CHAPTER 10

MULTIPLE CRITERIA AND FRAMING OF DECISIONS

1.Introduction

The purpose of this chapter is to show how a multi criteria approach to decisionmaking can be used as a general means of modelling the process of framing decisions. As examples the first part focuses on a number of well known classes of problems, in which frames have only been implicit, to make them explicit by means of preemptive goals in a multiobjective goal programming framework. These problems include the matching pennies game (Shubik 1982, Wang 1988), the distribution, or transportation, problem (Charnes and Cooper 1961, Shogan 1988), as well as linear and nonlinear programming formulations of the intertemporal peak load pricing problem (Turvey 1969, Littlechild 1970, Ryan 1992). In the second part of the paper non-premptive generalizations of the earlier analysis are considered and it is shown how the explicit introduction of frames can provide a means of resolving well known paradoxes. These include the more for less paradox in the distribution problem (Charnes and Klingman 1971, Szwarc 1971, Ryan 1980 Charnes, Duffuaa, Ryan 1980, Arshan 1992) and Allais' paradox (Machina 1993) with reference to individual decision-making under uncertainty.

The organization of the paper is as follows: Section 2 introduces the central idea of framing via preemptive goals with the context of explicit restrictions on the range of distributions and the probabilities of outcomes in the constant sum matching pennies game. In Section 3 I consider framing in the context of restrictions on supplies and demands in the distribution ("transportation") model and, in Section 4, of capacity restrictions in multiperiod peak load pricing problems. Sections 5, 6 and 7 then provide more subtle extensions in which the (choice of) frame is endogenized and the respectively game theoretic, distribution problem related, and intertemporal analyses extended to comprehend mixtures of non preemptive and preemptive frame related goals and decisions.

In every case the multi criteria decisionmaking nature of the problem will be evident for in every case the relevant problem may be broken into criteria relating to two distinct, yet related, parts of an overall problem, namely optimal selection of the relevant frame and optimal selection of outcomes within that particular frame.

2. Framing and constant sum games

The minimax-maximin specification of a two player constant sum matching pennies game with strategies j,k, payoffs π_{kj} and payoff probabilities p_i, q_j as a dual pair of linear programmes, as in (I),(I)', is well known:

$$\begin{array}{llll} & \text{Max } \rho & & \text{Min } \mu \\ \text{st} & \sum_{j\varepsilon J}\pi_{kj}p_j \geq \rho & \text{st} & \sum_{k\varepsilon K}\pi_{kj}q_k \leq \mu \\ & \sum_{j\varepsilon J}p_j=1 \quad (\text{I}) & & \sum_{k\varepsilon K}q_k = 1 \quad (\text{I})' \\ & p_j \geq 0 & & q_k, \geq 0 \end{array}$$

	H	**T**
H	**1**	**-1**
T	**-1**	**1**

TABLE 1

With j=1,2 and k=1,2 and payoffs as in Table 1, (I), (I)' give $\rho=\mu=0$ and $p_j=1/2$, $q_k=1/2$. This "fair" equiprobable solution might seem unsurprising - in the sense of consistency with prior beliefs concerning "nature's" strategy choices - given the symmetry of this particular specification. But notice, first, that if a player had such prior information/or beliefs both concerning the *range* of potential payoffs (in this case heads and tails) and concerning the *magnitudes* of potential probabilities, then properly they would be incorporated in the initial specification to *frame* the game accordingly. This can be done by introducing weights M and a preemptive goal programming formulation to generalize (I),(I)' to a *constrained game* formulation (see Charnes 1953, Owen 1982, Ryan 1994,1995) which pre-emptively restricts the range of outcomes to heads and tails and the magnitude of "nature's" probabilities, as in (II),(II)':

$$
\begin{array}{l}
\text{Max } \rho - Mp^{+} - Mp^{-} + \Sigma R_k q_k^{*} \\
\text{st} \quad \sum_{j\varepsilon J} \pi_{kj} p_j - R_k \geq \rho \\
\quad \sum_{j\varepsilon J} p_j + p^{+} - p^{-} = p^{*} \qquad \text{(II)} \\
\quad -M \leq \rho \leq M \\
p_j, p^{+}, p^{-} \geq 0,\ R_k \text{ unrestricted}
\end{array}
$$

$$
\begin{array}{l}
\text{Min } \mu + Mq^{+} + Mq^{-} \\
\text{st} \quad \sum_{k\varepsilon K} \pi_{kj} q_k \leq \mu \\
\quad \sum_{k\varepsilon K} q_k + q^{+} - q^{-} = q^{*} \qquad \text{(II)'} \\
\quad q_k + q_k^{+} - q_k^{-} = q_k^{*} \qquad k\varepsilon K \\
\quad -M \leq \mu \leq M \\
q_k, q^{+}, q^{-}\ q_k^{+}, q_k^{-} \geq 0
\end{array}
$$

Here R_k have interpretations as marginal informational values of prior probabilities - for coins, of relative *bias*. As a more general example of this same idea consider an analogous class of cases which follows Ryan 1994 and generalizes work by McInerney 1967, Hazell1970, Kawaguchi and Muruyama 1972 and in which π_{kj} j=1,2, k=1,2 in (II),(II)' represent contingent payoffs to a farmer engaged in a wet crop/dry crop:rain/no rain game against nature. In that case q_k represent probabilities with which nature adopts weather patterns k so that in (II)' q_k^{*}, R_k have interpretations as probabilities of elements of a prior weather forecast and potential rewards to relatively increased probabilities of relatively wet and dry weather.

Parenthetically, in this constrained game context the maximin-minimax criterion has the very nice *dominance* property that, if the relatively worst set of outcomes is improved by improving prior information, *all* outcomes are improved by improving that prior information. (For more on this see Ryan 1994,1995.)

3.Framing and the distribution model

The distribution problem is to minimize the cost of shipping available quantities a_i at origins i to meet demands b_j at deastinations j where $\Sigma a_i = \Sigma b_j$ and the unit shipping cost over route ij is c_{ij}. This problem can be expressed in a standard linear programming form as in (III) below.

$$
\begin{array}{l}
\text{Min } \sum_{ij} c_{ij} x_{ij} \\
\text{st} \quad \sum_{j} x_{ij} = a_i \qquad \text{(III)} \\
\quad \sum_{i} x_{ij} = b_j \\
x_{ij} \geq 0, \quad \sum_{i} a_i = \sum_{j} b_j
\end{array}
$$

But (III) implicitly restricts (i.e. implicitly *frames*) supply and demand to the equality case. Now introduce (IV) as an explicitly framed preemptive goal programming extension of (III). (For an introduction to goal programming see Charnes and Cooper 1961 or Shogan 1988.):

$$
\begin{array}{l}
\text{Min } \sum_{ij} c_{ij} x_{ij} - M\sum_{i}(x_i^{+} + x_i^{-}) - M\sum_{j}(x^{j+} + x^{j-}) \\
\text{st} \quad \sum_{j} x_{ij} + x_i^{+} - x_i^{-} = a_i \\
\quad \sum_{i} x_{ij} + x^{j+} - x^{j-} = b_j \qquad \text{(IV)} \\
\quad \sum_{i}(x_i^{+} - x_i^{-}) = \sum_{j}(x^{j+} - x^{j-}) \\
x_{ij}, x_i^{+}, x_i^{-}, x^{j+}, x^{j-}) \geq 0, \quad \sum_{i} a_i = \sum_{j} b_{j,}
\end{array}
$$

In (IV) preemptively large weights M explicitly prohibit supplies or demands greater or less than the prescribed amounts a_i, b_j. In particular this formulation explicitly prohibits exploitation of the conditions of the *more for less* or *more for nothing* paradoxes (see Charnes and Klingman 1971, Szwarc 1971, Ryan 1980, Charnes, Duffuaa and Ryan 1980, 1987).

In words the MFL paradox can be stated as:

> *Given an optimal solution to a distribution problem, it is possible in certain instances to ship more total goods for less total cost even if we ship at least the same amount from each origin and at least the same amount to each destination, and all shipping costs are nonnegative.* Charnes and Klingman 1971 p.11.

That is: given an optimal solution to (IV), and even if all shipping costs are strictly positive, it may be possible to *increase* at least one a_i to $a_i+\delta$ and one b_j to $b_j+\delta$ in such a way that the overall shipping cost is *reduced*. Interpreting a_i as dinner halls and b_j as groups of potential diners, the conditions of the more for less paradox appear to correspond, not just to the potential existence of a free dinner, but to one which one or more individuals could profitably be paid to eat!

In Section 6 I will show that explicitly framing the distribution problem by means of variously preemptive or non preemptive goal programming specifications can provide a new way of resolving this paradox, while at the same time comprehending a wide variety of other classes of network problems as special cases.

4.Framing and the peak load pricing problem

The peak load pricing problem as expressed by Turvey 1969 is to find the cost minimizing way of meeting demands x_t^* t=1,2..T from available capacity y, given that c_t will be the marginal variable cost of supply in period t and the marginal acquisition cost of units of capacity in an initial period is β. With these assumptions the optimal plan can be found as a solution to (V) and its dual (V)' below.

Notice first that this specification focuses essentially on the supply of output rather than on the value stemming from demand for it. Secondly, since in an optimal solution to (V), $x_t=x_t^*\forall t$ the specification (V) implicitly preemptively fixes period t demands at x_t^*, regardless of the associated dually optimal tariffs - from (V)' - namely ($\varphi_p=c_p+\mu_p$, $\Sigma\mu_p=\beta$) in peak periods and ($\varphi_t=c_t$,$t\neq p$) in off peak periods.

$$\begin{array}{llll} \text{Min } \sum_t c_t x_t + \beta y & & \text{Max } \sum_t \varphi_t x_t^* + \sum_t \mu_t y & \\ \text{st} \quad x_t \geq x_t^* & & \text{st} \quad \varphi_t \leq c_t+\mu_t & \\ \quad\quad x_t \leq y & \text{(V)} & \quad\quad \sum_t \mu_t \leq \beta & \text{(V)'} \\ \quad\quad x_t, y \geq 0 & & \quad\quad \varphi_t,\ \mu_t \geq 0 & \end{array}$$

But in general demand for output in periods t (eg electricity at different hours of the day) will not be independent of the prices charged for it. This point is clearer if demand is explicitly distinguished from supply and the preemptive framing of problem (V) is made explicit by means of the more general preemptive goal programming formulation (VI), (VI)':

$$\begin{array}{ll} \text{Max}\sum_t(p_t z_t - M z_t^+ - 0 z_t^-) - \sum_t c_t x_t - \beta y & \\ \text{st} \quad z_t + z_t^+ - z_t^- = z_t^* & \\ \quad\quad z_t \leq x_t & \text{(VI)} \\ \quad\quad x_t \leq y & \\ \quad\quad z_t, z_t^+, z_t^-, x_t, y \geq 0 & \end{array}$$

$$\begin{array}{ll} \text{Minimize } \sum_t \varphi_t x_t^* + \sum_t \mu_t y & \\ \text{st} \quad \varphi_t + \psi_t \geq p_t & \\ \quad\quad \varphi_t \leq c_t + \mu_t & \text{(VI)'} \\ \quad\quad \sum_t \mu_t \leq \beta & \\ \mu_t, \varphi_t \geq 0,\ M \geq \psi_t \geq 0 & \end{array}$$

In this extended formulation internal opportunity costs φ_t are not just explicitly related to variable and capital costs of production, but related, too, to target market prices p_t. In this way this explicitly framed extension opens up new and more direct relationships between Turvey's approach and Littlechild's nonlinear program-ming represent-ation (Littlechild 1970), in which the choice of constraint bounds, and in that sense the choice of the frame, is endogenously - as distinct from pre-emptively - determined. In its simplest form - Littlechild also considered multiple capacity and uncertainty related extensions. Littlechild's model is:

$$\begin{array}{ll} \text{Max } \sum\int p_t dz_t - \sum c_t x_t - \beta y & \\ \quad z_t \leq x_t & \text{(VII)} \\ \quad x_t \leq y_t & \\ \quad z_t, x_t, y \geq 0 & \end{array}$$

The objective of (VII) being concave, the Kuhn Tucker optimality conditions are:

$$\begin{array}{lcll} z_t \geq 0 & \Leftrightarrow & \varphi_t \geq p_t & \\ x_t \geq 0 & \Leftrightarrow & \varphi_t \leq c_t + \mu_t & \text{(VII)'} \\ y \geq 0 & \Leftrightarrow & \sum_t \mu_t \leq \beta & \end{array}$$

The form of conditions (VII)' is reminiscent of (VI)'. Indeed, if x_t and p_t are respectively equal in value in (VI) and (VII), (VI)' and (VII)' become equivalent and consistent with overall minim-ization of the cost of provision of optimal levels of output x_t^*, as in Turvey's approach. Here this result emerges entirely from explicit consider-ation of the process of framing Turvey's problem.

Although this preemptively framed class of multiperiod cases is of interest in itself, richer kinds of correspondences emerge once general nonpreemptively frame related cases are considered. Three classes of examples of these are the subjects of the next three sections

5.Nonpreemptive frames and constant sum games

Consider a nonpreemptive generalization of the constrained game (II),(II)'

$$
\begin{aligned}
\text{Max}\quad & \rho - c^{+}p^{+} - c^{-}p^{-} + \Sigma R_k q_k^{*} \\
& \text{st } \Sigma \pi_{kj} p_j - R_k \geq \rho \qquad j\varepsilon J \\
& \Sigma p_j + p^{+} - p^{-} = p^{*} \qquad\qquad \text{(VIII)} \\
& j\varepsilon J \\
& -d^{-} \leq \rho \leq d^{+}, -M^2 \leq R_k \leq M^2 \\
& p_j, p^{+}, p^{-} \geq 0,\ R_k \text{ unrestricted}
\end{aligned}
$$

$$
\begin{aligned}
\text{Min}\quad & \mu p^{*} + d^{+}q^{+} + d^{-}q^{-} + \Sigma M^2 q^{+} + \Sigma M^2 q^{-} \\
\text{st}\quad & \Sigma \pi_{kj} q_k \leq \mu \\
& k\varepsilon K \\
& \Sigma q_k + q^{+} - q^{-} = q^{*} \qquad\qquad \text{(VIII)'} \\
& k\varepsilon K \\
& q_k + q_k^{+} - q_k^{-} = q_k^{*} \qquad k\varepsilon K \\
& -c^{+} \leq \mu \leq c^{-} \\
& q_k, q^{+}, q^{-}\ q_k^{+}, q_k^{-} \geq 0
\end{aligned}
$$

For c^{+},c^{-},d^{+},d^{-} all positive and sufficiently large, (II),(II)' and (VIII), (VIII)' are equivalent. In that case, there is always an attainable optimal solution to (VIII) with $p_j^{+}=0$,$p_j^{-}=0$ and $p_j>0$ some j and consistent with conditions as if $-M<\mu<M$ in (VIII)'. If also $\Sigma q_k^{*} \leq q^{*}$ in (VIII)' a dual optimal solution, with $q_k^{+}=0$,$q_k^{-}=0$, $q_k>0$ some k would also be attained. Even if c^{+} is *negative,* and as long as it is not greater than $\Sigma \pi_{kj} q_k^{*}$, in general a mixed strategy solution to (VIII)' is optimal. But if c^{-} is then sufficiently reduced μ becomes reduced (via the final constraints of (VIII)') and thence, via the first constraints of (VIII)', q_k is reduced until ultimately the final constraints in (VIII)' become equivalent to $-c^{+}=\mu=c^{-}$. Even if initially $p_j>0$ some j is optimal for (VIII)', such a reduction of a relatively external magnitude is ultimately consistent with an optimal solution to (VIII) with $p_j=0$ all j, $p_j^{+}>0$ and/or $p_j^{-}>0$ on the primal side, and to (VIII)' with $q_k=0$, $q_k^{+}>0$ and/or $q_k^{-}>0$ on the dual side.

In the context of matching pennies-related interpretations this may correspond to the ultimate determination of a strict preference for non heads/tails outcomes $p_j^{+}>0$, $p_j=0$ $\forall j$ via a sufficient reduction in c^{-} (eg work related outcomes with the opportunity cost of leisure c_j^{+} equal to the marginal return to work c_j^{-}) over initially selected randomized heads/tails related outcomes with $p_j>0$ some j. Or, in the context of crop related interpretations of (VIII),(VIII)' this might imply the ultimate determination of a preference for non crop planting (eg leisure/fallow land related) alternatives $p_j^{+}>0$ over an initial choice of a randomized crop planting alternatives with $p_j>0$.

Summarizing: variants of (VIII),(VIII)' can correspond to a revealed preference for work over matching pennies related leisure, or to a revealed preferemce for crop related farm work over fallow land related activities, in each class of cases in response to correspondingly changed opportunity costs c_j^{+},c_j^{-} of alternative types of relatively interior or exterior outcomes and/or of relatively increased/reduced coin or crop related prior probabilities p_k^{*} .

In these two classes of cases the proportion of land (resp time) Σq_j devoted to crops (resp matching pennies) is dependent both on the weather forecast (resp prior heads/tails probabilities) q_k^{*} and on predictions c^{+},c^{-} for renting out or renting in increments of land (resp time) p^{+},p^{-}. In this context changes in forecast weather or prior heads/tails probabilities and/or in marginal evaluations c^{+},c^{-} may correspond- ingly change the optimal *frame* of the problem by changing the decision on how much land (resp time) to use, and not just on how to use it.

Clearly then (VIII),(VIII)' can be seen as one way of endogenizing multiple criteria relating not just to the the choice of activities given a particular frame, but as a way of endogenizing the choice of the relevant frame itself, respectively for a farm planning model and for a matching pennies game.

More subtly, such frame related implications of a change in a relatively *exterior* marginal eval- uation c^{-} for *switches in* individuals' relatively *interior* choices of mixes of relatively certain and uncertain opportunities can offer a class of endogenously optimally determined and frame related explanations for Allais like-preference reversals of kinds noted empirically variously by MacCrimmon 1968, MacCrimmon and Larsson 1979, Kahneman and Tversky 1979, and Chew and Waller 1986. To see this consider Machina's definition of Allais' paradox, (Machina 1993, pp23-24):

Given alternatives:

b_1: $x+(1-\alpha)P^{**}$ versus b_2: $P + (1-\alpha)P^{**}$

and

b_3: $x+(1-\alpha)P^{*}$ versus b_4: $P + (1-\alpha)P^{*}$

where:

x=prospect yielding x with certainty;

P involves outcomes both greater and less than x

P** stochastically dominates P*

Then Allais' paradox arises if subjects, when offered b_1 and b_2, prefer b_1 to b_2 (b_2 to b_1) but, when offered a choice between b_3 and b_4, prefer b_4 to b_3 (resp b_3 to b_4).

In contrast to earlier developments Machina's definition gives no explicit optimization framework for the choice of relevant frame determining relative interiority of relatively risky or riskless alternatives (b_1, b_2) vis a vis relatively exterior alternatives (b_3,b_4). Nor does that definition provide for the endogenous determ-ination of rewards/opportunity costs x, P, P*, P**, associated with these various alternatives. But in the context of (VIII),(VIII)', which explicitly and simultaneously model choices of frames and of frame related payoffs and opportunity costs, Machina's definition itself suggests corres-ponding classes of Allais related interpretations as follows:

Let $p^*=_{def}\alpha$ be the proportion of available time/land initially allocated to matching pennies/farming and $p^-=_{def}(1-\alpha)$ the proportion of time/land assoc-iated with relatively external activities. Then, given contingent payoffs π_{kj} to relatively interior and uncertain activities q_j, as in (VIII), (VIII)', an initially chosen solution to (VIII) might be such that $\Sigma p_j = p^*=\alpha$ some $p_j>0$, $p^+=0$, $\rho^*=\Sigma\pi_{kj}p_j$ and $p^-=(1-\alpha)>0$. But, if c^+ is itself negative and c^- is sufficiently reduced then, via (VIII)' and complementary slackness, ultimately cond-itions could obtain such that $p_j=0\ \forall\ j$ and $p^+=p^*=\alpha$ with $-c^+=\mu=c^-$ and $p^-=(1-\alpha)>0$. These circumstances are consistent both with Machina's definition and a frame related explanation of the common consequence version of Allais' paradox. In the first case, given a relatively external evaluator c^{-*} and probability $p^-=(1-\alpha)$, a randomized relatively interior outcome (b_2) is chosen with $\Sigma p_j=p^*=\alpha>0$, $\rho^*=\Sigma\pi_{kj}p_j$ in preference to a relatively nonrandomized interior outcome (b_1) with $p^+=\alpha>0$. And, if c^{-*} is sufficiently reduced, say to c^{-**}, and still with relatively exterior probability $(1-\alpha)$, the optimally chosen outcome may be such as to reveal a preference for a relatively interior nonrandomized out-come (b_3) with $p^+=\alpha>0$ to a relatively interior randomized outcome (b_4) with $\Sigma p_j = p^*=\alpha>0$. QED.

For more on constrained game approaches to the representation and resolution of Allais paradox in the different context of strategiacally equivalent transformations see Ryan 1996. (That paper includes a treatment of the common ratio version as well as the common consequence version considered here).

6.Nonpreemptive frames and the mfl/mfn paradoxes

The standard distribution model can be written as if embedded in a potentially nonpreemptively framed form as follows:

$$\operatorname{Min}\sum_{ij}c_{ij}x_{ij}+\sum_i(c_i^+x_i^+ + c_i^-x_i^-)+\sum_j(c^{j+}x^{j+}+c^{j-}x^{j-})$$

$$\text{st}\quad \sum_j x_{ij}+x_i^+-x_i^-=a_i$$

$$\sum_i x_{ij}+x^{j+}-x^{j-}=b_j \qquad \text{(IX)}$$

$$\sum_i(x_i^+-x_i^-)=\sum_j(x^{j+}-x^{j-})$$

$$x_{ij},x_i^+,x_i^-,x^{j+},x^{j-})\geq 0,\ \sum_i a_i=\sum_j b_j,$$

For $c_i^+,c_i^-,c^{j+},c^{j-}$ positive and sufficiently large (IX) is entirely equivalent to the premptive form (IV), which in turn becomes consistent with the as if preemptively framed (and standard) formulation (III). But, if (IX) is not preemptively framed, non zero overshoots x_i^-,x^{j-} and undershoots x_i^+,x^{j+}, respectively of supplies at origins and demands at destinations become potentially optimal. In particular conditions may obtain as if optimally $x_{ij}=0$ and $x_i^+=a_i$,$x^{j+}=b_j$ all ij. This suggests interpretations of $-c_i^-,c^{j-}$ in relation to potential acquisition costs associated with supplies and prices associated with demands. These and more general cases become evident if (IX) is rewritten as an equivalent maximizing problem (X) together with its dual as follows:

$$\text{Max} -\sum_j (c^{j-} x_i^{j-}) - \sum_{ij} c_{ij} x_{ij} - \sum_i (c_i^- x_i^-) - \sum_j (c^{j+} x^{j+}) - \sum_i (c_i^+ x_i^+)$$

$$\text{st} \quad \sum_j x_{ij} + x_i^+ - x_i^- = a_i$$

$$\sum_i x_{ij} + x^{j+} - x^{j-} = b_j \qquad (X)$$

$$\sum_i (x_i^+ - x_i^-) = \sum_j (x^{j+} - x^{j-})$$

$$x_{ij}, x_i^+, x_i^-, x^{j+}, x^{j-}) \geq 0, \qquad \sum_i a_i = \sum_j b_j$$

Associating dual variables R_i, K_j with the first two constraints of (X) and φ with the third, its dual becomes:

$$\text{Minimize } \sum_i R_i a_i + \sum_j K_j b_j)$$

$$\begin{array}{lll} x_{ij} \geq 0 & \text{st} & R_i + K_j \geq -c_{ij} \\ x_i^+ \geq 0 & & R_i + \varphi \geq -c_i^+ \\ x_i^- \geq 0 & & -R_i - \varphi \geq -c_i^- \;\; (=_{def} -p_i) \quad (X)' \\ x^{j+} \geq 0 & & K_j - \varphi \geq -c^{j+} \\ x^{j-} \geq 0 & & -K_j + \varphi \geq -c^{j-} \;\; (=_{def}\; p^j) \end{array}$$

If $c_i^+ = c_i^- = c^{j+} = c^{j-} = M$ conditions (X)' are consistent with optimal solutions for the standard implicitly preemptively framed case for which $x_i^+ = x_i^- = x^{j+} = x^{j-} = 0$ and $x_{ij} > 0$ and $R_i + K_j = c_{ij}$ for an optimally determined set of routes ij. But these conditions are also potentially consistent with various classes of explicitly framed and non premptively framed interpretations.

In particular, if p_i is the marginal cost of acquiring an additional unit at origin i and p^j is the revenue from selling a unit at destination j and if $x_i > 0$, $x^j > 0$ and $x_{ij} > 0$ for a particular route ij, the conditions of (X)' are consistent with interpretations as follows:

$$x_{ij} > 0 \Rightarrow R_i + K_j = c_{ij}$$
$$x_i^- > 0 \Rightarrow -R_i - \varphi = -c_i^-$$
$$x^{j-} > 0 \Rightarrow -K_j + \varphi = -c^{j-}$$

so that, for this class of cases:

$$R_i + K_j = p^j - p_i = c_{ij} \;\text{ or }\; p^j = p_i + c_{ij}$$

This in turn is consistent with spatially competitive decision rules according to which, for the optimally determined set of routes, destination price p^j is exactly sufficient to recoup marginal acquisition cost p_i plus unit transportation costs c_{ij}. Now consider the conditions of the more for less and more for nothing paradoxes in this context. First, define a degenerate* optimum to (X) as one with less that m+n-1 positive shipments x_{ij}. Then the MFL(MFN) paradox exists if, given a non degenerate* optimal solution to (X) with preemptive $c_i^+ = c_i^- = c^{j+} = c^{j-} = M$, at an optimum $R_r + K_s < 0$ (=0) for some non basic route rs. (Charnes and Klingman 1971 Theorem 1.1). In those circumstances it is possible to increase supplies at at least one origin and reduce demand at at least one market in such a way as to reduce (leave unchanged) the overall shipping cost.

With the context of (X), if this condition of the more for less (nothing) paradox is exploited in such a way as to leave the optimal basis unchanged, for the non basic MFL(MFN) route rs, $x_r > 0$ and $x^s > 0$, so that, from (X)', by complementary slackness $p^s - p_r = R_r + K_s$ (<0). From this it might seem that, via the MFL/MFN cell rs, it would be possible to ship more while reducing *overall* cost. While this is true in certain circumstances it certainly is not always so. In general exploitation of the conditions of the paradox inevitably involves acquisitions as well as shipments of additional quantities. Thus in general additional acquisition as well as additional shipment costs will be incurred and it may be that additional acquisition costs cancel out - or more than cancel out, any incremental shipping cost related savings.

In more detail: since by assumption the potentially MFL/MFN optimum is non degenerate*, $R_i + K_j = c_{ij}$ for each of the m+n-1 basic shipment cells ij. Noting that there are m+n-1 equations in m+n unknowns choose one measure $R_i = 0$. (Notice that this is potentially consistent with the idea that φ corresponds to a *base price* since with $R_i = 0$ from (X)' $x_i^- > 0 \Rightarrow -\varphi = -c_i^- =_{def} -p_i$.) With this initial value the m+n equations can be solved in such a way that each of the quantities R_i, K_j is expressed as a sum of differences of the basis related quantities c_{ij}. It follows that for any *non basic cell,* and in particular for a MFL (MFN) cell rs, $R_r + K_s = \sum c_{ij}^+ - \sum c_{ij}^-$ where the quantities c_{ij}^+, c_{ij}^- correspond to the relevant sum of differences of basis related measures c_{ij} for that cell rs. (For those familiar with the “stepping stone” solution method of Charnes and Cooper 1961 these entries correspond to “stones” in the stepping stone path associated with cell rs)

.

Thus, for any MFL (MFN) cell rs $R_r + K_s = \sum c_{ij}^+ - \sum c_{ij}^- < 0$ (resp=0). If the MFL/MFN opportunity is exploited by setting $x^{j-} = x_i^- = \delta > 0$ and increasing the overall quantity shipped by δ (as can always be arranged since a MFL/MFN solution is non degenerate* by definition) then, together the above conditions give:

$$(R_r + K_s)\delta = \Sigma c_{ij}^{+}\delta - \Sigma c_{ij}^{-}\delta$$
$$= (p^s - p_r)\delta < 0 \quad (\text{resp}=0)$$

so that, optimally:

$$p^j\delta = p_i\delta + \Sigma c_{ij}^{+}\delta - \Sigma c_{ij}^{-}\delta$$

It follows that, whether or not they are exploited, the conditions of the MFL (MFN) paradox may be consistent with conditions of spatial competition according to which, for each of the optimal set of shipments, marginal selling price is optimally equal to acquisition cost plus marginal shipping cost. In this case, even if a more for less (nothing) condition is exploited, incremental shipping cost reductions are exactly offset by a net contribution to expenditures on acquisition costs.

More generally, examples exploiting the MFL paradox can correspond to circumstances with effective subsidies for incremental shipments and/or supernormal profits or net losses for additional quantities shipped. Consider an interpretation in which a shipper determines an initially optimal solution for a client that exhibits the MFL paradox for a certain non basic route rs.

Assume that, by chance, the client wants an additional quantity δ shipped to destination s and will provide the increment to be shipped from origin r. Then, even if the shipper charges nothing for this additional shipment, if supply and market prices are as above, the shipper can make a superprofit of $(p^s-p_r)\delta$ and the client will effectively subsidize each additional unit supplied to s by an amount $(p^s - p_r)$ and make an overall loss of $(p^s - p_r)\delta$ on the revised shipping plan.. (For more on the MFL/MFN paradox and its relation to issues of degeneracy and decomposability, as well as extensions to non-linear cases see Ryan 1980, as well as our subsequent work in Charnes, Duffuaa, Ryan 1980, 1987.)

Apart from nonpreemptively framed distribution models and explicit treatments of MFL/MFN phenomena, the goal programming extension of the distribution model (X) can comprehend other types of network problems too. In distinction from others' work it not only potentially explicitly incorporates market related supply and demand prices: it also incorporates all forms of inequality cases as well as the standard preemptively fixed equality form as particular kinds of special cases. This in turn points to further frame related issues and generalizations pertaining to further constraints on x_i^{j-}, x_i^{-}, x^{j+}, x_i^{+} in the non preemptive case and so points to further generalizations to incorporate particular kinds of relatively externally price as well as cost constrained capacitated network problems. From another direction the form of the constraints in (X) suggest interpretations of (X) (X)' as local linearizations of nonlinear supply and demand relations and/or to demand or supply target related taxes and subsidies. I consider both of these kinds of possibilities and interpretations in the context of intertemporal models in the next section.

7. Nonpreemptive frames and intertemporal decisions

First consider a nonpremptively framed generalization of the systems (VI),(VI)' as in (XI),(XI)'

With $c_t^{+}=M$, $c_t^{-}=0$ (VI),(VI)' and (XI),(XI)' are equivalent. But the latter systems also include more general interpretations according to which φ_t are interpreted as optimally determined production costs and p_t as market prices and ψ_t correspond to relative output *taxes* and *subsidies* in response, respectively, to over-shoots and undershoots relative to externally determined levels of output z_t*.

$$\begin{aligned} \text{Maximize} \sum_t (p_t z_t - c_t^{+} z_t^{+} - c_t^{-} z_t^{-}) - \sum_t c_t x_t - \beta y \\ \text{st} \quad z_t + z_t^{+} - z_t^{-} = z_t^* \\ z_t \le x_t \qquad \text{(XI)} \\ x_t \le y \\ z_t, z_t^{+}, z_t^{-}, x_t, y \ge 0 \end{aligned}$$

$$\begin{aligned} \text{Minimize} \sum_t \varphi_t x_t^* + \sum_t \mu_t y \\ \text{st} \quad \varphi_t + \psi_t \ge p_t \\ \varphi_t \le c_t + \mu_t \qquad \text{(XI)'} \\ \sum_t \mu_t \le \beta \\ \mu_t, \varphi_t \ge 0 \quad -c_t^{-} \le \psi_t \le c_t^{+} \end{aligned}$$

Alternatively z_t^+, z_t^- (c_t^+,c_t^-) may correspond to elements of a local linearization of the demand relation and in that way act as if to correspond to a linearized form of Littlechild's model. In either case such interpretations suggest further frame related constraints corresponding to ranges over which associated taxes/subsidies/relative prices will be applicable.

Another direction for explicitly frame related extensions is one which explicitly incorporates price variations for elements of capacity, as well as for output:

$$\begin{array}{lll} \text{Max} & \sum_t (p_t z_t - c_t^+ z_t^+ - c_t^- z_t^-) - \sum_t c_t x_t - \beta y - \beta^+ y^+ - \beta^- y^- & \\ \text{st} & z_t + z_t^+ - z_t^- = z_t^* & \\ & z_t \le x_t & \text{(XII)} \\ & x_t \le y & \\ & y + y^+ - y^- = y^* & \\ & z_t, z_t^+, z_t^-, x_t, y \ge 0 & \end{array}$$

Each of these kinds of interpretations could be extended in turn to include multiperiod and uncertainty related cases of kinds considered by Littlechild in his original, but only implicitly framed, nonlinear programming formulations. (For more on these types of possibilities in the slightly different context of a collectively spatial and intertemporal decision-making framework see Ryan 1992.)

8. Concluding remarks

This paper has shown how, by making choices of frames explicit by means of appropriate pre-emptive goal programming models the way can be opened directly and immediately to the formulation and analysis of more general nonpreemptively framed classes of multicriteria decisions. In particular using this approach the choice of frame and the marginal evaluation of frame related phenomena are endogenized and the way is opened directly to new means of modelling and analysing various kinds of frame related phenomena, including Allais' paradox, the more for less paradox and production capacity determined peak load prices.

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