more for less results in Section 4, this point is

developed with reference to standardized monetary units, and in Section 6 with added emphasis on the potential significance of the fact that standard units of currency are not only self axiomatizing, but integer valued.

4. More for less principles and gains from exchange.

A general more for less (more for nothing) result:

THEOREM 1 *More for Less (Nothing)*

If: i) $z^s_{rjs}{}^*(t)$, $z^s_{sjr}{}^*(t)$, $U_s(C_{sk}{}^*(t))$ are feasible solutions to (Ia) given constraints (I); ii) $h_{rjs}{}^-(t)<M$, $\Sigma h_{rjs}{}^+(t)<M$, $h^s_{sjr}{}^-(t)<M$, $h^s_{sjr}{}^-(t)<M$ at least one $h^s_{rjs}{}^-(t)$, $h_{rjs}{}^+(t)$, $h^s_{sjr}{}^+(t)$, $h^s_{sjr}{}^-(t)$ in (I), then:

Max $U_s(C_{sk}(t))-\Sigma M z_{rjs}{}^+(t)-\Sigma M z_{rjs}{}^-(t)$ = $V_{1a}\leq V_{1b}$ =Max $U_s(C_{sk}(t))-\Sigma h^s_{rjs}{}^+(t)z^s_{rjs}{}^+(t)-\Sigma h^s_{rjs}{}^-(t)z^s_{rjs}{}^-(t)$

$-M z^s_{sjr}{}^+(t)-\Sigma M z^s_{sjr}{}^-(t) -g_{sk}{}^-(t)U_s(C_{sk}{}^-(t))$ **(Ia)**　　$-\Sigma h^s_{sjr}{}^+(t)z^s_{sjr}{}^+(t)-\Sigma h^s_{sjr}{}^-(t)z^s_{sjr}{}^-(t) -g_{sk}{}^-(t)U_s(C_{sk}{}^-(t))$ **(Ib)**

　　st　　　constraints (I)　　　　　　　　　　　st　　　constraints (I)

where constraints (I) are as follows:

$\varphi_{sk}(t)$　　　　st $C_{sk}(t)=\Sigma a_{sjk}y_{sj}(t) + \Sigma b_{sjk}x_{sj}(t) + \Sigma e^s_{rjsk}(t)z^s_{rjs}(t) - \Sigma f^s_{sjrk}(t)z^s_{sjr}(t)$　　j,k=1,2

$\omega_{sj}(t)$　　　　　　　$y_{sj}(t) + x_{sj}(t) + z^s_{sjr}(t) = x_{sj}(t-1) + z^s_{rjs}(t)$

$\psi^s_{rjs}(t)$　　　　　　$z^s_{rjs}(t) + z^s_{rjs}{}^+(t) - z^s_{rjs}{}^-(t) = z^r_{rjs}{}^*(t)$　　　　jεJ$_{1sr}$　　　　　(I)

$\psi^s_{sjr}(t)$　　　　　　$z^s_{sjr}(t) + z^s_{sjr}{}^+(t) - z^s_{sjr}{}^-(t) = z^r_{sjr}{}^*(t)$　　　　jεJ$_{2sr}$

$\lambda_s(t)$　　　　　　　$U_s(C_{sk}(t)) -U_s(C_{sk}{}^-(t))\geq U_s(C_{sk}{}^*(t))$

　　　　　　　　　　　　All variables nonnegative

PROOF

For M sufficiently large relative to $h^s_{sjr}{}^+(t)$, $h^s_{sjr}{}^-(t)$, any optimal solution to (Ia) will require $z^s_{rjs}{}^+(t) =z^s_{rjs}{}^-(t)=0$. But, while any such optimal solution to (Ia) is a feasible solution to (Ib), it is not necessarily an optimal solution to (Ib). That is, there may exist cases in which the strict inequality applies. **QED**

In (Ia),(Ib) $U_s(C_{sk}(t))$ is a Lancaster-like preference relation (see Lancaster 1966) for an individual s defined over characteristics k, where these charac-teristics stem from consumption $y_{sj}(t)$, from retained endowments $x_{sj}(t)$, and from potential transfers $z^s_{rjs}(t)$ from and $z^s_{sjr}(t)$ to another individual r. Quantities $x_{sj}(t-1)$ and $z^s_{rjs}(t)$ are initial endowments to s and transfers from individual r. Finally $z^r_{rjs}{}^*(t)$, $z^r_{sjr}{}^*(t)$ are elements, either of an initial state, or of an exchange proposed by a second individual and $U_s(C_{sk}{}^*(t))$ is a reference preference where if $g_{sk}{}^-(t)$ is sufficiently large, it can exclude imposition (or self imposition)

of states less preferred than the reference level $U_s(C_{sk}{}^*(t))$.

It follows from Theorem 1 that individual s may prefer a gift related solution with $z^s_{rjs}(t)>0$, or a barter related solution with $z^s_{rjs}(t)>0$, $z^s_{rjs}(t)>0$, as if via (Ib) to an initial state as represented by $z^s_{rjs}{}^+(t)=z^s_{rjs}{}^-(t)=0$ and $z^s_{sjr}{}^+(t)=z^s_{sjr}{}^-(t)=0$ as if via (Ia). Further, if $U_s(C_{sk}(t))$ is concave then, associating the indicated variables with its constraints, the Kuhn Tucker conditions are necessary and sufficient for an optimum to (Ib) as follows:

$C_{sk}(t)$　　　　　　　　　$(1-\lambda_s(t)) \delta U_s/\delta C_{sk}(t) \leq\varphi_{sk}(t)$

$y_{sj}(t)$　　　　　　　　　$\omega_{sj}(t) \geq \Sigma a_{sjk}\varphi_{sk}(t)$　　　　k=1,2

$x_{sj}(t)$　　　　　　　　　$\omega_{sj}(t) \geq \Sigma b_{sjk}\varphi_{sk}(t)$　　　　　　　　(I)'

$z^s_{rjs}(t)$　　　　　　　　$\omega_{sj}(t) \leq \Sigma e^s_{rjsk}\varphi_{sk}(t) + \psi^s_{rjs}(t)$

$z^s_{sjr}(t)$　　　　　　　　$\omega_{sj}(t) \geq \Sigma f^s_{sjrk}\varphi_{sk}(t) - \psi^s_{sjr}(t)$

$z^s_{rjs}{}^+(t),z^s_{rjs}{}^-(t)$　　　　$-h^s_{rjs}{}^+(t)\leq\psi^s_{rjs}(t)\leq h^s_{rjs}{}^-(t)$

$z^s_{sjr}{}^+(t),z^s_{sjr}{}^-(t)$　　　　$-h^s_{sjr}{}^+(t)\leq\psi^s_{sjr}(t)\leq h^s_{sjr}{}^-(t)$

$U(C_{sk}{}^-(t))$　　　　　　　　$-g_{sk}{}^-(t) \leq\lambda_s(t)$

If $C_{sk}(t), y_{sj}(t), x_{sj}(t), z^s_{rjs}(t)$ are optimally positive in (I) the corresponding condition in (I)' holds as a strict equality. Conversely, by complementary slackness, if any of conditions (I)' holds as a strict inequality the associated variable is optimally zero. In this way conditions (I)' interrelate potentials $\delta U_s/\delta C_{sk}(t), \omega_{sj}(t)$ relative to self and $\varphi_{sk}(t), \psi^s_{rjs}(t), \psi^s_{sjr}(t)$ relative to the system.

At an optimum conditions (I)' respectively yield optimal decision rules as follows:

Select characteristic $C_{sk}(t)$, if at all, then only to the point where the net marginal evaluation relative to self $(1-\lambda_s(t))\delta U_s/\delta C_{sk}(t)$ equates to the marginal evaluation, $\varphi_{sk}(t)$, relative to the system;

Select $y_{sj}(t), x_{sj}(t)$ if at all, then only to the point where the aggregated marginal contribution $\Sigma a_{sjk}\varphi_{sk}(t)$ (resp $\Sigma b_{sjk}\varphi_{sk}(t)$) from characteristics k to individual s equates to the marginal contribution, $\omega_{sj}(t)$, from commodity j relative to individual s,

Accept quantities $z^s_{rjs}(t)$ of commodity j from individuals s, if at all, then to the point where the sum $\Sigma e_{rjsk}\varphi_{sk}(t)+\psi^s_{rjs}(t)$ equates to the marginal contribution $\omega_{sj}(t)$ from commodity j relative to individual s.

Transmit quantities $z^s_{sjr}(t)$ of commodity j to individuals r, if at all, then to the point where the difference $\omega_{sj}(t)-\Sigma f_{sjrk}\varphi_{sk}(t)$ equates to the marginal opportunity cost $-\psi^s_{sjr}(t)$ from commodity j relative to individual s.

Acquisition potentials The sixth conditions of (I)' relate relatively internal, i.e. subjective and acquisition related, potentials $\psi^s_{rjs}(t)$ for commodities j to relatively external potentials $h^s_{rjs}{}^+(t), h^s_{rjs}{}^-(t)$ associated with deviations from target levels of acquisitions of commodity j.

Relinquishment potentials The seventh conditions of (I)' relate relatively internal, i.e. subjective and acquisition related, opportunity costs $-\psi^s_{sjr}(t)$ for commodities j to relatively external potentials $h^s_{sjr}{}^+(t), h^s_{sjr}{}^-(t)$ associated with deviations from target levels of relinquishments of commodity j.

Improvement? The final condition of (I)' signals whether or not an optimum to (I) attains an improvement relative to a reference preference measure $U_s(C_{sk}*(t))$. (If $U(C_{sk}{}^-(t))>0$ then $U(C_{sk}(t))< U_s(C_{sk}*(t)$ and the preference related measure $\lambda_s(t)$ becomes negative via. $-g_{sk}{}^-(t)=\lambda_s(t)$.)

Even in gift and barter related contexts the system related potentials $\varphi_{sk}(t), \psi^s_{rjs}(t), \psi^s_{sjr}(t)$ associated with the constraints of (I) would have informational value to another, or others. The significant point here is that $\psi^s_{rjs}(t), \psi^s_{sjr}(t)$ suggest a price related extension of (I) in which a second individual r, given price related potentials by an initial individual s, may set out to solve an explicitly budget constrained optimization problem.

In more detail: If a second individual r distinct from an initial individual s starts with the offer of a gift to individual s and, if that offer is accepted, both individuals thereby reveal themselves as at least as well off relative to themselves as they would have been in the absence of that offer. Further, if the offer of that gift is refused, then individual s reveals themselves as better off without the gift and, since individual r still has those commodities, if more is preferred for individual r, then individual r, too, is potentially better off relative to a reference preference evaluation $U_r(C_{rk}*(t))$ which would have obtained had the commodities been gifted.

Next: if individual s *conditionally* accepts a gift from individual r and reciprocates with a barter offer $z^s_{rjs}(t), z^s_{sjr}(t)$, as in (I), and if that offer is accepted by individual r, then both individuals again reveal themselves as at least as well off as they would have been relative to themselves, in this case in the absence of the barter offer.

If that same barter offer is *rejected* then individual r reveals themselves as better off by refusing and individual s, even though possibly worse off relative to a reference preference, say $U_s(C_{sk}**(t))$, contingent on acceptance of that barter offer by individual r, would generally be at least as well off as in the initial (reference) position $U_s(C_{sk}*(t))$, in the

absence of the initial gift. That is: individual r would be in a position to reach a state associated with a preference level $U_s(C_{sk}(t))$ and such that $U_s(C_{sk}**(t))\geq U_s(C_{sk}(t)) \geq U_s(C_{sk}*(t))$.

If, under the above conditions and in the absence of external duress, a barter offer from s to r was *accepted*, a state mutually (Pareto) preferred to that in the absence of the offer would become attainable. If that acceptance were unconditional then the barter related process would stop. Alternatively a barter process may continue with individual r reciprocating with a barter offer to s of a potentially Pareto preferred state in a manner wholly analogous to that from s to r above, and so on, until one or other of these individuals stops the process with

unconditional acceptance or rejection of an offer from the other. (For further development of gift and barter related cases see Chapter 4.)

5. Explicitly budget constrained processes

Rather than pursue gift and barter related cases further, now focus on an explicitly budget constrained process for individual r with reference to the system (II) in which $\psi^s_{rjs}(t-1)$, $\psi^s_{sjr}(t-1)$, which may (or may not) have been associated with a barter offer from individual s as in Section 4, take on roles as if perfect predictors of marginal values for individual r in relation to individual s as follows. (Compare the non budget constrained systems (Ia),(Ib),(I) for individual s.):

$$\text{Max } U_r(C_{rk}(t))-\Sigma h^r_{sjr}(t)z^r_{sjr}{}^+(t)-\Sigma h^r_{sjr}{}^-(t)z^r_{sjr}{}^-(t)-\Sigma h^r_{rjs}{}^+(t)z^r_{rjs}{}^+(t)-\Sigma h^r_{rjs}{}^-(t)z^r_{rjs}{}^-(t) -g_{rk}{}^-(t)U_r(C_{rk}{}^-(t))$$

$$\varphi_{rk}(t) \quad\quad \text{st } C_{rk}(t)=\Sigma a_{rjk}y_{rj}(t) + \Sigma b_{rjk}x_{rj}(t) + \Sigma e^r_{sjrk}(t)z^r_{sjr}(t) - \Sigma f^r_{rjsk}(t)z^r_{rjs}(t) \text{ j,k=1,2}$$

$$\omega_{rj}(t) \quad\quad y_{rj}(t) + x_{rj}(t) + z^r_{rjs}(t) = x_{rj}(t-1) + z^r_{sjr}(t)$$

$$\psi^r_{sjr}(t) \quad\quad z^r_{sjr}(t) + z^r_{sjr}{}^+(t) - z^r_{sjr}{}^-(t) = z^s_{sjr}{}^*(t) \quad\quad j\varepsilon J_{1rs} \quad\quad\quad \text{(II)}$$

$$\psi^r_{rjs}(t) \quad\quad z^r_{rjs}(t) + z^r_{rjs}{}^+(t) - z^r_{rjs}{}^-(t) = z^s_{rjs}{}^*(t) \quad\quad j\varepsilon J_{2rs}$$

$$\tau_r(t) \quad\quad \Sigma\psi^s_{rjs}(t-1)z^r_{rjs}(t)\geq\Sigma\psi^s_{sjr}(t-1)z^r_{sjr}(t)$$

$$\lambda_r(t) \quad\quad U_r(C_{rk}(t)) -U_r(C_{rk}{}^-(t))\geq U_r(C_{rk}*(t))$$

$$\text{All variables nonnegative}$$

In (II) $\psi^s_{rjs}(t-1),\psi^s_{sjr}(t-1)$ and $z^r_{rjs}(t),z^r_{sjr}(t)$ are under-stood as denominated in standard price units and quantity units respectively with $\psi^s_{sjr}(t-1)$ being a buying price and $\psi^s_{rjs}(t-1)$ a selling price relative to individual s and with $z^r_{sjr}(t)$, $z^r_{rjs}(t)$ being standard-izing quantities initially determined with reference only to individual r. If this offer to exchange these

quantities at these prices is accepted by individual s those prices and quantities become as if agreed - and in that sense standardized - with reference *both* to individual r and to individual s. Assuming that $U_r( )$ is concave, and associating the indicated dual variables with the constraints of (II) the associated Kuhn-Tucker conditions are:

$$C_{rk}(t) \quad\quad\quad (1-\lambda_r(t)) \,\delta U_r/\delta C_{rk}(t) \leq\varphi_{rk}(t)$$

$$y_{rj}(t) \quad\quad\quad \omega_{rj}(t) \geq \Sigma a_{rjk}\varphi_{rk}(t) \quad\quad k=1,2$$

$$x_{rj}(t) \quad\quad\quad \omega_{rj}(t) \geq \Sigma b_{rjk}\varphi_{rk}(t)$$

$$z^r_{sjr}(t) \quad\quad\quad \omega_{rj}(t) \geq \Sigma e^r_{rjk}\varphi_{rk}(t) + \psi^r_{sjr}(t) -\tau_r(t)\psi^s_{sjr}(t-1) \quad\quad\quad \text{(II)}'$$

$$z^r_{rjs}(t) \quad\quad\quad \omega_{rj}(t) \leq \Sigma f^r_{rjsk}\varphi_{rk}(t) - \psi^r_{rjs}(t) +\tau_r(t)\psi^s_{rjs}(t-1)$$

$$z^r_{sjr}{}^+(t),z^r_{sjr}{}^-(t) \quad\quad -h^r_{rsjr}{}^+(t)\leq\psi^r_{sjr}(t)\leq h^r_{sjr}{}^-(t)$$

$$z^r_{rjs}{}^+(t),z^r_{rjs}{}^-(t) \quad\quad -h^r_{rjs}{}^+(t)\leq\psi^r_{rjs}(t)\leq h^r_{rjs}{}^-(t)$$

$$U(C_{rk}{}^-(t)) \quad\quad\quad -g_{rk}{}^-(t) \leq\lambda_r(t)$$

Interpretations of conditions (II)' are wholly analogous to those for (I)' except that now additional budget constraint related terms appear in the fourth and fifth constraints of (II)' via a budget related multiplier $\tau_r(t)$. Since these latter conditions are central to the purposive contradiction and standard unit arguments that are the main subject of this chapter, I consider them in more detail. Consider the fourth, buying, constraints first. If $z^r_{sjr}(t)>0$ then, by complementary slackness:

$$\omega_{rj}(t)= \Sigma e^r_{rjk}\varphi_{rk}(t)+\psi^r_{sjr}(t)-\tau_r(t)\psi^s_{sjr}(t-1) \quad (5.1)$$

In (5.1) $\omega_{rj}(t)$ is a net evaluation of commodity j for individual r in period t relative to self. If $\omega_{rj}(t)$ is greater than the quantities on the right hand side of (5.1) then individual r will not offer to acquire this particular commodity at the margin. Conversely, if individual r does choose acquisition at the margin as if via an optimal solution to (II), then (5.1) applies and they will acquire that commodity up to the point where the marginal evaluation relative to self $\omega_{rj}(t)$ is equal to the effective marginal gain relative to self $\Sigma e_{rjk}\varphi_{rk}(t)$ net of any system related transfers $\psi^r_{sjr}(t)-\tau_r(t)\psi^s_{sjr}(t-1)$. In that way the marginal evaluation $\omega_{rj}(t)$ relates to the system related marginal opportunity cost $\psi^r_{rjs}(t)$ of consumption and/or of savings of that type foregone plus the effective transfer price $\tau_r(t)\psi^s_{sjr}(t-1)$ so, too, potentially equating $\omega_{rj}(t)$ to a net return as between acquired consumption or savings and relinquishment/exchange relative to others. Another way of interpreting (5.1) is that the difference $\omega_{rj}(t)-\Sigma e_{rjk}\varphi_{rk}(t)$ reflects a difference between the (subjective) evaluation of a commodity relative to a wider system and the (subjective) evaluation of that commodity relative to self at the margin.

Via the equality case (5.1) the fourth constraints of (II)' potentially yield three classes of special cases, each of which is potentially open to interpretation with reference to elements of transfers, exchanges or trades:

$z^r_{sjr}(t)>0 \Rightarrow \omega_{rj}(t)-\Sigma e^r_{rjk}\varphi_{rk}(t) <0$
$$\Leftrightarrow$$
$$\psi^r_{sjr}(t)>\tau_r(t)\psi^s_{sjr}(t-1) \quad (5.1a)$$

$z^r_{rjs}(t)>0 \Rightarrow \omega_{rj}(t)-\Sigma e^r_{rjk}\varphi_{rk}(t) >0$
$$\Leftrightarrow$$
$$\psi^r_{sjr}(t)<\tau_r(t)\psi^s_{sjr}(t-1) \quad (5.1b)$$

$z^r_{rjs}(t)>0 \Rightarrow \omega_{rj}(t)-\Sigma e^r_{rjk}\varphi_{rk}(t) =0$
$$\Leftrightarrow$$
$$\psi^r_{sjr}(t)=\tau_r(t)\psi^s_{sjr}(t-1) \quad (5.1c)$$

- Conditions (5.1a) are open to the interpretation that for individual r via $\omega_{sj}(t)-\Sigma e_{sjk}\varphi_{sk}(t) <0$ potentially an exchange related interaction relative to a system is preferred to ("consumption" related) interaction as if wholly and only relative to self. Equivalently, via $\psi^r_{sjr}(t)>\tau_s(t)\psi^s_{sjr}(t-1)$ a sufficient condition for acquisition of commodity j relative to individual r is that the system related potential $\psi^r_{sjr}(t)$ is strictly greater that its system related opportunity cost $\tau_r(t)\psi^s_{sjr}(t-1)$.

- Correspondingly condition (5.1b) is open to the interpretation that, if the marginal evaluation $\Sigma e^r_{rjk}\varphi_{rk}(t)$ relative to self is less than the marginal opportunity cost relative to self $\omega_{sj}(t)$ then $\psi^r_{sjr}(t)<\tau_r(t)\psi^s_{sjr}(t-1)$ is a sufficient condition for willingness by individual r to acquire elements of commodity j at that margin.

- Under condition (5.1c) individual r is indifferent at the margin between consumption and relinquishment of commodity j at the margin. Equivalently conditions (5.1c) imply coincidence of conditions under which an individual will act to agree relative to self as if because agreeing relative to a system relative to self (and vice versa). Correspondingly conditions (5.1a,b) are consistent with interpretations according to which *disagreement* relative to a system is potentially generated by *disagreement* relative to self (and vice versa). [In these cases a higher potential relative to a system connotes a relatively lower potential relative to self (and vice versa). Correspondingly less relative to a wider system implies more relative to self. These are crucial observations in wider contexts pertaining to principles and processes of potentially mutually advantageous gain through price related exchange.]

- Note that, if any of the (mutually exclusive) conditions (5.1a-c) is consistent with (5.1), then that pair of conditions implies conditions of *degeneracy* (i.e. as if redundancy of (5.1)) as if *because* then consistent with (5.1). *A fortiori,* if derived via (5.1) as above, then

conditions (5.1a-c) necessarily imply this species of degeneracy. Under these conditions determination relative to self $\Leftrightarrow$ determination relative to a wider system relative to self, *even if constituent parts of the relevant conditions are not equalities.*]

In the immediately preceding developments I have focused on *buying*. In a similar way the fifth conditions and associated interpretations of (II)' relate to *selling* at the margin. More generally actions as if to agree on quantities relative to another as if via (I) and $z^r_{rjs}{}^*(t), z^r_{sjr}{}^*(t)$ (resp (II) and $z^s_{sjr}{}^*(t), z^s_{rjs}{}^*(t)$) yield *subsequent* potentials to (dis)agree on relative prices $\psi^r_{rjs}(t), \psi^s_{sjr}(t)$ via (I)', (II)', and vice versa.

In each of these contexts it is crucially significant that, if individuals engaged in processes of exchange *agree* on quantities, then in general they will *disagree* on potentials. Further, if individuals engaged in a process of disagreement relative to budget related offers on quantities then act as if to *agree* on such quantities, such actions imply cancellation/contradiction of processes of disagree-ment relative to (changes in) quantities and thence cancellations/contradictions of processes of disagreement relative to (changes in) associated potentials/relative prices. That in turn implies actions as if to secure relative *turning* in the sense of relative *reversal* of such (changes in) quantities and relative prices. (Actions as if to stop are equiv-alent to actions as if to agree *not* to continue.)

Principles of as if purposive contradiction are fundamental here. Specifically: processes potent-ially generating gifts or exchanges are open to interpretation as potentially stemming from more fundamental mathematical/ principles and processes according to which less quantity (and thence relatively higher price/potential) relative to self may systematically generate more quantity (and

thence relatively lower price/potential) relative to a wider system, and vice versa.

In a language of relative polarities, relatively negative relative to self implies relatively positive relative to a system relative to self and relatively positive relative to a system relative to self implies potential for another/others to gain relative to that system by generating a consequence relatively negative to that system and consequently relatively positive relative to themselves. Such a sequence of relatively negative to relatively positive and thence relatively positive to relatively negative events can be seen as open to interpretation as if corresponding to a process of (e.g. gift related) change generated and propagated via principles and processes of contradiction and self contradiction.

For a process of exchange *two* sequence of this type will be required corresponding to processes of donation and receipt relative to two distinct individuals. On this see the earlier quantity related developments via (I),(II) and the associated relative price/potential related developments via (I)',(II)'.

[For example, in (I) $z^s_{sjr}(t)$ represents a potential transfer from s to r and in that sense relative less for s and more for r as if via an intervening system. More subtly, via $z^s_{sjr}{}^+(t), z^s_{sjr}{}^-(t)$ and associated potentials $h^s_{sjr}{}^+(t), h^s_{sjr}{}^-(t)$ the system (I) incorporates mechanisms according to which s may potentially secure agreement /disagreement relative to another/others via an intervening system.]

Now consider a class of cases in which individual s, given an offer and associated prices by individual r, e.g. as if via (I),(I)', chooses a response by solving the optimization problem (III) below. Assuming, as before, that $U_s(\ )$ is concave, and associating the indicated dual variables with the constraints of (III) the associated Kuhn-Tucker conditions are as in (III)'. (Compare with (I) and (Ia) and/or contrast the case for individual r via (II) earlier in this section.):

$$\text{Max}\, U_s(C_{sk}(t)) - \Sigma h^s_{rjs}{}^+(t)z^s_{rjs}{}^+(t) - \Sigma h^s_{rjs}{}^-(t)z^s_{rjs}{}^-(t) - \Sigma h^s_{sjr}{}^+(t)z^s_{sjr}{}^+(t) - \Sigma h^s_{sjr}{}^-(t)z^s_{sjr}{}^-(t) - g_{sk}(t)U_s(C_{sk}{}^-(t))$$

$\varphi_{sk}(t)$ st $C_{sk}(t) = \Sigma a_{sjk}y_{sj}(t) + \Sigma b_{sjk}x_{sj}(t) + \Sigma e^s_{rjsk}(t)z^s_{rjs}(t) - \Sigma f^s_{sjrk}(t)z^s_{sjr}(t)$ $j,k=1,2$

$\omega_{sj}(t)$ $y_{sj}(t) + x_{sj}(t) + z^s_{sjr}(t) = x_{sj}(t-1) + z^s_{rjs}(t)$

$\psi^s_{rjs}(t)$ $z^s_{rjs}(t) + z^s_{rjs}{}^+(t) - z^s_{rjs}{}^-(t) = z^r_{rjs}{}^*(t)$ $j\varepsilon J_{1sr}$ (III)

$\psi^s_{sjr}(t)$ $z^s_{sjr}(t) + z^s_{sjr}{}^+(t) - z^s_{sjr}{}^-(t) = z^s_{sjr}{}^*(t)$ $j\varepsilon J_{2sr}$

$\tau_s(t)$ $\Sigma\psi^r_{sjr}(t-1)z^s_{sjr}(t) \geq \Sigma\psi^r_{rjs}(t-1)z^s_{rjs}(t)$

$\lambda_s(t)$ $U_s(C_{sk}(t)) - U_s(C_{sk}{}^-(t)) \geq U_s(C_{sk}{}^*(t))$

All variables nonnegative

$C_{sk}(t)$ $(1-\lambda_s(t))\,\delta U_s/\delta C_{sk}(t) \leq \varphi_{sk}(t)$

$y_{sj}(t)$ $\omega_{sj}(t) \geq \Sigma a_{sjk}\varphi_{sk}(t)$ $k=1,2$

$x_{sj}(t)$ $\omega_{sj}(t) \geq \Sigma b_{sjk}\varphi_{sk}(t)$

$z^s_{rjs}(t)$ $\omega_{sj}(t) \geq \Sigma e^s_{rjsk}\varphi_{sk}(t) + \psi^s_{rjs}(t) - \tau_s(t)\psi^r_{rjs}(t-1)$ (III)'

$z^s_{sjr}(t)$ $\omega_{sj}(t) \leq \Sigma f^s_{sjrk}\varphi_{sk}(t) - \psi^s_{sjr}(t) + \tau_s(t)\psi^r_{sjr}(t-1)$

$z^s_{rjs}{}^+(t), z^s_{rjs}{}^-(t)$ $-h^s_{rjs}{}^+(t) \leq \psi^s_{rjs}(t) \leq h^s_{rjs}{}^-(t)$

$z^s_{sjr}{}^+(t), z^s_{sjr}{}^-(t)$ $-h^s_{sjr}{}^+(t) \leq \psi^s_{sjr}(t) \leq h^s_{sjr}{}^-(t)$

$U(C_{sk}{}^-(t))$ $-g_{sk}(t) \leq \lambda_s(t)$

Three possible classes of solution to (III) are:

i) as if via choice of $z^s_{rjs}{}^+(t) = z^s_{rjs}{}^-(t) = 0$ and $z^s_{sjr}{}^+(t) = z^s_{sjr}{}^-(t) = 0$ in (III) *potentially accept* an offer $z^r_{rjs}{}^*(t)$, $z^r_{sjr}{}^*(t)$ which, via $U_s(C_{sk}{}^-(t)) = 0$, yields $U_s(C_{sk}{}^{**}(t))$ and so is at least weakly preferred relative to the reference preference level $U_s(C_{sk}{}^*(t))$.

ii) Given i) as if via choice of at least one $z^s_{rjs}{}^+(t)$, $z^s_{rjs}{}^-(t), z^s_{sjr}{}^+(t), z^s_{sjr}{}^-(t) \neq 0$ in (III) with a reference prefer-ence level $U_s(C_{sk}{}^{***}(t))$ with $U_s(C_{sk}{}^{***}(t)) \geq U_s(C_{sk}{}^{**}(t))$ *conditionally accept* an offer $z^r_{rjs}{}^*(t)$, $z^r_{sjr}{}^*(t)$.

iii) If an offer is such that it corresponds to a solution of (III) with $U_s(C_{sk}{}^-(t)) > 0$ where $U_s(C_{sk}{}^*(t))$ is an attainable alternative, then *reject* that offer.

If used in combination with analogous cases for individual r these three classes of possible cases can be used both to motivate and to analyze various kinds of exchange relations between individuals.

For example: Assume that individual s starts with endowments $x_{sj}(t-1)$ and a reference preference $U_s(C_{sk}{}^*(t))$. By selecting $z^s_{rjs}{}^+(t) = z^s_{rjs}{}^-(t) = 0$ and $z^s_{sjr}{}^+(t) = z^s_{sjr}{}^-(t) = 0$ that individual may choose not to (further) interact with any others and in that way accept an outcome corresponding to case i) above. By selecting *non zero* values for one or more of

these quantities individual s may be able to select an outcome potentially meeting the conditions of ii) above so potentially attaining an at least weakly preferred state as if via interaction with another /others. Such a potential for gain may in turn be either accepted, conditionally accepted, or rejected, by another /others in that way potentially variously leaving individual s either better off or indifferent relative to his/her initial reference preference level. In that context not only do such processes potentially yield potentially Pareto improving gift/exchange related mechanisms but, in doing so via elements of dual relations analogous to (III)' above, they potentially accord with interpretations and associated mechanisms analogous to those associated variously with prices, taxes and subsidies. These and other types of types of possibilities are considered in more detail in Chapter 4.

6. Determinacy, Indeterminacy and Potentially Beneficial Exchanges.

By contrast with developments in the previous section, in which individuals were setting prices for each other, now consider a case in which prices are given via a relatively external agent. In such cases there would be the possibility of an exchange and bargaining mechanism isomorphic with that stemming from (III) and (III)' for individual s above for

each of agents r and s and possibly, too for this additional agent, say agent g. In addition to the possibility that agents r and s respond only to each other's offers $z^s_{sjr}{}^*(t)$, $z^s_{sjr}{}^*(t)$ and associated prices $\psi^s_{sjr}(t-1), \psi^s_{rjs}(t-1)$ (resp $z^r_{rjs}{}^*(t), z^r_{rjs}{}^*(t)$ and associated prices $\psi^r_{rjs}(t-1), \psi^r_{sjr}(t-1)$) further possibilities may then arise. Two examples are:

i) cases in which the third agent g acts as if *simultaneously* to set offers for r and s via common prices $\psi^g_{sjr}(t-1) = \psi^r_{sjr}(t-1) = -\psi^s_{sjr}(t-1)$ and $\psi^g_{rjs}(t-1) = -\psi^r_{rjs}(t-1) = \psi^s_{rjs}(t-1)$. [Notice that for a seller the marginal evaluation equals the marginal opportunity cost and is associated with less at that margin whereas for a buyer the marginal valuation os associated with relative gain at that margin, Hence the relatively opposed signs of buying and selling prices.]

ii) cases in which agent g acts to make offers of quantities and prices *sequentially* in a manner analogous to those of r and s. These two classes of cases might be termed *government* cases, in which g a government (which might be variously chosen by or imposed upon individuals r, s) and; *additional agent* cases in which g is an additional agent (which might be variously chosen by, or imposed upon, individuals r,s).

Government cases and additional agent cases are not necessarily mutually exclusive. Agents r and s might choose to act *as if* creating a third (government) agency g with a view to being ruled by that agent with reference the specification of price and quantity information. In that case the additional agent case may include an additional role for r or s but additional agent cases also include other possibilities in which one or both of agents r and s discover or accept a third agent g and choose to interact with that agent by means of gifts, barter or trade.

In any case a relatively external agent g is in effect an additional agent potentiating new opportunities to either or both of the existing agents r,s according to which each of those agents may (dis)agree relative to self as if to (dis)agree relative to g through a wider system relative to self, and so on. In that way a system may potentially *grow* via offers from additional agents such that some or all are potentially better off. In particular, if a freely made offer, either from an existing agent or from an additional agent, is accepted by one or more individuals and the status quo is freely accepted by others, *all* are potentially better off with reference to knowledge and/or to commodities than they would have been in the absence of that offer. In that way, to an economy may *grow* in a manner potentially consistent with extensions of processes inherent in systems (I) through (III).

In a context in which a relatively external agent g fixes relative prices, currency and standard units potentially have key roles as elements of mechanisms mediating between bargaining (and in that sense relatively indeterminate) individuals. Such a mediating role potentially exploits a property of money/currency that is not usually stressed namely that it is typically *self axiomatizing*, with the validity of such an axiomatization guaranteed by the resources of a government supported central bank.

For example, a British five pound note bears the statements "£5" and "FIVE POUNDS" as well as the statement: "I promise the bearer on demand the sum of five pounds" and the signature of the Chief Cashier on behalf of the Governor of the Bank of England.

In practice units of currency will not exchange for other identical units of that currency. So, interpreted literally, the Governor's promise to exchange £5 notes appears empty; it would seem irrational for an individual to seek to exchange a five pound note for a strictly identical note.

But, viewed as a guarantee of relative worth, the governor's promise becomes more meaningful. It can then be seen as a guarantee of value to individuals potentially foregoing their own evaluations of five pounds' "worth" of other currency, other financial instruments, or of commodities in exchange for a self axiomatizing Five Pounds. [Even if understood as a guarantee against forgery, a

strict 1:1 interpretation would not appear meaningful since a counterfeit note would then only guarantee exchange with another counterfeit note.]

A self axiomatizing five pound note thus constitutes a measure of potential mathematical/physical determinacy relative variously to itself, to the Governor of the Bank of England and to other five pounds', and a measure of potential *indeterminacy* relative to (elements of) potential processes of exchange relative to other units of currency, other financial instruments and other commodities. It subsumes a guarantee of potential determinacy, inter alia, in the sense of potential for as if 1:1 physical correspond-ences, which is to be useful essentially only for circumstances when a five pound note does not physically exchange for an identical note. Evidently an individual can gain from a transaction if choosing to exchange units of currency of a particular denomination for a subjectively more highly valued number of units of a commodity. Conversely, another individual could *also* gain from that same transaction, if potentially choosing to exchange those same units of currency for those "same" units of relatively less highly valued commodity.

It follows that both individuals may guarantee gains relative to units of currency because they (e.g. five pounds') axiomatize themselves as the same, not only relative to themselves, but as if "the same" relative to individuals whose differences of subjective evaluations of the exchanged commodity are themselves the basis of that transaction. In this way self axiomatizing units of currency can be useful because potentially guaranteeing system related determinacy of commodity and/or currency exchanges, while also potentially *guaranteeing* relative indeterminacy of relatively subjective evaluations of currency-commodity exchanges.

Particularly with reference to (5.1a)-(5.1c) it has already been noted that standard units in general and currency units in particular have potentially significant roles according to which agents might meaningfully agree upon such

standards as if purposively thereby to disagree relative to them. In that way, paradoxically, *guarantees* seemingly secured via agreements on values of currency relative to standard units of currency potentially enable gains to be made via *disagreements* on values of commodities relative to standard units of currency. In that way, too, with contexts of economic exchanges relative to standard units, individuals may be understood as acting rationally when acting as if to agree upon a class of standard units as if only with the purpose of acting as if to disagree via them.

More subtly, an exchange of commodities for currency (and vice versa) may not only take place at the conclusion of a single iteration of a bargaining process between individuals: it may be the means of securing determinacy for an otherwise potentially indeterminate sequence of potentially mutually beneficial processes of disagreement, (such as that implicit in standard "offer curve" arguments, see Baumol, 1977, p.207), each potentially yielding a bargain between individuals and themselves and individuals and each other.

In such contexts, as in contexts of n person exchange more generally, currency and standard units potentially have key roles as elements of *absolutely determinate* mechanisms intermediating between two bargaining (and in that sense *relatively indeterminate*) individuals.

7. International trade

While only one standard currency unit is needed for a closed economy analysis, for open economy cases there may be multiple currencies with consequent need for mathematical and/or physical conditions relating and interrelating elements of those different currencies. This suggests not only currency-currency exchange rates interrelating different currencies, but a variety of (potential) exchange rates as if potentially determining and/or potentially determined via measures in relation to different Governors of Central Banks.

With this context, interpret individuals r and s in the previous sections as not just representative of different countries via buying and selling prices $\psi^r_{sjr}(t)$, $\psi^s_{rjs}(t)$ and $\psi^r_{rjs}(t)$, $\psi^s_{sjr}(t+)$ for quantities j and time periods t, but of different currencies via the implicit exchange rates τ_r, τ_s.

Together with developments in Section 7 these remarks suggest interpretations of national currencies as integer denominated measures of relative value which

  i) provide means of securing coordinated and determinate opportunities for relative gain or loss between individual economic agents in each nation and;

  ii) via non-integer relative evaluations provide means of securing coordinated opportunities for continuing and, in that sense indeterminate, processes of relative barter or trade related bargaining between individuals in any particular nation, as well as those of others.

It might seem that, to the extent that precision in currency transactions is desirable, axiomatically integer denominations for elements of currency would become inherently desirable. But, as has been seen in a context of potential exchanges of other commodities, other financial instruments and/or other currencies relative to a particular currency, relative *imprecision* of measurement is arguably also desirable in the sense, for example, that a margin of difference between prices constituting as if potentially objective exchange evaluations and potentially subjective use evaluations, may be construed and/or may construe itself as if a measure of the margin of relative gain/loss in a transaction.

There is significance here for the potential in units of currency to generate principles and processes pertaining both to integer and to non-integer arithmetic. (On this point see also see Ryan 1992.) With reference to integer arithmetic, as noted in Section 6 each unit of a currency typically axiom-atizes itself as a standard unit (e.g. a five pound note as Five Pounds). In this respect units of currency constitute a special class of standard units. Further: standard units of currency are

typically denominated in integers and therefore each potentially relates directly to principles and processes pertaining to integer arithmetic.

Although particular currencies are determined with reference to integers and potentially associated integer arithmetic principles and processes, the specification and determination of extranational exchange rates $\tau_r(), \tau_s(\ )$ (including supranational as well as international exchange rates) may employ non-integer magnitudes and associated non-integer arithmetic principles and processes in that way potentially ensuring indeterminacy of choice unless or until conditions of overall equality obtain.

To illustrate this reconsider the buying related condition (5.1) for individual r together with a corresponding buying related condition (5.2) for individual s:

From (II)' and complementary slackness $z^r_{sjr}(t) \geq 0 \Rightarrow$

$$\omega_{rj}(t) \geq \Sigma e^r_{rjk}\varphi_{rk}(t) + \psi^r_{sjr}(t) - \tau_r(t)\psi^s_{sjr}(t-1) \quad (5.1)$$

and from (III)' and complementary slackness $z^s_{sjr}(t) \geq 0 \Rightarrow$

$$\omega_{sj}(t) \leq \Sigma f^s_{sjrk}\varphi_{sk}(t) - \psi^s_{sjr}(t) + \tau_s(t)\psi^r_{sjr}(t-1) \quad (5.2)$$

If conditions of exchange are such that optimally $z^r_{sjr}(t) = z^s_{sjr}(t) > 0$ then (5.1) and (5.2) hold as equalities, viz:

$$\omega_{rj}(t) = \Sigma e^r_{sjrk}\varphi_{rk}(t) + \psi^r_{sjr}(t) - \tau_r(t)\psi^s_{sjr}(t-1) \quad (5.1a)$$

$$\omega_{sj}(t) = \Sigma f^s_{sjrk}\varphi_{sk}(t) - \psi^s_{sjr}(t) + \tau_s(t)\psi^r_{sjr}(t-1) \quad (5.2a)$$

If also $\psi^g_{sjr}(t-1) = \psi^r_{sjr}(t-1) = -\psi^s_{sjr}(t-1)$, $\psi^g_{rjs}(t-1) = -\psi^r_{rjs}(t-1) = \psi^s_{rjs}(t-1)$, as in case i) of Section 6, then (5.1a), (5.1b) in turn imply:

$$\omega_{rj}(t) = \Sigma e^r_{sjrk}\varphi_{rk}(t) + \psi^g_{sjr}(t) - \tau_r(t)^g_{sjr}(t-1) \quad (5.1b)$$

$$\omega_{sj}(t) = \Sigma f^s_{sjrk}\varphi_{sk}(t) - \psi^g_{sjr}(t) - \tau_s(t)^g_{sjr}(t-1) \quad (5.2b)$$

It follows that in general $\omega_{rj}(t) \neq \omega_{sj}(t)$ at an optimum even if both individuals r and s agree

both on the nature and quantities of exchanged commodities via conditions $z^r_{sjr}(t)=z^s_{sjr}(t)>0$ *and* on the nature and magnitude of exchange related prices via conditions $\psi^g_{sjr}(t-1)=\psi^r_{sjr}(t-1)= -\psi^s_{sjr}(t-1)$, $\psi^g_{rjs}(t-1)=-\psi^r_{rjs}(t-1)= -\psi^s_{rjs}(t-1)$. That is: even given this apparently complete agreement with reference to the nature, quantities and valuations of exchanged commod-ities, only exceptionally will conditions obtain according to which private evaluations $\omega_{rj}(t),\omega_{sj}(t)$ of commodity j are equal at that margin for individuals r and s. [Given conditions yielding (5.1b), (5.2b) sufficient conditions for $\omega_{rj}(t)=\omega_{sj}(t)$ are a common exchange rate $\tau_s(t)=\tau_r(t)$ and conditions such that at that optimum the marginal incremental gain $\Sigma e^r_{sjrk}\varphi_{rk}(t)$ to individual r from increasing j to individual r equates to the marginal incremental loss of $\Sigma f^s_{sjrk}\varphi_{sk}(t)$ to individual s from increasing j to individual r. Clearly these conditions are very stringent since they imply inter alia that r gains by gaining $z^r_{sjr}(t)\geq0$ relative to r and also s loses by losing $z^s_{sjr}(t)\geq0$ relative to s. [I.e. for r possession of an increment of commodity j is a "good" and for s *dis*possession of that same increment is not also a "good".] More generally s may gain relative to self by a transfer $z^s_{sjr}(t)$ so that $-\Sigma f^s_{sjrk}\varphi_{sk}(t)>0$. And/or conditions may obtain such that e.g. $\omega_{rj}(t)=\Sigma e^r_{sjrk}\varphi_{rk}(t)$ and $\psi^r_{sjr}(t)=\tau_r(t)\psi^s_{sjr}(t-1)$ for individual r and $\omega_{sj}(t) > -\Sigma f^s_{sjrk}\varphi_{sk}(t)$ with $-\psi^g_{sjr}(t) +\tau_s(t)\psi^g_{sjr}(t-1)>0$ for individual s.

Related issues with reference to degeneracy and decomposability, to timing, to subjectivity/ objectivity and to learning with reference to processes of interaction between individuals r and s are the subjects of the next section.

8. Framing, decentralization and exchange related Pareto improvements

In preceding sections distinct optimizations (II) and (III) have been developed for agents r and s. Now consider a more comprehensive formulation (IV) which conditionally decomposes to yield (II) and (III) as special cases.

In effect (IV) potentially connects (II) and (III) by means of the final constraints which by means of terms in the third line of the objective to (IV) potentially equate quantities $z^s_{rjs}{}^{*}(t)$ to quantities $z^r_{rjs}{}^{*+}(t)$ and quantities $z^r_{sjr}{}^{*}(t)$ to quantities $z^s_{sjr}{}^{*}(t)$. Models comprehending more subtle processes of interrelation and timing are the subjects of Chapter 4.

If in (IV) $U_r(C_{rk}(t))$ and $U_r(C_{rk}(t))$ are concave the Kuhn-Tucker conditions are sufficient for optimality. With the exception of relations associated with variables in the last two constraints those conditions are as for (II) and (III) so that, writing those remaining conditions explicitly, the Kuhn-Tucker conditions associated with (IV) at an optimum are those given above as conditions (II)' (III)' *and conditions* (IV)' below.

$$\text{Max} U_r(C_{rk}(t))-\Sigma h^r_{sjr}(t)z^r_{sjr}{}^{+}(t)-\Sigma h^r_{sjr}(t)z^r_{sjr}{}^{-}(t)-\Sigma h^r_{rjs}{}^{+}(t)z^r_{rjs}{}^{+}(t)-\Sigma h^r_{rjs}{}^{-}(t)z^r_{rjs}{}^{-}(t) -g_{rk}(t)U_r(C_{rk}{}^{-}(t))$$

$$+U_s(C_{sk}(t))-\Sigma h^s_{rjs}{}^{+}(t)z^s_{rjs}{}^{+}(t)-\Sigma h^s_{rjs}{}^{-}(t)z^s_{rjs}{}^{-}(t)-\Sigma h^s_{sjr}{}^{+}(t)z^s_{sjr}{}^{+}(t)-\Sigma h^s_{sjr}{}^{-}(t)z^s_{sjr}{}^{-}(t)-g_{sk}{}^{-}(t)U_s(C_{sk}{}^{-}(t))$$

$$\mathbf{-\Sigma d^s_{sjr}{}^{+}(t)z^s_{sjr}{}^{*+}(t)-\Sigma d^s_{sjr}{}^{-}(t)z^s_{sjr}{}^{*-}(t)--\Sigma d^s_{sjr}{}^{+}(t)z^s_{sjr}{}^{*+}(t)-\Sigma d^s_{sjr}{}^{-}(t)z^s_{sjr}{}^{*-}(t)}$$

$\varphi_{rk}(t)$	st $C_{rk}(t)=\Sigma a_{rjk}y_{rj}(t) + \Sigma b_{rjk}x_{rj}(t) + \Sigma e^r_{sjrk}(t)z^r_{sjr}(t) - \Sigma f^r_{rjsk}(t)z^r_{rjs}(t)$ j,k=1,2
$\omega_{rj}(t)$	$y_{rj}(t) + x_{rj}(t) + z^r_{rjs}(t) = x_{rj}(t-1) + z^r_{sjr}(t)$
$\psi^r_{sjr}(t)$	$z^r_{sjr}(t) + z^r_{sjr}{}^{+}(t) - z^r_{sjr}{}^{-}(t) = z^s_{sjr}{}^{*}(t)$ $j\varepsilon J_{1rs}$
$\psi^r_{rjs}(t)$	$z^r_{rjs}(t) + z^r_{rjs}{}^{+}(t) - z^r_{rjs}{}^{-}(t) = z^s_{rjs}{}^{*}(t)$ $j\varepsilon J_{2rs}$
$\tau_r(t)$	$\Sigma\psi^s_{rjs}(t-1)z^r_{rjs}(t)\geq\Sigma\psi^s_{sjr}(t-1)z^r_{sjr}(t)$
$\lambda_r(t)$	$U_r(C_{rk}(t)) -U_r(C_{rk}{}^{-}(t))\geq U_r(C_{rk}{}^{*}(t))$

(IV)

$$\varphi_{sk}(t) \qquad C_{sk}(t)=\Sigma a_{sjk}y_{sj}(t) +\Sigma b_{sjk}x_{sj}(t) + \Sigma e^s_{rjsk}(t)z^s_{rjs}(t) - \Sigma f^s_{sjrk}(t)z^s_{sjr}(t) \quad j,k=1,2$$

$$\omega_{sj}(t) \qquad y_{sj}(t) + x_{sj}(t) + z^s_{sjr}(t) = x_{sj}(t-1) + z^s_{rjs}(t)$$

$$\psi^s_{rjs}(t) \qquad z^s_{rjs}(t) + z^s_{rjs}{}^+(t) - z^s_{rjs}{}^-(t) = z^r_{rjs}{}^*(t) \qquad j\varepsilon J_{1sr}$$

$$-\psi^s_{sjr}(t) \qquad z^s_{sjr}(t) + z^s_{sjr}{}^+(t) - z^s_{sjr}{}^-(t) = z^s_{sjr}{}^*(t) \qquad j\varepsilon J_{2sr}$$

$$\tau_s(t) \qquad \Sigma\psi^r_{sjr}(t-1)z^s_{sjr}(t)\geq\Sigma\psi^r_{rjs}(t-1)z^s_{rjs}(t)$$

$$\lambda_s(t) \qquad U_s(C_{sk}(t)) -U_s(C_{sk}{}^-(t))\geq U_s(C_{sk}{}^*(t))$$

$$\psi^g_{rjs}(t-1) \qquad \mathbf{z^r_{rjs}{}^*(t) + z^r_{rjs}{}^{*+}(t) - z^r_{rjs}{}^{*-}(t) = z^s_{rjs}{}^*(t)}$$

$$\psi^g_{sjr}(t-1) \qquad \mathbf{z^s_{sjr}{}^*(t) + z^s_{sjr}{}^{*+}(t) - z^s_{sjr}{}^{*-}(t) = z^r_{sjr}{}^*(t)}$$

All variables nonnegative

$$z^r_{rjs}{}^*(t) \qquad \psi^g_{rjs}(t)\geq\psi^s_{rjs}(t)$$

$$z^s_{rjs}{}^*(t) \qquad -\psi^r_{rjs}(t)\geq\psi^g_{rjs}(t)$$

$$z^s_{sjr}{}^*(t) \qquad \psi^g_{sjr}(t)\geq\psi^r_{sjr}(t) \qquad (IV)'$$

$$z^r_{sjr}{}^*(t) \qquad -\psi^g_{rjs}(t)\geq\psi^s_{rjs}(t)$$

Six observations re (IV), (IV)':

- **Re Theorem 1:** If a solution such that $z^s_{sjr}{}^{*+}(t) =z^s_{sjr}{}^*(t)=0$ and $z^s_{sjr}{}^{*+}(t)=z^s_{sjr}{}^*(t)=0$ all s.j.r is feasible in (IV) then if $M=d^s_{sjr}{}^+(t)=d^s_{sjr}{}^-(t)$ and $d^s_{sjr}{}^+(t)=d^s_{sjr}{}^-(t)=M$ a solution with $z^s_{sjr}{}^{*+}(t)=z^s_{sjr}{}^*(t) =0$ and $z^s_{sjr}{}^{*+}(t)=z^s_{sjr}{}^*(t)=0$, all s.j.r will be optimal in (IV). But if $d^s_{sjr}{}^+(t),d^s_{sjr}{}^-(t)<<M$ and/or $d^s_{sjr}{}^+(t),d^s_{sjr}{}^-(t)<<M$ for at least one r.j.s, then solutions corresponding to (further) gifts or exchanges via conditions such that $z^r_{rjs}{}^*(t)\neq z^s_{rjs}{}^*(t)$ and/or $z^s_{sjr}{}^*(t)\neq z^s_{sjr}{}^*(t)$ at least one r.j.s may be optimal to (IV). (This is an application of a variant of Theorem 1.)

- *Re subjectivity/objectivity:* Throughout the prec-eding developments quantities $C_{rk}(t),C_{sk}(t)$ and $y_{rj}(t),x_{rj}(t),y_{sj}(t),x_{sj}(t)$ together with the associated quantities $a_{rjk},b_{rjk},a_{sjk},b_{sjk}$ have been implicitly *subjective* because not subject to exchange between individuals. More subtly, throughout those develop-ments, via the superscripts r and s, the quantities $z^r_{rjs}(t),z^s_{rjs}(t)$,$z^r_{sjr}(t),z^s_{sjr}(t)$ and associated coefficients $e^s_{rjsk}(t)$, $z^s_{rjs}(t)$, $f^s_{sjrk}(t)$ have been treated as also subjectively measured. With that context the conditional nature of equations $z^r_{rjs}{}^*(t)=z^s_{rjs}{}^*(t)>0$ and $z^s_{sjr}{}^*(t)=z^s_{sjr}{}^*(t)>0$ via the final two constraints of (IV) and thence the explicitly conditional nature of the associated equalities $\psi^g_{sjr}(t-1)=\psi^r_{sjr}(t-1)= -\psi^s_{sjr}(t-1)$, $\psi^g_{rjs}(t-1)=-\psi^r_{rjs}(t-1)= -\psi^s_{rjs}(t-1)$ stemming from (IV) and complementary slackness. are open to interpret-ation, not only with reference to relatively objective evaluations but to conditional agreements (or requirements) to exchange as if via standard quantity and currency units. [Note that if they do agree on these potentials then this is as if one individual is as if perfectly predicting the other's behaviour in that respect. But, note, too, that in general, if individuals agree on quantities they will *not* agree on potentials. This is because, for a seller, the marginal evaluation is for a loss whereas for a buyer it is for a gain. Thus, even if they agree with reference to the value of the marginal unit they will be opposed as to its direction. (The sum of a relatively positive price p and a relatively negative price -p is zero. But the difference is 2p. From that perspective apparent agreement may correspond to significant disagreement.)

- *Re degeneracy and decomposability:* A special class of solutions to (IV) are those in which conditions obtain as if $d^s_{sjr}{}^+(t)=d^s_{sjr}{}^-(t)=0$ and $d^s_{sjr}{}^+(t)=d^s_{sjr}{}^-(t)=0$ all s.j.r.t. In that case in effect the last two conditions of (IV) become redundant and (IV) decomposes to systems (II) and (III). Equivalently, if conditions $z^r_{sjr}(t)=z^s_{sjr}{}^*(t)$ and $z^r_{rjs}(t)=z^r_{rjs}{}^*(t)$ are selected via $h^s_{rjs}{}^+(t),h^s_{rjs}{}^-(t),h^s_{sjr}{}^+(t)h^s_{sjr}{}^-(t)$ in (II) and (III), then those conditions are consistent with the last two conditions of (IV) if also conditions obtain as if *implicitly* $z^r_{sjr}(t)=z^r_{sjr}{}^*(t)$ and $z^r_{rjs}(t)=z^r_{rjs}{}^*(t)$ in (II) and (III). Those conditions in turn are consistent with an optimal solution to (IV) generated inter alia via weights $d^s_{sjr}{}^+(t)=d^s_{sjr}{}^-(t)=0$ and $d^s_{sjr}{}^+(t)=d^s_{sjr}{}^-(t)=0$. [Notice that in either of these cases the final constraint would in effect be redundant and the optimal solution to (IV) would be *degenerate* as well as decomposable and in that way decentralizable.]

9. Conclusion

In this chapter it has been shown how both standard units of quantity abnd standard units of value can be useful in facilitating exchanges between individuals. With that context it has been explained how the usefulness for currency can derive, inter alia, from its self axiomatizing potential to impute precise, integer, evaluations to elements of potential exchanges for contexts where processes of exchange will generally be motivated by relative differences of subjective evaluations - including ultimately unknowable differences between individuals' different preferences relative to commodities.

Stressing this: not only is relative imprecision of subjective measurement apparently inevitable, in the Heisenberg sense, but, in contrast, say, to contexts of particle physics research, in economic contexts of trade it may also be desired - and hence *desirable* - in that sense.

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