CHAPTER 4

PURPOSIVE CONTRADICTION AND GAINS FROM EXCHANGE

1. Introduction

Standard approaches to competition and trade, such as those cited in Section 2 of chapter 3 using Edgeworth box based analyses, focus on exchange under conditions of complete information, and more particularly on conditions where differences between individuals' marginal evaluations of preferences, information and/or endowments ultimately become exhausted. This is also true of Bayesian gaming approaches founded on common knowledge and rationality axioms (see Harsanyi 1968, Aumann 1987, Brandenburger and Dekel 1993), and of evolutionary games (see Mailath 1992, Friedman 1996). In all of these cases the principal focus is on optimal *stopping* rules for agents with *known* opportunity sets.

But, if others' endowments of commodities are initially unknown, an Edgeworth Box or its n person analogue is not definable by any of the individuals concerned - its dimensions are not common knowledge. And, if a purpose of individuals' interactions is to expand their own and others' opportunity sets with a view to mutual gain, there is no reason to suppose that individually and collectively known and agreed opportunity sets of kinds considered via Edgeworth boxes in standard two person cases would ever be determined. Indeed processes of exploration and discovery not only potentially generate bigger quantities (and thence correspondingly bigger Edgeworth boxes) with reference to initially known commodities, but may generate further dimensions and in that sense newly defined "boxes" through discoveries of initially unknown commodities.

From these latter perspectives analyses using Edgeworth Box and associated offer curves (which focus on instances where differences between individuals' marginal evaluations of information and/or endowments become exhausted) can be seen as focussing on optimal *stopping* rules. But such marginal conditions do not in general correspond either to optimal

starting rules, or to optimal continuation rules. In these latter senses such marginal conditions are clearly not desirable in themselves.

By contrast, in this chapter the primary focus will not be on equilibrium conditions within given opportunity sets, but on potentially Pareto superior positions associated with points initially beyond them.

With these contexts the central purpose of the chapter is to focus on growth and exploration oriented processes of mutually advantageous exchange. The emphasis throughout will be on principles and processes of self contradiction according to which individual economic agents act as if purposively to choose less relative to self as if thereby to potentiate more relative to themselves or others differently located in space and time.

Even though the central themes and examples are economic the mathematical and physical principles involved are more fundamental. On reflection it will be clear that purposive self contradiction of kinds considered here is consistent with mathematical and physical principles of action, reaction and interaction which will be familiar to readers in other contexts. For example, at first it might seem that walking from A to B would involve strictly forward progress but, even though "the longest journey begins with a single step", the impulse for that step would necessarily be relatively *backwards*, as the reader may verify by trying it. For economic agents, as for physical objects in general, relatively forward motions are generated in reaction to relatively backward impulses.

2. A class of exchange related examples

Given complete information concerning the nature and quantities of available commodities, gains from exchange between individuals - if any - will stem from differences in those individuals' preferences relative both to those commodities

and relative to each other. For example, if two individuals were the same with respect to preferences, information and endowments then, in the absence of production, and assuming that their survival was certain, under any reasonable set of individual or collective choice criteria, they would not exchange elements of their initial endowments with each other. Strictly, if each was aware of their "sameness", under the conditions of the previous sentence they would be indifferent to exchanges of elements of endowments at the margin.

But how are individuals to discover others' endowments, and more particularly their willingness or otherwise to exchange elements of them? Among other things any process of discovery must initially involve purposive self contradiction and incompleteness in the following sense: potential explorers initially determined wholly and only relative to their own relatively abstract preferences and their own endowments of commodities, if to explore beyond those initial preferences and endowments, must act, at least initially, as if purposively to propose less of at least one characteristic relative to self, even if preferring more of all commodity related characteristics relative to self.

So principles and processes of exploration, including those potentially yielding knowledge and mutually advantageous exchanges relative to other persons, are in any case inextricably interrelated with principles and processes of indeterminacy and incompleteness. Underlining this: even if two individuals were by chance initially identically determined relative to themselves in some otherwise generally accepted sense, if one of them were to seek to verify that as a fact it would no longer be so. In particular, if one were to initiate a process potentially leading to exchange or trade, for example by making an offer of less of one commodity relative to self in exchange for more of another relative to another, by initiating that process, that individual would necessarily become not just incompletely determined relative (solely) to self, but differently determined relative to the other.

Notice that for anyone initiating a process intended to potentiate conditions of mutually advantageous exchange, a relevant *starting* rule is

to act as if purposively to disturb an initial state relative to self as if thereby to potentiate opportunities relatively outside an initial opportunity set determined relative to another via an offer of less to self as if thereby to potentiate more relative to another or others.

That is: principles and processes of as if purposive self contradiction and incompleteness are necessarily involved in any process potentiating exchange between persons. Indeed, although they have not been explicitly recognized in those contexts, on reflection the reader will see that all are implicit in standard competitive Edgeworth Box related approaches, as well as in Bayesian and evolutionary gaming approaches to equilibrium.

In the present explorer-related context it is especially important to recognize the fundamental role of as if purposive uncertainty and incompleteness with respect to initially given constraints on individual and collective choice sets. In such circumstances the very purpose of interactions between individuals may be to vitiate such initial conditions - and *a fortiori* to vitiate any possibility of long run equilibria within the initial endowments of commodities for the individuals concerned.

That is, far from desirably *converging* to some as if unanimously agreed optimum within initially given individual and/or collective constraint sets, (as for example in standard competitive equilibrium, or evolutionary game or Bayesian equilibrium analyses), exchange related offers may desirably start and continue with *divergences* from initial conditions, among other things via processes explicitly directed toward expansions of initially given individual and collective resource constraints and opportunity sets.

Informally, other things equal, the greater the difference between individuals' knowledge and/or preferences with reference to their endowments of commodities, the greater the opportunities for gain from exchanges of knowledge and/or endowments relative to others. This becomes especially significant with the context of exploration since then ignorance concerning others' knowledge or endowments may be complete, and so opportunities to expand the

initial boundaries of the choice sets of the individuals concerned may be considerable.

3. Gift related exploration examples

Exploration processes are not just directed to fuller determinations of others' preferences and/or endowments, but may be interpreted, too, with reference to expeditions abroad.

Consider an example emphasizing the second type of exploration: A first individual equips a second as an explorer. The second has a successful expedition and returns with sufficient rewards, not just to return the first's investment with interest, but to equip the first as an explorer. The first, in turn, has a successful expedition and returns with sufficient gains, not just to reward the second, but to equip them for yet another exploration, and so on.

Clearly gains, including information gains, from such a (fortunate) sequence of transactions stem fundamentally from expansions of opportunity sets and are potentially unbounded. Evidently in such cases decisions will be directed *outside* currently attainable collective opportunity sets.

Less obviously, by choosing to equip an expedition, an individual initially selects a position strictly *inside* his/her opportunity set. That individual will act as if to prefer less relative to self, albeit with the objective of potentiating more relative to another /others and, in due course, to self. Principles and processes of self contradiction are apparent here according to which individuals may act as if rationally to prefer less to more.

A more formal example may help here: Assume that Individual 1 has Lancaster type preferences (see Lancaster 1966, Ryan 1992) defined over characteristics and that those characteristics may be yielded by expenditures of an initial endowment $z_{111}(0)$ of commodity 1 to individual 1 variously on: i) present consumption $y_{11}(1)$; ii) deferred endowments $x_{11}(1)$ and; iii) commitments to exploration $z_{112}(1)$ by individual 2, where individual 2 is a prospective partner, known or unknown. Then individual 1's choice of that exploration related plan over non-exploration related alternatives can be expressed in an extremal form as in (I). (The examples that follow refine ideas in Chapter 3 of this book and Chapter 4 of Ryan 1992):

$$\text{Max } U_1(C_{11}(1), C_{12}(1)) - d_{11}^+C_{11}^+(1) - d_{11}^-C_{11}^-(1) - d_{12}^+C_{12}^+(1) - d_{12}^-C_{12}^-(1)$$

$$\varphi_{11}(1) \quad \text{st } C_{11}(1) = a_{111}y_{11}(1) + b_{111}x_{11}(1) - f^1_{1121}(1)z_{112}(1)$$

$$\varphi_{12}(1) \quad C_{12}(1) = a_{112}y_{11}(1) + b_{112}x_{11}(1) - f^1_{1122}(1)z_{112}(1)$$

$$\omega_{11}(1) \quad y_{11}(1) + x_{11}(1) + z_{112}(1) = z_{111}(0) \qquad \text{(I)}$$

$$\nu_{11}(1) \quad C_{11}(1) + C_{11}^+(1) - C_{11}^-(1) = C_{11}^*(1)$$

$$\nu_{12}(1) \quad C_{12}(1) + C_{12}^+(1) - C_{11}^-(1) = C_{12}^*(1)$$

$$\lambda_1(1) \quad U_1(C_{11}(1), C_{12}(1)) \geq U_1(C_{11}^*(1), C_{12}^*(1))$$

$$\text{All variables nonnegative}$$

The first two constraints of (I) incorporate a Lancaster-like linear consumption technology. The third equates planned uses to initial endowments for individual 1. The fourth and fifth constraints relate chosen levels of characteristics k, $C_{1k}(1)$, to target levels $C_{1k}^*(1)$. The last constraint relates chosen preferences $U_1(\ )$ to a reference preference $U_1(\ ^*)$. Finally the objective of (I) is to maximize a function of preferences less a weighted sum of deviations from target levels for those character-istics.

Assuming that $U_1(\ )$ is concave and differentiable, the Kuhn Tucker conditions are sufficient for an optimal solution to (I) for any given values of its parameters. Such conditions might be used to determine and evaluate an exploration related plan for individual 1 with $y_{11}(1){>}0$, $x_{11}(1){>}0$, $z_{112}(1){>}0$ vis a vis a non exploration-related reference plan with $y_{11}(1){>}0$, $x_{11}(1){>}0$, $z_{112}(1){=}0$. Associating the indicated dual variables with the constraints of (I), the Kuhn Tucker conditions are:

$C_{ik}(1)$	$(1-\lambda_1(1))\, \delta U_1/\delta C_{1k}(1) \leq \varphi_{1k}(1) + \nu_{1k}(1)$		
$y_{11}(1)$	$\omega_{11}(1) \geq \Sigma a_{11k}\varphi_{1k}(1)$	$k=1,2$	
$x_{11}(1)$	$\omega_{11}(1) \geq \Sigma b_{11k}\varphi_{1k}(1)$		(I)'
$z_{112}(1)$	$\omega_{11}(1) \geq \Sigma f^1_{112k}\varphi_{1k}(1)$		
$C_{ik}^+(1), C_{ik}^-(1)$	$-d_{1k}^+ \leq \nu_{1k}(1) \leq d_{1k}^-$		

If at an optimum to (I) $C_{1k}(1)>0$, k=1,2 and $y_{11}(1)>0$, $x_{11}(1)>0$, $z_{112}(1)>0$, the first four constraints of (I)' hold with equality by complementary slackness,. Consider the first in more detail:

$C_{1k}(1)>0 =>$

$$(1-\lambda_1)\, \delta U_1/\delta C_{1k}(1) = \varphi_{1k}(1) + \nu_{1k}(1) \quad (3.1)$$

- **Indifference** If $\lambda_1 \neq 0$ optimally $U_1(C_{1k}(1)) = U_1(C_{1k}*(1))$ in (I). So, as a special case, the chosen position and the reference position may be identical.

- **As if indifference** Whether or not $\lambda_1(1) \neq 0$ at an optimum, conditions may obtain with $\delta U_1/\delta C_{1k}(1) \neq 0$. Yet conditions may simultaneously obtain *as if* $\delta U_1/\delta C_{1k}(1) = 0$ in (I). In particular, if $\lambda_1(1) = 1$ then $(1-\lambda_1(1))\delta U_1/\delta C_{1k}(1) = 0$. These conditions in turn are consistent with as if indifference on the part of individual 1 via $C_{1k}^+(1)=C_{1k}^-(1)=0$ k=1,2 and $\nu_{1k}(1) = -\varphi_{1k}(1) \neq 0, -d_{1k}^- \leq \nu_{1k}(1) < 0$. More subtly, in the context of processes of change these conditions are consistent with indifference to any (further) change relative to the system, i.e to a *stopping* rule for individual 1.

- **Less Preferred** Cases for which a proposed state is *less* preferred than the reference state, are not feasible in (I). But those possibilities can be comprehended, too, by extending the final conditions of (I) to $U_1(C_{1k}(1))-U_1(C_{1k}^-(1)) \geq U_1(C_{1k}*(1))$ and associating a large weight $-g_{1k}^-(1)$ with $U_1(C_{1k}^-(1))$ in the objective. This extension is open to choice-of-frame related interpretations (see Chapter 10 below). More narrowly it introduces the possibility that there may be self imposed and/or externally imposed *compulsion* for an individual to adopt a less preferred outcome than the reference alternative. [If indeed there is compulsion then $z_{112}(1)>0$ implies $U_1(C_{1k}^-(1))>0$ and, by complementary slackness, $\lambda_1(1)= -g_{1k}^-(1)<0$. For example, if a preferred alternative was $z_{112}(1)=0$ this case would correspond to an interpretation that the offer of $z_{112}(1)>0$, when less preferred, is in that sense a relative "bad" for individual 1 - and would not be made except under duress.]

- **Strict Preference** By contrast conditions $\lambda_1(1)=0$ are consistent with strict preference via $U_1(C_{1k}(1)) > U_1(C_{1k}^*(1))$ in (I) and thence $C_{1k}^+(1)$, $C_{1k}^-(1)>0$ some k=1,2. That in turn is consistent with $\nu_{1k}(1) \neq 0$ some k, so generally $\omega_{11}(1)$ would *not* optimally equate to weighted sums of individual 1's marginal preferences. For example if $\lambda_1(1)=0$:

$z_{1112}(1)>0 =>$

$$\omega_{11}(1)x_{12}(1)=\Sigma f^1_{112k}\varphi_{1k}(1)x_{12}(1)$$
$$=\Sigma f^1_{112k}\delta U_1/\delta C_{1k}(1)-\nu_{1k}(1))x_{12}(1) \quad (3.2)$$

And, unless $d_{1k}^+, d_{1k}^-=0$ for such cases, evaluations $\nu_{1k}(1)=0$ for all relevant $C_{1k}^-(1)>0$ are inconsistent with $U_1(C_{1k}(1)) > U_1(C_{1k}^*(1))$, so that:

$$\omega_{11}(1) \neq \Sigma f^1_{112k}\delta U_1/\delta C_{1k}(1) \quad (3.3)$$

Summarizing: If optimally $U_1(C_{1k}(1)) >,=,< U_1(C_{1k}^*(1))$ with $z_{112}(1)>0$ and $\omega_{11}(1)=\Sigma f^1_{112k}\varphi_{1k}(1)$, conditions (3.3) are consistent with a free choice by individual 1 respectively to prefer, be indifferent to, or disprefer a gift of $z_{112}(1)$ to another vis a vis retention by himself/herself. These cases are respectively consistent with (I) and conditions as if $\lambda_1(1)=0$, $\lambda_1(1)=1$ and $\lambda_1(1)<0$ in (I)'.

4. Exploration, discovery and reciprocated gifts

Assume next that a second individual has Lancaster-like preferences together with an initial endowment $z_{222}(1)$ of commodity 2. Assume that individual 2 has been discovered via an offer $z_{112}(1)$ from individual 1 and has knowledge of that offer from individual 1. The optimization for individual 2 in period 2 then becomes:

$$\text{Max } U_2(C_{2k}(2)) - \Sigma d_{2k}^+ C_{2k}^+(2) - \Sigma d_{2k}^- C_{2k}^-(2) - g_{1k}^-(1)U_1(C_{1k}^-(1))$$

$$\varphi_{2k}(2) \quad \text{st } C_{2k}(2) = \Sigma a_{2jk} y_{2j}(2) + \Sigma b_{2jk} x_{2j}(2) - \Sigma f^2_{2j1k}(2)z_{2j1}(2) \quad j,k=1,2$$

$$\omega_{21}(2) \quad y_{21}(2) + x_{21}(2) + z_{211}(2) = z_{112}(1)$$

$$\omega_{22}(2) \quad y_{22}(2) + x_{22}(2) + z_{221}(2) = z_{222}(1) \quad\quad (II)$$

$$\nu_{2k}(2) \quad C_{2k}^+(2) + C_{2k}^+(2) - C_{2k}^-(2) = C_{2k}^*(2)$$

$$\lambda_2(2) \quad U_2(C_{2k}(2)) - U_2(C_{2k}^-(2)) \geq U_2(C_{2k}^*(2))$$

$$\text{All variables nonnegative}$$

(In contrast to (I), via $U_i(C_{ik}^-(t))$, the final constraints of (I) explicitly comprehend alternatives potentially less preferred than the reference alternative.)

Since otherwise $z_{222}(1)$ would have been the only endowment to individual 2 at the beginning of period 2, the offer $z_{112}(1)$ expands individual 2's opportunity set. Nevertheless, if all activities $y_{2j}(2)$, $x_{2j}(2)$, $z_{2j1}(2) > 0$ were inevitably associated with less preferred states, individual 2 would reject the offer $z_{112}(1)$ (unless compelled to accept it). Otherwise, via $z_{112}(1)$, individual 2 will be able to choose states strictly preferable to those attainable without accepting that offer. Accordingly assume that individual 2 conditionally accepts the offer of $z_{112}(1)$ in the

belief that it will not lead to a dispreferred state and proposes a preferred, or indifferent, state by solving (II) given $z_{112}(1)$. Assuming, too, that $U_2(\ )$ is concave and differentiable, the Kuhn Tucker conditions are necessary and sufficient for an optimal solution to (II) for any given values of its parameters. In a manner analogous to the analysis of (I) with reference to individual 1 such conditions might be used to determine and evaluate an exploration related plan for individual 2 with $y_{2j}(2) > 0$, $x_{2j}(2) > 0$, $z_{2j1}(2) > 0$ vis a vis a non exploration related reference plan with $y_{2j}(2) > 0$, $x_{2j}(2) > 0$, $z_{2j1}(2) = 0$. In any case, associating the indicated dual variables with its constraints, the Kuhn Tucker conditions associated with (II) are as follows:

$$C_{2k}(2) \quad (1-\lambda_2(2))\delta U_2/\delta C_{2k}(2) \leq \varphi_{2k}(2) + \nu_{2k}(2)$$

$$y_{2j}(2) \quad \omega_{2j}(2) \geq \Sigma a_{2jk}\varphi_{2k}(2) \quad k=1,2$$

$$x_{2J}(2) \quad \omega_{2j}(2) \geq \Sigma b_{2jk}\varphi_{2k}(2) \quad\quad (II)'$$

$$z_{2j1}(2) \quad \omega_{2j}(2) \geq \Sigma f^2_{2j1k}\varphi_{2k}(2)$$

$$C_{2k}^+(2), C_{2k}^-(2) \quad -d_{2k}^+ \leq \nu_{2k}(2) \leq d_{2k}^-$$

$$U_2(C_{2k}^-(2)) \quad -g_{1k}^-(1) \leq \lambda_2(2)$$

In a manner analogous to the analysis of (3.1) with reference to (I), via $\lambda_2(2)=0$ or $\lambda_2(2)=1$ (II) may lead respectively to acceptance and preference for the offer $z_{112}(1)$ from individual 1, or to indifference to that offer. In the absence of compulsion (when $\lambda_2(2)<1$) that offer would either be accepted or rejected. Further, from (II), acceptance of $z_{112}(1)$ by individual 2 may be represented as conditional on reciprocation with an offer $z_{211}(2)$, $z_{221}(2)$ to individual 1. In that case, if *that* offer is accepted, then individual 2 attains a correspondingly preferred or indifferent position, say $U_2(C_{2k}^{**}(2))$, initially stemming from individual 1's gift.

More subtly, even if an offer $z_{211}(2)$, $z_{221}(2)$ is refused, individual 2 may nevertheless attain a

preferred, state $U_2(C_{2k}(2)) > U_2(C_{2k}^*(2))$, either by making an alternative offer to 1, or by retaining that remaining part of the initial gift for their own current or future consumption. In that way *both* individuals can freely choose states at least indifferent to, and possibly mutually preferable to, those in the absence of offers of gifts, *whether or not such gifts are accepted*.

The role of the reference preference is central and subtle here. In the example just given, if the offer $z_{211}(2)$, $z_{221}(2)$ is rejected, then the attained preference is $U_2(C_{2k}(2)$ where $U_2(C_{2k}^{**}(2)) > U_2(C_{2k}(2)) > U_2(C_{2k}^*(2))$ and individual 2 would be *better off* with reference to a reference preference $U_2(C_{2k}^*(2))$ but *worse off* relative to a still more

preferable (yet in fact unattainable) reference preference $U_2(C_{2k}**(2))$.

In any case, given an offer from individual 2, individual 1 may form a new offer relative to individual 2, and so on. Such offers potentially constitute a process of sequential giving and receiving with both persons revealing themselves as better off relative both to themselves and each other at each stage, by freely making and freely accepting such offers.

Of course, at any stage either individual i can stop as if via $\lambda_i(t)=1$ and $(1-\lambda_i(t))\delta U_i/\delta C_{ik}(t)=0$, i.e. where perceived marginal gain to continuing equals the marginal opportunity cost of continuing (the perceived marginal gain to stopping). Importantly, if an individual i *does* stop at any stage (including the first), without accepting an offer, then what was offered is retained and, as long as $a_{rjk}>0$ or $b_{rjk}>0$ some k, both offeror and stopping individual will reveal themselves as potentially better off, respectively relative to that rejected offer and to that retained offer.

Summarizing: with qualifications concerning realized values of a_{rjk}, b_{rjk} and/or f^r_{rjsk} as above, and in the absence of external compulsion, a process of giving and receiving as if sequentially via solutions to (I),(I)' and (II),(II)' can generate Pareto improve-ments relative to the initial states of the individuals concerned, whenever they choose to start, to continue, or to stop that process.

As well as continuation and stopping principles *learning* processes are implicit in (I) and (II) with reference to new commodities and potentially new characteristics. In that context I emphasize that, although preference relations here are Lancaster-like, it has *not* been assumed - as Lancaster does - that characteristics are intrinsic and objective properties of commodities. Indeed at any stage, t, coefficients a_{rjk}, b_{rjk} and/or f^r_{rjsk} may be specific to agent i and that particular point in time.

Even though learning related features are likely to be empirically highly significant, in what follows for simplicity I will largely ignore them. I note only that acceptance of any offer, be it of a gift via (I) or (II), or through a barter or trading relationship as in subsequent sections, may lead in turn to subjective confirmations, or re-determinations, of parameters a_{rjk}, b_{rjk} and/or f^r_{rjsk}. In that way these parameters may be learned or revised as if in response to a consequent experience of others' potentials $\Sigma f^r_{rjsk}\varphi_{rk}(t)$ on offered commodities.

For example knowledge and/or experience of a previously unknown commodity may predispose an individual both to value its attributes (more) posit-ively - to acquire a (more) positive taste for it - and to seek more of it from another or others by offering more in exchange. In any case an offer is the offer of an opportunity to experience otherwise unavailable increased amounts of a commodity and, at least initially, of increased amounts of a hitherto *unknown* commodity. In no case would an individual freely accept such an offer unless perc-eiving positive values a_{rjk}, b_{rjk} and/or f^r_{rjsk} for attributes in it. [In (I)' and (II)' $\Sigma f^r_{rjsk}\varphi_{rk}(t)$ is wholly preference related. If in practice there are also resource related implications of processes of relinquishment, they would add to this difference via appropriate extension of those systems.]

In more detail: with the context of gifts, via $\Sigma f_{rjsk}\varphi_{rk}(t)$, an offer from individual r may project a *potential* relative to individual s. If benignly made by individual r (i.e. if this term is perceived as positive) such an offer will be anticipated (possibly erroneously) by individual s as intended to yield positive attributes relative to individual s, even if apparently yielding a net negative potential relative to r. (Of course if $\omega_{rj}(t)$ is negative and $\omega_{sj}(t)$ is positive a transfer from r to s of elements of commodity j would represent an increased opportunity for mutual gain relative to the case in which the marginal evaluations $\omega_{rj}(t),\omega_{sj}(t)$ were both positive.)

As already noted, processes of sequential giving may continue as if via (I),(I)',(II),(II)' to the advantage of both individuals concerned. Or, they may stop via acceptance (or rejection) of freely made offers, again potentially to the advantage of both in the manner developed above. If these indiv-iduals do continue beyond one period, the relevant optimizations would be modified by the fact that, after the first period, both individuals will have endowments from two sources, namely

retentions from the preceding period ("savings") and transfers from another /others.

Thus, for period t, the analogues of the individual problems (I) and (II) take the form shown in (III). With $x_{rj}(t-1)$, $z_{sjr}(t-1)$ given (i.e. known), the associated Kuhn Tucker conditions correspondingly take the form shown in (III)'. (Contrast the sequential nature of the timing structures here with those in Chapter 3 where the emphasis was on simultaneous exchange):

$$\text{Max } U_r(C_{rk}(t)) - \Sigma d_{rk}^{+}C_{rk}^{+}(t) - \Sigma d_{rk}^{-}C_{rk}^{-}(t) - g_{rk}^{-}(t)U_r(C_{rk}^{-}(t))$$

$\varphi_{rk}(t)$ st $C_{rk}(t) = \Sigma a_{rjk}y_{rj}(t) + \Sigma b_{rjk}x_{rj}(t) - \Sigma f^{r}_{rjsk}z_{rjs}(t)$

$\omega_{rj}(t)$ $y_{rj}(t) + x_{rj}(t) + z_{rjs}(t) = x_{rj}(t-1) + z_{sjr}(t-1)$

$\nu_{rk}(t)$ $C_{rk}^{+}(t) + C_{rk}^{+}(t) - C_{rk}^{-}(t) = C_{rk}^{*}(t)$ (III)

$\lambda_r(t)$ $U_r(C_{rk}(t)) - U_r(C_{rk}^{-}(t)) \geq U_r(C_{rk}^{*}(t))$

         All variables nonnegative

$$(1-\lambda_r(t)) \, \delta U_r/\delta C_{rk}(t) \leq \varphi_{rk}(t) + \nu_{rk}(t)$$

$y_{rj}(t)$ $\omega_{rj}(t) \geq \Sigma a_{rjk}\varphi_{rk}(t)$ k=1,2

$x_{rj}(t)$ $\omega_{rj}(t) \geq \Sigma b_{rjk}\varphi_{rk}(t)$ (III)'

$z_{rjs}(t)$ $\omega_{rj}(t) \geq \Sigma f^{r}_{rjsk}\varphi_{rk}(t)$

$C_{rk}^{+}(t), C_{rk}^{-}(t)$ $-d_{rk}^{+} \leq \nu_{rk}(t) \leq d_{rk}^{-}$

$U_r(C_{rk}^{-}(t))$ $-g_{rk}^{-}(t) \leq \lambda_r(t)$

Except insofar as both contribute to $C_{rk}(t)$ there is no necessary connection in (III) or (III)' between $z_{rjs}(t)$ and $z_{sjr}(t-1)$. More generally there are no explicit manifestations of intertemporal time preferences in (III) or (III)'. Nevertheless the form of the third constraints in (III) suggests developments in which $x_{rj}(t)$ and $z_{rjs}(t)$ would take on interpretations as relatively internal and non interest bearing and relatively external and interest bearing *net savings*, and $z_{rjs}(t-1)$ as among other things including real returns (if any) from relatively external *investments*. Corresponding developments of the objective of (III) and of the constraints of (III)' might then take on interpretations in relation to intertemporal consistency conditions in general, and in relation to savings and investment related time preference rates in particular. That in turn suggests development of (III) and (III)' to explicitly incorporate interpretations in relation to *conditional offers* between individuals. One such development - and one alternative to continuation of reciprocal and unconditional characteristics related gifts between individuals is the more explicitly commod-ity related process of *barter*.
I emphasize that, if considered in the context of preceding developments a process of barter, if chosen, would stem from the choices of those concerned. In what follows, only for special cases (of kinds already considered with reference to gifts in this section), is there compulsion of any kind, let alone compulsion to use barter processes in preference to processes of giving gifts and reciprocating with further gifts.

5. Exploration, discovery and barter

Given experience of gift based systems of exchange stemming from (I),(II) and (III), and thence knowledge of the types of endowments of individual s via an accepted offer of $z_{sjr}(t-1)$ and associated weights $f^{r}_{rjsk}(t)$, individual r might consider a *barter* arrangement as follows: Individual r might simultaneously offer specified amounts $z_{rjs}^{**}(t)$ of his/her own endowment of commodities $j\epsilon J_{1rs}$ to individual s in exchange for specified amounts $z_{sjr}^{**}(t)$ of commodities $j\epsilon J_{2rs}$ from individual s, where J_{1rs}, J_{2rs} are external exchange related sets for individual r. If this offer is freely made by an individual r choosing relative to the relevant reference alternative (e.g. a no exchange case with $z_{rjs}^{**}(t)=0, z_{sjr}^{**}(t)=0$) then, in order to be preferred, conditions must obtain such that $U_r(C_{rk}(t)) \geq U_r(C_{rk}^{*}(t))$. The problem for

individual r with reference to a barter offer of commodities $z_{rjs}^{**}(t-1)>0$ $j\varepsilon J_{1rs}$ from r to s in exchange for commodities $z_{sjr}^{**}(t-1)>0$ $j\varepsilon J_{2rs}$ from s to r is shown in (IV). Analogously to the derivation of (III)' via (III), at an optimum (IV) leads to (IV)'. (Compare developments via (I),(II),(III) in Chapter 3.):

$$\text{Max } U_r(C_{rk}(t)) - \Sigma h_{rjs}^{+}(t)z_{rjs}^{+}(t) - \Sigma h_{rjs}^{-}(t)z_{rjs}^{-}(t)$$
$$-\Sigma h_{sjr}^{+}(t)z_{sjr}^{+}(t) - \Sigma h_{sjr}^{-}(t)z_{sjr}^{-}(t) - g_{rk}^{-}(t)U_r(C_{rk}^{-}(t))$$

$\varphi_{rk}(t)$	st $C_{rk}(t) = \Sigma a_{rjk}y_{rj}(t) + \Sigma b_{rjk}x_{rj}(t) + \Sigma e_{sjrk}^{r}(t)z_{sjr}(t) - \Sigma f_{rjsk}^{r}(t)z_{rjs}(t)$	j,k=1,2
$\omega_{rj}(t)$	$y_{rj}(t) + x_{rj}(t) + z_{rjs}(t) = x_{rj}(t-1) + z_{sjr}(t-1)$	
$\psi_{sjr}^{r}(t)$	$z_{sjr}^{r}(t) + z_{sjr}^{+}(t) - z_{sjr}^{-}(t) = z_{sjr}^{**}(t)$	$j\varepsilon J_{1rs}$ (IV)
$\psi_{rjs}^{r}(t)$	$z_{rjs}^{r}(t) + z_{rjs}^{+}(t) - z_{rjs}^{-}(t) = z_{rjs}^{**}(t)$	$j\varepsilon J_{2rs}$
$\lambda_r(t)$	$U_r(C_{rk}(t)) - U_r(C_{rk}^{-}(t)) \geq U_r(C_{rk}^{*}(t))$	
	All variables nonnegative	

$C_{rk}(t)$	$(1-\lambda_r(t))\, \delta U_r/\delta C_{rk}(t) \leq \varphi_{rk}(t)$	
$y_{rj}(t)$	$\omega_{rj}(t) \geq \Sigma a_{rjk}\varphi_{rk}(t)$	k=1,2
$x_{rj}(t)$	$\omega_{rj}(t) \geq \Sigma b_{rjk}\varphi_{rk}(t)$	(IV)'
$z_{sjr}(t)$	$\omega_{rj}(t) \leq \Sigma e_{sjrk}^{r}\varphi_{rk}(t) + \psi_{sjr}^{r}(t)$	
$z_{sjr}(t)$	$\omega_{rj}(t) \geq \Sigma f_{rjsk}^{r}(t)\varphi_{rk}(t)$	
$h_{sjr}^{+}(t), h_{sjr}^{-}(t)$	$-h_{sjr}^{+}(t) \leq \psi_{sjr}^{r}(t) \leq h_{sjr}^{-}(t)$	
$h_{rjs}^{+}(t), h_{rjs}^{-}(t)$	$-h_{rjs}^{+}(t) \leq \psi_{rjs}^{r}(t) \leq h_{rjs}^{-}(t)$	
$C_{rk}^{+}(t), C_{rk}^{-}(t)$	$-d_{rk}^{+} \leq v_{rk}(t) \leq d_{rk}^{-}$	
$U_r(C_{rk}^{-}(t))$	$-g_{rk}^{-}(t) \leq \lambda_r(t)$	

As in the gift giving case there are a number of possible types of solution to (IV). In particular:

• in the absence of compulsion and for appropriate values of $h_{sjr}^{+}(t), h_{sjr}^{-}(t), h_{rjs}^{+}(t), h_{rjs}^{-}(t)$ optimally $z_{sjr}(t) = z_{sjr}^{**}(t)$, $z_{rjs}(t) = z_{rjs}^{**}(t)$ and $U_r(C_{rk}(t)) = U_r(C_{rk}^{**}(t)) \geq U_r(C_{rk}^{*}(t))$ in (IV).

• using a specification entirely analogous to (IV), individual s may *either* accept or reject this offer (made one period previously) by setting $z_{sjr}(t) = z_{sjr}^{*}(t) = z_{sjr}^{**}(t-1)$ and $z_{rjs}(t) = z_{rjs}^{*}(t) = z_{rjs}^{**}(t-1)$ *or* respond by means of a solution to the optimization (V) together with (V)' :

$$\text{Max } U_s(C_{sk}(t)) - \Sigma h_{rjs}^{+}(t)z_{rjs}^{+}(t) - \Sigma h_{rjs}^{-}(t)z_{rjs}^{-}(t)$$
$$-\Sigma h_{sjr}^{+}(t)z_{sjr}^{+}(t) - \Sigma h_{sjr}^{-}(t)z_{sjr}^{-}(t) - g_{sk}^{-}(t)U_r(C_{rk}^{-}(t))$$

$\varphi_{sk}(t)$	st $C_{sk}(t) = \Sigma a_{sjk}y_{sj}(t) + \Sigma b_{sjk}x_{sj}(t) + \Sigma e_{rjsk}^{s}(t)z_{sjr}(t) - \Sigma f_{sjrk}^{s}(t)z_{rjs}(t)$	j,k=1,2
$\omega_{sj}(t)$	$y_{sj}(t) + x_{sj}(t) + z_{sjr}(t) = x_{sj}(t-1) + z_{rjs}(t-1)$	
$\psi_{rjs}^{s}(t)$	$z_{rjs}(t) + z_{rjs}^{+}(t) - z_{rjs}^{-}(t) = z_{rjs}^{*}(t)$	$j\varepsilon J_{1sr}$ (V)
$\psi_{sjr}^{s}(t)$	$z_{sjr}(t) + z_{sjr}^{+}(t) - z_{sjs}^{-}(t) = z_{sjr}^{*}(t)$	$j\varepsilon J_{2sr}$
$\lambda_s(t)$	$U_s(C_{sk}(t)) - U_s(C_{sk}^{-}(t)) \geq U_s(C_{sk}^{*}(t))$	
	All variables nonnegative	

$C_{sk}(t)$	$(1-\lambda_s(t))\, \delta U_s/\delta C_{sk}(t) \leq \varphi_{sk}(t)$	
$y_{sj}(t)$	$\omega_{sj}(t) \geq \Sigma a_{sjk}\varphi_{sk}(t)$	k=1,2
$x_{sj}(t)$	$\omega_{sj}(t) \geq \Sigma b_{sjk}\varphi_{sk}(t)$	(V)'
$z_{rjs}(t)$	$\omega_{sj}(t) \leq \Sigma e_{rjsk}^{s}\varphi_{sk}(t) + \psi_{rjs}^{s}(t)$	
$z_{rjs}(t)$	$\omega_{sj}(t) \geq \Sigma f_{sjrk}^{s}(t)\varphi_{sk}(t)$	
$h_{rjs}^{+}(t), h_{rjs}^{-}(t)$	$-h_{rjs}^{+}(t) \leq \psi_{rjs}^{s}(t) \leq h_{rjs}^{-}(t)$	
$h_{sjr}^{+}(t), h_{sjr}^{-}(t)$	$-h_{sjr}^{+}(t) \leq \psi_{sjr}^{s}(t) \leq h_{sjr}^{-}(t)$	
$C_{sk}^{+}(t), C_{sk}^{-}(t)$	$-d_{sk}^{+} \leq v_{sk}(t) \leq d_{sk}^{-}$	
$U_s(C_{sk}^{-}(t))$	$-g_{sk}^{-}(t) \leq \lambda_s(t)$	

If the reference preference for individual s is $U_s(C_{sk}*(t)$ in the absence of the barter offer from individual r then, given a barter offer $z_{rjs}**(t-1)=z_{rjs}*(t)$ and $z_{sjr}**(t-1)=z_{sjr}*(t)$, individual s might *disprefer* and reject it as if via $U_s(C_{sk}^-(t))>0$. If $U_s(C_{sk}**(t)) \geq U_s(C_{sk}*(t))$ individual 2 might *prefer* and accept the offer, *or,* using $U_s(C_{sk}**(t))$ as a new *lower* bound reference preference related state s in (V), individual s may offer both themselves and individual r a state potentially preferable to it. That offer in turn may be accepted or rejected or may elicit an alternative offer in turn from individual r, and so on.

There is no guarantee that such a process of freely made barter offers would stop. But, if a freely made offer was freely accepted, then the party making it would reveal themselves as preferring it to the alternative of stopping without making it, whereas the party accepting it would reveal themselves as preferring accepting it to the alternative of making a preferred offer relative to the other party. In that way both parties would have revealed a preference for that offer to the alternatives available to them when they made it. More subtly, if such an offer is rejected then the individual rejecting it would reveal themselves as *dispreferring* that offer to the alternative - and implicitly preferring the state associated with their reference preference - whereas the individual making the subsequently rejected offer would also be restored to a reference preference. So, by making an offer each party is potentially at least as well off as (and potentially better off than) they would have been had they not made such offers. *That is: they are potentially better off whether or not their offers are rejected.*

In every case implicit system related potentials $\psi^s_{sjr}(t)$, $\psi^s_{rjs}(t)$ are generated via (V),(V)'. Specifically, in (V)' $\psi^s_{sjr}(t)$, $\psi^s_{rjs}(t)$ respectively take on interpretations as relatively external acquisition and relinquishment related *potentials* for commodities j relative to individual s. Thus, from the fifth relations of (V)' and complementary slackness, if optimally $z_{sjr}(t)>0$ then $\omega_{sj}(t)=\Sigma f^s_{sjrk}\varphi_{rk}(t)-\psi^s_{sjr}(t)$. In that way a relatively internal potential is equated to the relatively external opportunity cost $-\psi^r_{rjs}(t)$ plus or minus a difference stemming from the preferences of individual s with reference to chosen

relinquishments (if any) of commodity j. (Here, via other conditions of (V)', the relatively internal potential $\omega_{sj}(t)$ also reflects potentially alternative uses of j for current consumption and/or for saving relative to individual r.) And, from the fourth conditions of (V)' and complementary slackness, if optimally $z_{rjs}(t)>0$, the relatively internal potential $\omega_{sj}(t)$ will equate to $\psi^s_{rjs}(t)$ and a quantity $\Sigma f^s_{rjsk}(t)$ stemming from the preferences of individual s with reference to chosen acquisitions (if any) of commodity j in period (t+1) from individual s at the margin.

Only exceptionally will acquisition related opportunity costs $\psi^s_{rjs}(t)$ equate to internal potentials $\omega_{sj}(t)$ for individual s. More generally, unless there are negative associations with such acquisitions, (i.e. unless $\Sigma e^s_{rjsk}\varphi_{rk}(t)$ is negative), from the fourth constraints of (V)' and complementary slackness, relatively internal potentials $\omega_{sj}(t)$ will be relatively *higher* than relatively external potentials $\psi^s_{rjs}(t)$.

In any case it would be exceptional for relatively external acquisition and relinquishment related potentials $\psi^r_{rjs}(t)$, $\psi^s_{sjr}(t)$ to be equal since:

  i) acquisitions and relinquishments would take place not just with reference to different individuals, but at different times since physically relinquishments from individual r would precede acquisitions by individual s, and;
  ii) in general individuals r and s would be *either* an acquirer *or* a disposer of any chosen exchange related commodity at the margin.

It follows that, even in the absence of transaction related inputs, in general exchange related *rents* will accrue to one or both parties to any barter related transaction.

Summarizing: given a barter related offer from individual r, individual s may accept it, reject it, or propose a still more preferred position via a further barter related process, until that process stops with an individual accepting or rejecting an offer. (I.e. not making a further offer.) But again there are other possibilities. In particular as if via (III) and (IV), both parties now have information concerning others' endowments as well as

knowledge of their willingness or otherwise to exchange, and, via $\psi^r_{rjs}(t)$, $\psi^r_{sjr}(t)$ and $\psi^s_{rjs}(t)$, $\psi^s_{sjr}(t)$, of implicit offer prices. With these ingredients they may turn to *trade*.

6. Purposive self contradiction and Trade

Developments via (IV),(V) in the previous section have shown that optimal solutions may yield a barter related offer of relatively less of commodities j, $j\varepsilon J_{1sr}$ for individual s and relatively more of commodities j, $j\varepsilon J_{2sr}$ relative to another.

But, via the fourth constraints of (IV)' and complementary slackness, if optimally $z_{rjs}(t)>0$ then $\omega_{rj}(t)= \Sigma e^s_{rjsk}\varphi_{rk}(t)+\psi^s_{rjs}(t)$ and the terms $\Sigma e^s_{rjsk}\varphi_{rk}(t)$ lead to relatively higher potentials $\omega_{sj}(t)$ relative to self and, via $\omega_{sj}(t)-\Sigma e^s_{rjsk}\varphi_{rk}(t)=\psi^s_{rjs}(t)$, to relatively lower potentials relative to the system.

In conjunction offers $z_{sjr}(t)$, $z_{rjs}(t)$ and relatively external potentials $\psi^i_{sjr}(t)$, $\psi^i_{rjs}(t+1)$ i=r,s provide essential ingredients for trade. Specifically: given a barter related offer and associated potentials via (IV) and (IV)' from individual r, a second individual, s, might use potentials *previously determined relative to r* as relatively external rates of exchange (offer prices) and, either conditionally accept that offer, or select a preferred trade related offer $z_{rjs}(t)>0$ $j\varepsilon J_{1sr}$, $z_{sjr}(t)>0$ $j\varepsilon J_{2sr}$ via the budget constrained extension of (V) in (VI) below. That system maximizes a measure of preferences over characteristics subject to a trade related budget constraint interrelating exchanged commodities, given relatively external prices. Assuming, as before, that $U_s( )$ is concave, and associating the indicated dual variables with the constraints of (VI), the associated Kuhn-Tucker conditions are modified versions of (V)' as in (VI)':

$$\text{Max } U_s(C_{sk}(t))-\Sigma h_{sjr}{}^+(t)z_{sjr}{}^+(t)-\Sigma h_{sjr}{}^-(t)z_{sjr}{}^-(t)$$
$$-\Sigma h_{rjs}{}^+(t)z_{rjs}{}^+(t)-\Sigma h_{rjs}{}^-(t)z_{rjs}{}^-(t)-g_{sk}{}^-(t)U_s(C_{sk}{}^-(t))$$

	st	constraints of (V) *and*		(VI)
$\tau_s(t)$		$\Sigma\psi^r_{rjs}(t-1)z_{rjs}(t)\leq\Sigma\psi^r_{sjr}(t-1)z_{sjr}(t)$		

$C_{sk}(t)$	$(1-\lambda_s(t))\delta U_s/\delta C_{sk}(t) \leq\varphi_{sk}(t)$	
$y_{sj}(t)$	$\omega_{sj}(t) \geq\Sigma a_{sjk}\varphi_{sk}(t)$ k=1,2	
$x_{sj}(t)$	$\omega_{sj}(t) \geq\Sigma b_{sjk}\varphi_{sk}(t)$	(VI)'
$z_{sjs}(t)$	$\omega_{sj}(t) \leq \Sigma e^s_{sjrk}\varphi_{sk}(t)+\psi^s_{sjr}(t)-\tau_s(t)\psi^r_{sjr}(t-1)$	
$z_{rjs}(t)$	$\psi^s_{rjs}(t)\geq \Sigma f^s_{rjsk}(t)\varphi_{sk}(t)+\tau_s(t)\psi^r_{rjs}(t-1)$	
$h_{rjs}{}^+(t), h_{rjs}{}^-(t)$	$-h_{rjs}{}^+(t)\leq\psi^s_{rjs}(t)\leq h_{rjs}{}^-(t)$	
$h_{sjr}{}^+(t), h_{sjr}{}^-(t)$	$-h_{sjr}{}^+(t)\leq\psi^s_{sjr}(t)\leq h_{sjr}{}^-(t)$	
$C_{sk}{}^+(t), C_{sk}{}^-(t)$	$-d_{sk}{}^+\leq v_{sk}(t)\leq d_{sk}{}^-$	
$U_s(C_{sk}{}^-(t))$	$-g_{sk}{}^-(t)\leq\lambda_s(t)$	

As in the barter case that was analyzed in part via (V), there are a number of possible types of solution to (VI). Three classes of examples are:

- First: In the absence of compulsion in period t an individual s may *accept* an offer from r in period t as if selecting $z_{rjs}*(t)=z_{rjs}(t-1)$, $z_{sjr}{}^*(t) =z_{rjs}(t-1)$ via preemptively large weights on potential deviations from $z_{rjs}{}^*(t)$ and $z_{srj}{}^*(t)$ in (V) where $z_{rjs}(t-1)=z_{rjs}{}^{**}(t-1)$ and $z_{sjr}(t-1)=z_{sjr}{}^{**}(t-1)$ relative to individual r. In that way individual s may s *stop* a process of interaction as if via preemptively large

weights on potential deviations from $z_{sjr}{}^*(t)$ and $z_{rjs}{}^*(t+1)$.

- Second; an offer $z_{rjs}(t-1)=z_{rjs}{}^{**}(t-1)$ and $z_{sjr}(t-1)= z_{sjr}{}^{**}(t-1)$ relative to individual r may be *provisionally accepted* relative to an individual s via a similar process to that of the first case except now with nonpreemptive weights $h_{rjs}{}^+(t)$, $h_{rjs}{}^-(t)$,$h_{sjr}{}^+(t)$,$h_{sjr}{}^-(t)$ relative to individual s, in that way potentially leading for individual s to one or more conditions $z_{rjs}(t)\neq z_{rjs}{}^*(t)$ and $z_{sjr}(t)\neq z_{sjr}{}^*(t)$ and states $U_s(C_{sk}{}^{**}(t))$ relatively preferred or indifferent to a reference state $U_r(C_{rk}(t))$ itself

preferred or indifferent to an initial reference state $U_s(C_{sk}*(t))$.

- A refinement of the second class of cases gives a third class of possibilities in which acceptance by individual s of an offer $z_{rjs}(t-1)$, $z_{sjr}(t-1)$ from individual r would be conditional and lead to the on the proposal by individual s of a subsequent *trade* $z_{sjr}(t)$, $z_{rjs}(t)$ based not on the prices $\psi^r_{rjs}(t-1)$, $\psi^r_{sjr}(t-1)$ and implicit budget constraint associated with the offer $z_{rjs}(t-1)$, $z_{rjs}(t-1)$ from individual r to individual s in the previous period, but on prices stemming from that subsequent offer from s to r.

In the third class of cases the revised offer to individual r stemming from an optimal solution to (VI) would lead, via (VI)', to *revised* offer prices and to a (revised) budget constraint $\Sigma\psi^r_{rjs}(t-1)z_{rjs}(t)\leq\Sigma\psi^r_{sjr}(t-1)z_{sjr}(t)$ for individual r. As one class of special cases, if $\omega_{sj}(t)=\Sigma e^s_{sjrk}\varphi_{sk}(t)$ then $\psi^s_{rjs}(t)=\tau_s(t)\psi^r_{rjs}(t-1)$ If *that* offer was freely accepted by individual r then both r and s would be acting as if to respect *the same budget constraint*. In that case, too, one's offer is associated with a state *preferred* to their reference preference related state and the other's acceptance is at least *indifferent* to their reference level. If that offer was freely refused then individual r may take the prices of individual s and recompute a relatively preferred offer via systems wholly analogous to (VI) and (VI)'.

A crucially important point is that in general, even if individuals concerned *agree* on quantities, they will in general *disagree* on prices (and if they *agree* on prices in general they will *disagree* on quantities).

Underlining this: not only are the prices in question temporally different but, even if $\omega_{sj}(t)=\Sigma e^s_{sjk}\varphi_{sk}(t)$ and conditions obtain as if all relative prices stay the same, nevertheless if $\tau_{sk}(t)\neq1$ absolute prices are different in periods t,t-1. In any case, unless all relative prices are invariant between periods, the budget constraint of individual r may correspond to an inequality when that for individual s does not - in that way inviting the discovery of further opportunities for potentially mutually advantageous trade. [A potentially mutually advantageous budget inequality is one such that a responding individual potentially chooses to gain by operating *inside*

their budget when evaluating exchanges at the proposer's prices, in that way potentially offering that proposer a strict gain. If that offer is accepted both potentially gain. If it is not accepted the second individual may simply stop. Alternatively, the second individual may make an offer preferable to themselves and yet strictly inside *their* budget constraint as measured by the prices associated with the preceding offer by the first individual. In that way the second individual acts as if to prefer an offer of continued and potentially preferable offers of trade to no trade and/or to prefer acceptance of a previously available and potentially mutually advantageous opportunity to trade. Conversely; a budget constraint may become unequal in a relatively *disadvantageous* sense by effectively *compelling* a respondent to exchange at potentially disadvantageous prices relative to self (and advantageous prices relative to another/others). (Note here that, if prices rise with relative scarcity and fall with relative plenty, an offer of less relative to self raises prices relative to self and lowers them relative to another/others. Conversely, an offer of more relative to self lowers prices relative to self and increases them relative to another/others. In the context of trade an offer of net gain to another given that other's prices implies the opportunity for that other to achieve a net gain, whereas an offer of net gain given the offeror's prices may not promise an opportunity for net gain. Thus a crucial issue here is whether or not an individual is compelled to accept an offer from another be it by means of a price mechanism or by any other means. I have considered this kind of case in detail with contexts of potentially mutually annihilatory tatonnement adjustment processes and processes potentially determining competitive equilibria in Ryan 1992.)]

Three important remarks:

First: in general two individuals will not agree on an identical plan. Even if they do agree on quantities then relatively external prices and relatively internal potentials will generally be *different* relative to any given individual and so different as between any two individuals concerned at any stage.

Second; in general individuals engaged in processes of change may either gain or lose according to their own preferences and would only agree to undertake

less preferred exchanges instead of conditions of no (further) exchange under conditions of compulsion.

Third; this approach can be extended to further individuals by considering a sequence with proposer r_1 responder s_1, subsequent proposer r_2 and so on with proposers and responders potentially generating gains for themselves and each other until all stop. In this context stopping may occur either by a prospective proposer refusing to make (further) proposals or with a responder's agreement to therefore mutually agreed and for that reason mutually preferred exchanges. By contrast with tatonnement-like adjustment process in the standard competitive equilibrium literature in general such a sequence will not converge to equivalent solution parameters relative to any two individuals. [For example agreement to a proposer's quantities and prices where those prices are potentials on those previously proposed quantities in general will not be consistent with equivalence both of those preceding quantities to the responder's (subsequent) choice of quantities and the responder's (subsequent) choice of prices. As noted above, in general agreement on quantities will imply disagreement on prices and vice versa. (Informally, in general a mutually beneficial exchange may be such that one individual gains - e.g. altruistically - by losing and the other gains by gaining with reference to any transferred commodity, and thence gains a fortiori with reference to any exchange of commodities by means of a price mechanism.]

More subtly, a relatively external agent is not needed as "auctioneer" in any of the gift or exchange related processes considered in this chapter except in the sense that all agents are relatively external to each other. In fact the chapter started with a relatively external agent (an explorer) who discovered a second. But of course exploration and discovery here are relative states. More generally in a world of two individuals r and s either can start and either can finish a sequence of gifts, of reciprocated gifts, of barters or of trade. In that sense either or both may be relatively external to the other (i.e. determine quantities or prices relatively externally to the other).

In any case proposed offers $z_{rjs}(t)$, $z_{sjr}(t)$ will emerge from (VI) and quantities potentially equating to trade related prices $\psi^s_{rjs}(t), \psi^s_{sjr}(t)$ will emerge for individual r from (VI)'. In that way an offer and associated prices may in turn be put to individual r different from s who, using a system

analogous to (VI), may either accept that offer or put a different offer to individual s, and so on until an offer is accepted - or rejected and no further offer made in return.

[If an offer was refused individual s may be worse off via $U_s(C_{sk}(t))$ in his/her own estimation than had it been accepted leading to $U_s(C_{sk}**(t))$. Never-theless, because $U_s(C_{sk}(t)) \geq U_s(C_{sk}*(t))$, that refusal would leave individual s at least as well off in his/her own estimation as compared with the reference state in the absence of the offer $z_{rjs}(t-1)$, $z_{sjr}(t-1)$ from individual r. In these circumstances, and in the absence of compulsion, both parties will always be at least as well off as a consequence of a barter offer as they would have been had that barter offer not been made - *whether or not that barter offer was refused.*]

So, variously under conditions of reciprocated gifts and barter and of trade there are roles for relative contradiction and uncertainty, even for already discovered commodities and choices relative to given opportunity sets. But the emphasis in this chapter is on more inclusive classes of cases which relate to the choice of opportunity sets themselves. To illustrate this I provide a more extensive and structured treatment of the explorer example in the next Section.

7. Purposive contradiction, as if perfect prediction and exchange

Another way of considering processes of exchange is by starting with two individuals who, except for their different locations, are identical, as illustrated schematically in Figure 1:
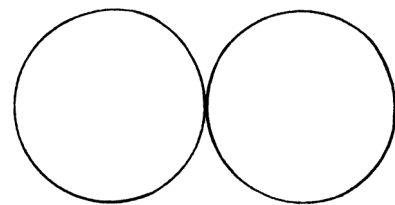

Figure 1

Assume next that Individual 1 starts a process of exchange according to the rule: offer less relative to self so as to potentiate more, via $z_{1js}^+(t)$ to
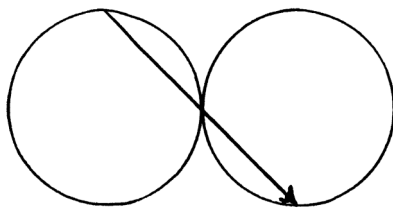
another (in this case individual s=2), as in Figure 2:

Figure 2

If individual 2 responds similarly and continues by dominating preferences relative to self to potentiate gains $x_{2kr}^{+}(t)$ relative to individual r=1, the position becomes:
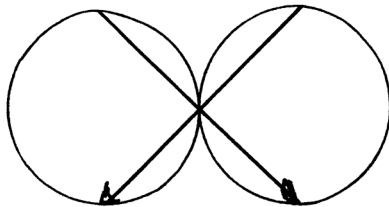

Figure 3

The sequence illustrated in Figures 1 through 3 constitutes a sequence of *reciprocal revealed preferences* (see also Ryan 1992) according to which individuals, starting with a first, choose less relative to a relatively abstract self as if thereby to reveal a (relatively non abstract) preference for more relative to another or others, who, in turn may choose, either to stop, or to choose less relative to self as if thereby to potentiate further opportunities for gain relative to another or others. Such a sequence does not require complete information. At this level of abstraction it would work, too, under conditions of incomplete information. Indeed in principle and, if outcomes of the two individuals' explorations were consistently favourable, such a sequence could continue to work to the mutual advantage of the individuals concerned, even under conditions of incomplete information until, for whatever reason, either chose to stop.

In principle individual 1 would not even need to know of the existence of individual 2 in order to make an initial offer. Individual 1 could make a conditional offer of the form: "If you are there and if you are willing to exchange I would be willing to exchange x with you." (Examples include NASA type broadcasts into outer space seeking extra-terrestrial life forms and, more prosaically, screen based computer trading systems.) In this way individual 2 both receives an offer of a previously unknown commodity and simultaneously discovers the existence of that commodity (and possibly also the existence of the agent offering it). If individual 2 responds with an offer, they reveal their existence to individual 1 and also confirm their knowledge of what was offered. At the same time individual 2 reveals a preference for what was offered. Individual 2 may then reciprocate with an offer concerning a commodity hitherto unknown to individual 1. In turn individual 1 may accept this offer from the, now more completely known individual 2 and, either stop, or continue with a (revealed preference for) a revised offer to individual 2, and so on.

By these means each individual may choose to become different relative to self and relative to a wider system as if to generate gains, including gains, of knowledge as well as potentially of commodities, relative to that wider system.

Such principles and processes appear both rational and feasible under conditions where individuals are initially differently endowed and have incomplete information relative to each other. But, if two individuals were initially determined wholly and only relative to themselves and identical with respect both to preferences and to endowments it might be asked, *how could* such individuals engage in exchange related processes of interaction except by processes generating relative indeterminacy and incompleteness relative to themselves. Indeed, *why would* any such individual seek to engage in exchange since apparently there would be no prospect of gains either of information or of commodities by doing so? In such circumstances apparently a no exchange outcome would be both as if perfectly predictive of, and as if perfectly predicted via, initial conditions of no trade. (In those circumstances Figure 3 appears open to interpretation as consistent with a revealed preference based global stability theorem, such as Theorem 7 Quirk and Saposnik 1968, p179 to the effect that, if equilibrium is unique in the pure trade case and, if there is no trade at equilibrium, then equilibrium is globally stable.)

This is not surprising. Under conditions of complete prior information, initially given and finally identical optimal allocations would be not only as if unanimously preferred to others, but as if inform-ationally identical to each other. Indeed, viewed in isolation, Figure 3 might appear to be a closed and cyclic system consistent with conditions of as if perfect prediction according to which what was is as if potentially perfectly predictive of what will be, not only relative to self, but relative to another/others. That is: as if conditions of change may be as if wholly consistent with conditions of no change, not just with reference to self, but with reference to another - and so with as if unanimous indifference between change and no change.

8. Purposive contradiction, indeterminacy and gains from exchange

If it is assumed that Figure 3 stems from initial assumptions to the effect that the two individuals concerned are mirror images of each other and in that sense the same relative to each other, it would not be surprising that, if both started identically and simultaeneously, they would reflect indifference (no exchange) relative to each other.

But, given the same initial assumptions, if the story is understood as sequential with one individual starting and the other responding in sequence (as earlier in Section 5) the accompanying process becomes essentially *asymmetric* in space and time. In that case, even if the parties to such interactions were initially identical, they would not finally be so.

Incidentally for economic as for other kinds of scientifically oriented interactions the fundamental purpose of offers here may be to *change* elements of an observed system - here economic agents - or to be changed by them, even if only to learn from them. In such contexts the emphasis may not be just on recognitions of Heisenberg-like effects according to which experimenters inevitably induce change relative to their experiments, but with the context of mutually advantageous exchanges, on potentials for deliberate generation and exploit-ations of such relatively unpredictable effects by individuals relative to themselves and each other.

Now reconsider a less restrictive classes of examples in which two non identical individuals potentially gain by exploring each other - even if only by gaining knowledge of each other's preferences and endowments, by considering a stylized example in which one initially has only apples and the other only oranges.

In that case an initial invitation to explore might be: Would you like some of this apple? (showing one) - then, accepting (knowledge of) apples, the second reciprocating with: Yes, would you like (some of) this orange (showing one).. In this way the range of preferences and information and opportunities increases for both and, insofar as these increases in ranges of opportunities lead to states which are mutually revealed preferred, may in turn lead to initially individually unattainable but collectively Pareto preferred outcomes.

Stressing this: if one starts with an offer, and for further improvements if the second reciprocates with an exchange related offer there are potentials for essentially altruistic and otherwise unattainable Pareto improvements. More subtly there are potentials for Pareto improvements relative to the initial conditions, whether or not such offers are accepted. This is because revealed preferred offers will either be accepted or rejected but, if rejected, then, relative to that offer, the offeror will not only gain information concerning the other's preferences, but retain commodities which would otherwise have been transferred.

Further, under conditions of incomplete information concerning endowments, Pareto preferred opportunities may be attainable if one or both individuals use part of their endowments to underwrite successful explorations by the other. (For instance leading to the discovery of previously unknown quantities and/or varieties of fruit.) More formally, two individuals may seek to potentiate gains relative to each other by discovering otherwise unattainable states in conjunction with a schematic representation as in Figure 4.

Let X_1, X_2 and Y_1, Y_2 respectively represent distinct endowments and distinct consumptions (e.g. of apples and oranges) for two individuals 1 and 2 and let a relatively central location 0 represent a region to be explored initially relatively unknown to both of them. Assume that

Individual 1 uses elements of X_1 to provision him/herself with Y_1 to explore and to return gains to individual 2, who accepts those gains as an (augmentation of an) initial endowment X_2 and uses this increment wholly or partly for consumption commodities Y_2 and exploration and returns gains to individual 1, who accepts those gains as an augmentation of his/her initial endowments and uses this increment wholly or partly for consumption commodities Y_1, and so on. In this way a directed path $X_1 \rightarrow Y_1 \rightarrow 0 \rightarrow X_2 \rightarrow Y_2 \rightarrow 0 \rightarrow X_1$. is determined, as in Figure 4:
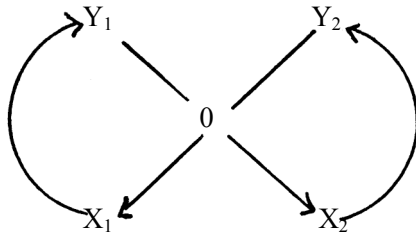

Figure 4

In a more narrowly physical context Figure 4 suggests as if perfectly self predictive electromagnetic explanations and interpretations according to which relatively negative is potentially as if perfectly predictive of relatively positive, and conversely. (For more on this see Ryan 1992.) But here the emphasis is on individualistically oriented economic gain. Whereas physical scientists emphasize *collective* gains to increases in their collective knowledge, here the primary emphasis is on *individual* gains via perturbations relative to elements of otherwise as if perfectly self predictive subsystems and contexts. In this way even the stylized facts associated with Figure 4 make it evident that

individually generated "errors" ΔX_i, ΔY_i relative to individuals' initially determinate endowments and consumptions X_i, Y_i - may become individually and so collectively preferred to such initially determinate states. In certain cases such "errors" may correspond to quantities $z_{rjs}^{+}(t)$, $z_{rjs}^{-}(t)$ $z_{rsr}^{+}(t)$, $z_{sjr}^{-}(t)$ in Sections 6 and 7 together with processes of as if perfectly predicted (i.e. as if completely informed) exchange between individuals. Figure 5 gives a graphical representation of such a case.
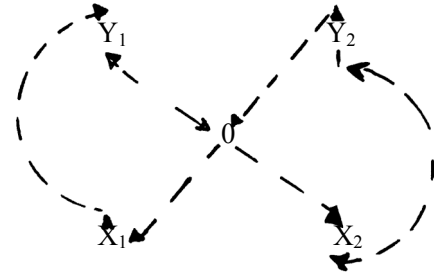

Figure 5

9. As if perfect prediction and processes of exchange

In the previous two sections I have considered as if perfect prediction with reference to graphically represented processes of change for individuals and for processes of exchange between them. Now consider this in a way that relates back to algebraic developments in Chapter 3 as well as in Section 6 of this chapter: If optimally $z_{rjs}^{-}(t)=z_{rjs}^{-}(t)=0$ and $z_{sjr}^{+}(t)=z_{sjr}^{-}(t)=0$ and $z_{rjs}^{*}(t)=z_{rjs}^{*}(t-1)$, $z_{sjr}^{*}(t)=z_{sjr}^{**}(t-1)$ in (VI), then that system (reproduced below) is potentially also consistent with an optimal solution both with $U_s(C_{sk}(t))=U_s(C_{sk}*(t))$ and with $z_{rjs}(t)= z_{rjs}^{**}(t-1)$ and $z_{sjr}(t) =z_{sjr}^{**}(t-1)$.

$$\text{Max } U_s(C_{sk}(t))-\Sigma h_{rjs}^{+}(t)z_{rjs}^{+}(t)-\Sigma h_{rjs}^{-}(t)z_{rjs}^{-}(t)$$
$$-\Sigma h_{sjr}^{+}(t)z_{sjr}^{+}(t)-\Sigma h_{sjr}^{-}(t)z_{sjr}^{-}(t)-g_{sk}^{-}(t)U_r(C_{rk}^{-}(t))$$

$\varphi_{sk}(t)$ st $C_{sk}(t)=\Sigma a_{sjk}y_{sj}(t) + \Sigma b_{sjk}x_{sj}(t) + \Sigma e_{rjsk}^{s}(t)z_{sjr}(t) - \Sigma f_{sjrk}^{s}(t)z_{rjs}(t)$ j,k=1,2

$\omega_{sj}(t)$ $y_{sj}(t) + x_{sj}(t) + z_{sjr}(t) = x_{sj}(t-1) + z_{rjs}(t-1)$

$\psi_{rjs}^{s}(t)$ $z_{rjs}(t) + z_{rjs}^{+}(t) - z_{rjs}^{-}(t) = z_{rjs}^{*}(t)$ $j\varepsilon J_{1sr}$ (VI)

$\psi_{sjr}^{s}(t)$ $z_{sjr}(t) + z_{sjr}^{+}(t) - z_{sjs}^{-}(t) = z_{sjr}^{*}(t)$ $j\varepsilon J_{2sr}$

$\tau_s(t)$ $\Sigma\psi_{rjs}^{r}(t-1)z_{rjs}(t)\leq\Sigma\psi_{sjr}^{r}(t-1)z_{sjr}(t)$

$\lambda_s(t)$ $U_s(C_{sk}(t)) -U_s(C_{sk}^{-}(t))\geq U_s(C_{sk}*(t))$

                All variables nonnegative

In order for a solution with $U_s(C_{sk}(t))=U_s(C_{sk}*(t))$ to be consistent with an optimum for (VI) the

budget conditions $\Sigma\psi_{rjs}^{r}(t-1)z_{rjs}(t)\leq\Sigma\psi_{sjr}^{r}(t-1)z_{sjr}(t)$ must obtain, in which case in effect individual s

agrees to potentials *previously decided* relative to individual r. In that sense individual s does not necessarily agree either with themselves or with individual r with reference to potentials, because in that case in general $\psi^s_{rjs}(t-1) \neq \psi^s_{rjs}(t) \neq \psi^r_{rjs}(t)$ and $\psi^s_{sjr}(t-1) \neq \psi^r_{sjr}(t) \neq \psi^s_{sjr}(t)$. That is, not only are these variables then quantitatively different, they are then also *temporally* different. (Because in the example just considered $z_{rjs}(t)=z_{rjs}^*(t)=z_{rjs}^{**}(t-1)$ and $z_{sjr}(t)=z_{sjr}^*(t)=z_{sjr}^{**}(t-1)$, similar temporal disagreements will also apply to quantities. In effect individual s agrees to a *previous* offer from individual r and, a fortiori, individual r potentially agrees not just to a *previous* acceptance from individual s, *but as if to a previous acceptance from individual r* (i.e. from themselves).

10. Conclusion

By using trade and explorer related examples it has been shown how individuals may systematically gain by means of principles and processes of contradiction, not just relative to their individual opportunity sets, but by cooperating in such a way as to generate initially unknown and potentially mutually advantageous possibilities relative to the collective opportunity set.

The chapter has shown how gains may be made variously by processes of giving, bartering and/or trading. It has been shown, too, how such processes could evolve from each other so that each type of exchange builds on knowledge and experience gained from the previous one. In this way a process of giving need presuppose no knowledge of the preferences or the endowments of the prospective recipient, but gifts, once made and accepted (or refused), will convey information concerning both the preferences and the endowments of the recipient - if only because gifts once accepted constitute new endowments to the recipient of kinds already known to the donor. If gifts are reciprocated by a process such as that in Section 4, both parties will gain some personal knowledge, not just concerning each others preferences and endowments, but concerning each other's willingness or otherwise to engage in processes of potentially mutually advantageous processes of exchange. Given such mutual awareness of others' endowments and their perceived willingness to exchange, the way is

then open to conditional barter based offers, as in Section 5. That kind of offer reveals not just the offering individual's willingness to engage in conditional exchanges, but the rates at which that individual is willing to exchange a commodity, or group of commodities, for another. If the other individual accepts such an offer or proposes an alternative, that, too, will convey information, not just about potentially offered commodities, but about that individual's preferred rate of exchange between offered and potentially received commodities. This knowledge in turn can provide essential background to the establishment and refinement of a relative price based trading relationship as in Section 6.

These various types of interaction, and associated processes of learning, may lead to evolution with learning on the basis of one kind of process leading to the introduction of another to another. But there is no reason to suppose that barter would wholly replace gifts and reciprocated gifts, or that trade would wholly replace barter. In practice, too all three types of exchange may coexist. In that way, at least, developments in this paper are consistent with everyday experience in which individuals may give to others (e.g. parents ot infants), barter with others (e.g. parents with dependent children concerning household chores) and trade with others - for example employers with employees concerning hours and rewards associated with employment.

I close with three observations. First: evolutions from processes of giving through processes of bartering to processes of trading would involve evolutions from processes associated with relatively personalized knowledge, exchange and learning, through relatively personalized barter related processes of exchange, to relatively depersonalized processes associated with price based processes of trade. Such an evolution has clear parallels with stages of personal and social development e.g. from infant, to child, to adult; or from a charity dependent individual to a member of a society in which dependency relations and interrelations are bound by barter-like customs (you help me in specific ways with my crops and I will help you in specific ways with yours or; I aid you in specific ways with your crops and you aid me in specific ways under specified types of disaster conditions). Given prior knowledge and

experience based on such gift giving and barter arrangements it is then a smaller step to establish and enforce trade based market relations and interrelations.

Secondly, at no stage in the preceding developments was it essential to the arguments used for individuals initiating or responding to processes of giving, bartering, or trading that in fact have Lancaster-like preferences or any particular specialization of them. In every case individuals may not only choose whether or not to initiate processes of giving, bartering and/or of trading - or whether or not to respond to such processes if initiated by another (including how they might express their preferences vis a vis others' actions), - but they may choose *how* they will do so and how they might represent their preferences for that purpose. At no stage have individual preference relations been assumed to be common knowledge. Further, even if personal preferences are in fact initially consistent with differentiable Lancaster like characteristics based preference relations, preceding developments have shown how, in general, measures on commodities may be modified in response to contact with and learning from others, and how in practice such modifications may include the possibility that new characteristics - and in that way new preferences - may be generated for individuals from contacts through gifts, barter and trade with others.

Thirdly, preceding developments have not explicitly comprehended overall quantity-based externalities. But the Lancaster-like formulations considered here could easily be extended so that individual characteristic were measured on others' provision (e.g. reflecting reputational or snob effects) and/or on aggregate provision, as well as personally owned, consumed and exchanged quantities of commodities. Extensions in those directions would lead naturally to the generation of taxes and subsidies and market related social organizations of more sophisticated kinds than those associated with a narrowly self interested and trade based economy. (For developments in those directions in a slightly different context see Ryan 1992.)

Summarizing: it has been demonstrated that each of the types of variously gift, barter and trade related processes which have been considered here can potentially yield Pareto improvements over states which would obtain in the absence of gifts, barter or trade for the individuals concerned. From that perspective it is not obvious that trade is in any sense preferable to mutual gifts or to barter, except that processes of trade are essentially anonymous ie less personalized. Indeed all of these types of exchange between persons might optimally coexist and be to the mutual advantage of distinct types of economic agent (e.g. respectively a parent in their home, a worker in a partnership and a worker in a firm) and in particular parts of society. A fortiori it is not obvious that processes of self interested and price based trade within an initially given opportunity set is in any sense preferable to a framework in which individuals may choose to expand those sets for themselves and others.

It is possible to argue, as I have done here, that gains may stem from incomplete information in general. More significantly it can be argued, as again I have done here, *how* such gains can flow from processes of exploration, discovery and exchange designed to capitalize on such incomplete information to the advantage of those both funding and those undertaking the explorations and discoveries involved.

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