

# CHAPTER 1

## MORE FOR LESS AND MORE FOR NOTHING

### 1. Introduction

The More for Less Paradox was first analyzed in Charnes and Klingman 1971 and in Swarcz 1971 with the context of the distribution (or transportation) problem of linear programming. In Ryan 1980 I extended this analysis to more for nothing cases and provided interpretations in relation to spatial competition and spatial monopoly and in Charnes et al 1987 we extended more for less (nothing) and degeneracy-decomposability results to more general linear programming cases.

More recent developments include those by Arsham 1992, Gupta and Puri 1995 and others who have investigated the more for less paradox in the distribution model using various forms of post optimality analysis (but without any reference to economic interpretations) and Ryan 1998 with reference to economies of scale and scope and to contestability for nonlinear cases.

In this chapter I build on that earlier work to obtain still more general more for less and more for nothing results and to show how these theorems can be applied to wide classes of economic problems, including individual and collective choice problems and exchange and trade related regulatory and bargaining problems.

Among results following directly from these theorems are demonstrations of the potential Pareto superiority of conditions of exchange over conditions wholly or partly prohibiting exchange between individuals and the potential Pareto superiority of regulated over non regulated optima.

### 2. Three general more for less results

#### THEOREM 1 *More for Less/Nothing*

If an optimal solution exists for (I) then:

$$\begin{aligned} \text{Min } f(x) &= z \geq z' = \text{Min } f(x) \\ \text{st } g(x) &= b \quad (I) \quad \text{st } g(x) - S = b \quad (Ia) \\ x &\geq 0 & x, S &\geq 0 \end{aligned}$$

#### PROOF

Any optimal solution to (I) is a feasible solution for (Ia), but not conversely. Thus any optimal solution to (I) is feasible but not necessarily optimal for (Ia). So there may exist optimal solutions to (Ia) such that  $z' < z$  or  $z' = z$  with  $S_i > 0$  at least one  $i$ .

#### THEOREM 2 *Less for More/Nothing*

If an optimal solution exists for (II) then:

$$\begin{aligned} \text{Max } f(x) &= z \leq z' = \text{Max } f(x) \\ \text{st } g(x) &= b \quad (II) \quad \text{st } g(x) + S = b \\ (IIa) & & x &\geq 0 & x, S &\geq 0 \end{aligned}$$

#### PROOF

Similar to Theorem 1.

#### THEOREM 3

With  $M$  arbitrarily large and if an optimal solution exists for (III) then:

$$\begin{aligned} \text{Max } f(x) - Ms^+ - Ms^- &= z \leq z' = \text{Max } f(x) - h^+(s^+) - h^-(s^-) \\ \text{st } g(x) + s^+ - s^- &= b \quad (III) \quad \text{st } g(x) + s^+ - s^- = b \\ (IIIa) & & x, s^+, s^- &\geq 0 & x, s^+, s^- &\geq 0 \end{aligned}$$

#### PROOF

An optimal solution to (III) is a feasible solution to (IIIa). But any optimal solution to (III) with all  $s^+, s^- = 0$  is a feasible but not necessarily an optimal solution to (IIIa). It follows that there may exist optimal solutions to (IIIa) such that  $z' > z$  or  $z' = z$  with  $s_i^+, s_i^- > 0$  some  $s_i^+, s_i^-$ . (Notice that if variables  $s_i^+, s_i^-$  appear in each of the constraints  $i=1,2,..m$  of (III) then there is *always* a feasible solution to that system.)

#### REMARKS

Theorems 1 and 2 first appeared in Ryan 1997. Theorem 3 includes Theorems 1 and 2 as special cases.

### 3. Some special cases of theorems 1 and 2.

#### 3.1 LINEAR PROGRAMMING CASES

Theorem 1 comprehends more for less (nothing) cases in the distribution problem viz:

**THEOREM 1A More for Less (Nothing) in the Distribution Problem**

If an optimal solution exists for (IV) then:

$$\begin{aligned} \text{Min } \sum c_{ij}x_{ij} &= z \geq z' = \text{Min} \sum c_j x_j \\ \text{st } \sum x_{ij} &= a_i \quad (\text{IV}) \quad \text{st } \sum x_{ij} - S_i = a_i \quad (\text{IVa}) \\ \sum x_{ij} &= b_j & \sum x_{ij} - S_j &= b_j \\ \sum a_i &= \sum b_j, \\ x_{ij} &\geq 0 & \sum (a_i + S_i) &= \sum (b_j + S_j), \quad x_j, S_i, S_j \geq 0 \end{aligned}$$

**PROOF**

As for Theorem 1.

As another class of special cases Theorems 1 and 2 comprehend more for less and more for nothing cases in linear programming. Specifically, with both a linear objective and linear constraints Theorem 1 becomes:

**THEOREM 1B More for Less/Nothing in Linear Programming**

If an optimal solution exists for (V) then:

$$\begin{aligned} \text{Min } \sum c_j x_j &= z \geq z' = \text{Min} \sum c_j x \\ \text{st } \sum a_{ij} x_j &= b_i \quad (\text{V}) \quad \text{st } \sum a_{ij} x_j - S_i = b_i \quad (\text{Va}) \\ x_j &\geq 0 & x_j, S_i &\geq 0 \end{aligned}$$

**PROOF**

As for theorem 1.

This is a simplified variant of the result in Charnes et al 1987. A similar less for more (nothing) result follows from a linear specialization of Theorem 2.

**3.2 Goal programming cases**

If  $f(x)$ ,  $g(x)$  are linear and  $h^+(s^+) =_{\text{def}} h^+ s^+$ ,  $h^-(s^-) =_{\text{def}} h^- s^-$  then (III) and (IIIa) respectively take on the form of preemptive and nonpreemptive goal programming specifications for the linear case. (See Charnes and Cooper 1961.) If  $f(x)$ ,  $g(x)$  are nonlinear and  $h^+(s^+)$ ,  $h^-(s^-)$  are nonlinear (III),(IIIa) take on interpretations as correspondingly nonlinear extensions of goal programming formulations of that more standard linear type.

**3.3. Nonlinear programming cases**

With  $f(x)$  concave and  $g(x)$  convex,  $h^-(s^-)$  arbitrarily large and  $h^+(s^+) =_{\text{def}} 0$ , (IIIa) becomes isomorphic with (IIa) and takes the form of a concave program of the standard maximization type. Similarly, with  $f(x)$  concave and  $g(x)$  convex,  $h^-(s^-) =_{\text{def}} 0$ ,  $h^+(s^+)$  arbitrarily large and  $f(x) =_{\text{def}} -f(x)$ , (IIIa) becomes isomorphic with (Ia)

and takes on the form of a concave program of the standard minimization type. So, with these interpretations (IIIa) includes both standard concave programming maximization and concave programming minimization cases as special cases of a more general nonlinear goal programming formulation.

**3.4 Neoclassical constrained maximization cases**

With  $f(x)$  concave and  $g(x)$  convex, program (I) becomes isomorphic with the specifications of budget constrained and cost constrained optimization models for individuals and firms in standard neoclassical microeconomic analyses.

**REMARK**

Theorem 1 can be seen as interrelating standard neoclassical and standard nonlinear programming formulations of minimizing constrained choice models, and Theorem 2 as formally interrelating standard neoclassical and standard nonlinear programming formulations of constrained choice maximizing models. With that context Theorem 3 can be seen as potentially interrelating neoclassical cases (via (III)) and nonlinear programming cases (via (IIIa)) in general.

**3.5 More general cases**

While Theorems 1,2 and 3 comprehend linear programming, nonlinear programming and neoclassical optimisation cases as three classes of special cases, they include other cases too. In particular there is no requirement of continuity or of differentiability for  $f(x)$ ,  $g(x)$ ,  $h^+(s^+)$ ,  $h^-(s^-)$ . Nor is there any requirement of connectedness for constraint sets (opportunity sets) in programs (III),(IIIa). The only prerequisite of these theorems is that of feasibility. This requirement is very weak: in the context of the modelling of economic decisions and associated phenomena it might be interpreted loosely as an assumption of *empirical plausibility*. (Such an assumption is arguably an essential requirement for *any* theory intended to model and predict empirical phenomena. In that sense arguably the prerequisite of feasibility in the preceding theorems corresponds more to a *preemptive objective* of the formulation of these theorems than to a restrictive constraint on their applicability.)

## 4. Some economic applications and interpretations of Theorems 1 and 2

### 4.1 The distribution model

Charnes and Klingman's verbal statement of their more for less theorem, (which is equivalent in its effect to Theorem 1A above) indicates one class of examples, viz:

Given a non degenerate optimal solution to (IV) it is possible to ship more total product at less total cost while shipping at least as much from each origin and to each destination if and only if  $R_i + K_j < 0$  for some non basic route  $ij$ . (Charnes and Klingman 1971.)

[ $R_i, K_j$  are the dual variables associated with (IV).]

Other instances include labour market related examples where supplies  $a_i$  and demands  $b_j$  in (IV), (IVa) refer to availabilities and requirements for various kinds of skilled workers and costs  $c_{ij}$  refer to retraining costs. In that instance the more for less paradox takes on the interpretation that in certain circumstances more workers may be retrained while overall retraining costs are reduced (resp unchanged). For more on this type of example see Ryan 1997.

### 4.2 Linear programming cases

An example here is the well known diet problem. With that context (V) takes on the interpretation of the minimization of the cost of meeting specified minimum dietary requirements  $b_i$  where  $a_{ij}$  are unit outputs of diet characteristics per unit input of food  $j$ . The more for less (nothing) possibility presented by program (Va) in Theorem 1B then states that in certain circumstances it may be cheaper (as cheap) to provide a diet *exceeding* minimal daily requirements as to provide a diet exactly meeting those requirements. (For more on this case see Charnes, Duffuaa, Ryan 1987.)

### 4.3 Nonlinear programming cases

Examples here include those in which  $f(x)$  corresponds to an individual preference relation and  $b$  to endowments to that individual, or in which  $f(x)$  corresponds to a measure of corporate performance (e.g. profit or sales revenue) and  $b$  refers to constraints on available inputs.

In the nonlinear programming case the scarcity (or otherwise) of endowments of inputs  $b_i$  is

endogenous. In particular there is no implicit requirement that all available resources be used in every period. For example, due to seasonality of productivity and/or of demand, a farmer would in general *not* use all his/her time and resources for farm related purposes in every period. Or, a firm (such as an electricity generator) subject to variable demands on its capacity over time, would in general *not* optimally use all of its available capacity in every time period.

### 4.4 Neoclassical economic cases

Here problems are of the form (see for example Samuelson 1948):

$$\begin{array}{ll} \text{Max } f(x) & \\ \text{st } g(x)=b & \text{(VI)} \\ x \geq 0 & \end{array}$$

Examples are those in which  $f(x)$  represents an individual preference relation and  $g(x)=b$  an individual budget constraint, or in which  $f(x)$  represents a firm's production function and  $g(x)=b$  that firm's cost equation. In distinction from the nonlinear programming formulations in subsection 4.3 such formulations implicitly assume given scarcity and given and efficient production and consumption processes too. That is: in (VI) there is neither endogenous choice of optimal production opportunity sets by producers, nor of consumption opportunity sets by consumers.

Neoclassical models of type (VI) implicitly assume nonsatiety and optimally determined conditions of factor scarcity and associated factor opportunity costs. They typically take the form of partial equilibrium models with exogenous prices together with implicit assumptions of full utilization of resources and/or of budget or cost constraints. In both production and consumption contexts such assumptions are typically associated with an assumption that firms and individuals will optimally accommodate themselves to stationary state conditions in "the long run". This is despite the fact that in reality supply and demand for many commodities including electricity and gas, for agricultural crops and for vacation related services, are cyclical so that non full utilization of these and other types of production capacity can be optimal at certain periods even in the long run.

#### 4.5 Ambiguous cases

A nonlinear extension of the distribution model provides an example here. Consider the nonlinear distribution model (VII) below:

$$\begin{aligned}
 & \text{Max } z_1 = f_1(x_1) + f_2(x_2) \\
 & - c_{11}x_{11} - c_{12}x_{12} - c_{21}x_{21} - c_{22}x_{22} - c_1\Delta a_1 - c_2\Delta a_2 \\
 \text{st} \quad & x_{11} + x_{21} = x_1 \\
 & x_{12} + x_{22} = x_2 \quad \text{(VII)} \\
 & x_{11} + x_{12} = a_1 + \Delta a_1 \\
 & x_{21} + x_{22} = a_2 + \Delta a_2 \\
 & \sum x_j = \sum a_i + \Delta a_i, \quad x_{ij} \geq 0, x_{ij}, x_{ij} \geq 0
 \end{aligned}$$

Clearly a feasible solution exists to (VII). (Consider  $x_1 = x_{11} = a_1$ ,  $x_2 = x_{22} = a_2$ .) Further, if  $c_1$  and  $c_2$  are arbitrarily large (VII) becomes a special case of (III), whereas if  $c_1$  and  $c_2$  are *not* arbitrarily large, then (VII) becomes a special case of (IIIa) and the more for less (nothing) Theorem 2 applies. To see this, assume that the  $f(x)$  in (VII) refer to demand relations at markets  $j$ ,  $a_i + \Delta a_i$  refer to supplies at factories  $i$ ,  $c_{ij}$  refer to potential unit costs of shipments from factories  $i$  to markets  $j$  and  $c_i$  refer to incremental costs of production at factory  $i$ . Assume finally that initially  $c_i$  are arbitrarily large. Then in general the optimal shipping pattern will *either* be to ship commodities exclusively to local destinations *or* the optimal path will be associated with a “cross country” shipment.

In a two origin, two destination example, unless cross-country shipments are prohibitively expensive, an optimum will generally correspond to a pattern with two locally positive shipments and one cross-country shipment. (Exceptionally it may be optimal *not* to ship to a local market, and/or not to make any cross-country shipments. But in every case, due to the logical requirement that shipments be from relatively lower cost to relatively higher cost areas an optimum will *never* connect both markets to both factories.)

Now consider a case in which initially arbitrarily large costs  $c_i$  of incremental outputs  $\Delta a_i$  are reduced. If they are reduced sufficiently, more sales may be made. That is: more total output may be shipped in such a way that marginal returns equals or exceeds the marginal shipment costs of the additional quantities shipped. As a class of special more for less (nothing) cases circumstances may be such that additional shipments actually *reduce* total costs by replacing cross country shipments with relatively lower

cost local shipments. (For more on this see Ryan 1997 on the MFL/MFN paradox in relation to economies of scale and scope.)

#### 5. Further economic interpretations of Theorem 3

##### 5.1 An exchange related example

##### THEOREM 3A Weak Preference for Gifts or Barter

If a feasible solution exists to (VIII), then:

$$\begin{aligned}
 z_1 = \text{Max} \quad & \theta_1 U_1(x_{11}, x_{12}, x_{21}, x_{22}) + \\
 & \theta_2 U_2(x_{21}, x_{22}, x_{11}, x_{12}) - M \sum x_j^{ri} - M \sum x_j^{ir} \\
 \text{st} \quad & x_{ij} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad i \neq r \quad \text{(VIII)} \\
 & x_{ij}, x_j^{ri}, x_j^{ir} \geq 0 \\
 & \leq
 \end{aligned}$$

$$\begin{aligned}
 z_1' = \text{Max} \quad & \theta_1 U_1(x_{11}, x_{12}, x_{21}, x_{22}) + \\
 & \theta_2 U_2(x_{21}, x_{22}, x_{11}, x_{12}) - \sum c_j^{ri} x_j^{ri} - \sum c_j^{ir} x_j^{ir} \\
 \text{st} \quad & x_{ij} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad i \neq r \quad \text{(VIIIa)} \\
 & x_{ij}, x_j^{ri}, x_j^{ir} \geq 0
 \end{aligned}$$

##### PROOF

Analogous to that for Theorem 3.

Now interpret (VIII),(VIIIa) as referring to a two individual, two commodity case in which  $\theta_i$  are system parameters,  $U_i(x_{ij})$  represent preferences of individuals  $i=1,2$  over their own and the other's endowments of two commodities  $j=1,2$ ,  $x_{ij}^*$  are initial endowments of commodities  $j$  to individuals  $i$  and  $x_j^{ri}$ ,  $x_j^{ir}$ ,  $i=1,2$  are shipments of commodities  $j$  from individual  $r$  to individual  $i$ . Then (VIII),(VIIIa) determine preference maximizing allocations to individuals  $r,i$  such that gifts and/or exchanges are never preferred (via VIII) and potentially preferred (via (VIIIa)) to exclusive consumption of their own endowments by the individuals concerned.

With these interpretations, if  $c_j^{ri}, c_j^{ir}$  are preemptively large, (VIII) and (VIIIa) become equivalent. Conversely, if some or all  $c_j^{ri}, c_j^{ir}$  are non preemptive, there is potential for individual and/or mutual gains stemming from solutions with  $x_j^{ri} > 0, x_j^{ir} > 0$  at an optimum to (VIIIa) vis a vis  $x_j^{ri} = x_j^{ir} = 0$  in (VIII). Such less for more solutions via (VIIIa) are open to interpretation as reflecting *altruistic* gift related behaviours (in which individuals act as if preferring less to self in order to potentiate more for another) and, if reciprocated, to mutually beneficially oriented processes of exchange. This result is very general. No assumptions have been made in Theorems 3 or Theorem 3A concerning continuity or differen-

tiability for  $U_i(x_{ij})$ , or connectedness or otherwise of the constraint sets. (Indeed disjoint “no gift/exchange” objectives and constraints are potentially consistent with optimal solutions to both problems.)

### 5.2. Pareto improvements and gains from exchange

Assume that  $U_1^*(\cdot), U_2^*(\cdot)$  are consistent with optimal solutions to (VIII) with  $x_j^{rs}=0$  all  $r,s$ . Then a refinement of the conditions of Theorem 3A applies as follows:

#### THEOREM 3B Weak Pareto Preference for Gifts or Barter

If an optimal solution exists to (VIII) with  $U_1(\cdot)=U_1^*(\cdot), U_2(\cdot)=U_2^*(\cdot)$  then:

$$z_1 = \text{Max } \theta_1 U_1(x_{11}, x_{12}, x_{21}, x_{22}) + \theta_2 U_2(x_{21}, x_{22}, x_{11}, x_{12}) - M \sum x_j^{ri} - M \sum x_j^{ir}$$

$$\text{st } x_{ij} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad i \neq r \quad \text{(VIII)}$$

$$x_{ij}, x_j^{ri}, x_j^{ir} \geq 0$$

$$\leq$$

$$z_1' = \text{Max } \theta_1 U_1(x_{11}, x_{12}, x_{21}, x_{22}) + \theta_2 U_2(x_{21}, x_{22}, x_{11}, x_{12}) - \sum c_j^{ri} x_j^{ri} - \sum c_j^{ir} x_j^{ir}$$

$$\text{st } x_{ij} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad i \neq r \quad \text{(VIIIwp)}$$

$$x_{ij}, x_j^{ri}, x_j^{ir} \geq 0$$

$$U_1(\cdot) \geq U_1^*(\cdot), U_2(\cdot) \geq U_2^*(\cdot)$$

#### PROOF

Analogous to that for Theorem 3.

#### COROLLARY

Since zero gift and zero exchange solutions are feasible for (VIII) and (VIIIwp) given the (weak) conditions of Theorem 3B, in general gifts or exchanges will be (weakly) Pareto preferable to no gifts or exchanges.

### 5.3 Gains from exchange and regulation

#### THEOREM 3C Weakly preferred regulation

If an optimal solution exists to (IX) with

$U_1(\cdot)=U_1^*(\cdot), U_2(\cdot)=U_2^*(\cdot)$ , then:

$$z_1 = \text{Max } \theta_1 U_1(x_{11}, x_{12}, x_{21}, x_{22}) + \theta_2 U_2(x_{21}, x_{22}, x_{11}, x_{12}) - \sum M x_{ij}^+ - \sum M x_{ij}^- - M \sum x_j^{ri} - M \sum x_j^{ir}$$

$$\text{st } x_{ij} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad \text{(IX)}$$

$$x_{ij} + x_{ij}^+ - x_{ij}^- = x_{ij}^{**}$$

$$x_{ij}, x_j^{ri}, x_j^{ir}, x_{ij}^+, x_{ij}^- \geq 0 \quad i \neq r$$

$$\leq$$

$$z_1' = \text{Max } \theta_1 U_1(x_{11}, x_{12}, x_{21}, x_{22}) + \theta_2 U_2(x_{21}, x_{22}, x_{11}, x_{12}) - \sum c_{ij}^+ x_{ij}^+ - \sum c_{ij}^- x_{ij}^- - \sum c_j^{ri} x_j^{ri} - \sum c_j^{ir} x_j^{ir}$$

$$\text{st } x_{ij} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad \text{(IXa)}$$

$$x_{ij} + x_{ij}^+ - x_{ij}^- = x_{ij}^{**}$$

$$x_{ij}, x_j^{ri}, x_j^{ir}, x_{ij}^+, x_{ij}^- \geq 0 \quad i \neq r$$

$$U_1(\cdot) \geq U_1^*(\cdot), U_2(\cdot) \geq U_2^*(\cdot)$$

#### PROOF

Analogous to Theorem 3.

#### REMARK

(IXa) allows regulation in markets  $i$  via goal related penalties  $c_{ij}^+, c_{ij}^-$  and/or via potential entries  $x_j^{ri}$  of commodities from markets  $r$ , where  $i \neq r$ .

#### COROLLARIES

- With  $c_{ij}^+, c_{ij}^-$  interpreted as goal related penalties and  $x_j^{ri}$  as potential entries from spatially distinct markets  $r$ , Theorem 3C implies that *regulated* exchanges between individuals will be at least as preferable as preemptively prohibited exchanges.
- Theorem 3C contains Theorem 3B as a class of no externality special cases for which conditions obtain as if  $x_{ij}^* = x_{ij}^{**}$  and specific distinctions between consumption related goals and exchange related goals are redundant. Conversely Theorem 3C implies that only exceptionally will a socially preferred state be such that there will be no explicit regulation and no gift, theft or other exchange related externalities.

### 5.4 More general exchange related examples

Consider a result which extends Theorem 3C to comprehend production by interpreting  $x_{ij}^*$  as initial endowments of resources  $j$  (e.g. of time and of raw materials) to individuals  $i$ , which are then used to produce quantities  $y_{ik}$  of outputs  $k$  by means of production relations  $g_k(x_{ijk})$ :

#### THEOREM 3D

If an optimal solution exists to (X) with  $U_1(\cdot)=U_1^*(\cdot), U_2(\cdot)=U_2^*(\cdot)$ , then:

$$\text{Max } \theta_1 U_1(y_{1k}, y_{2k}, x_{1j}, x_{2j}) + \theta_2 U_2(y_{1k}, y_{2k}, x_{1j}, x_{2j}) - M \sum x_j^{rs} - M \sum x_j^{ir}$$

$$\text{st } \sum x_{ijk} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad i \neq r$$

$$y_{1k} + y_{2k} \leq g_k(x_{ijk}) \quad \text{(X)}$$

$$y_{ik}, x_{ij}, x_j^{ri}, x_j^{ir} \geq 0$$

$$= z_1 \leq z_1'$$

$$\begin{aligned}
& \text{Max } \theta_1 U_1(y_{1k}, y_{2k}, x_{1j}, x_{2j}) + \\
& \theta_2 U_2(y_{1k}, y_{2k}, x_{1j}, x_{2j}) - \sum c_j^{ri} x_j^{ri} - \sum c_j^{ir} x_j^{ir} \\
& \text{st } \quad \sum x_{ijk} + x_j^{ri} - x_j^{ir} = x_{ij}^* \quad i \neq r \\
& \quad y_{1k} + y_{2k} \leq g_k(x_{ijk}) \quad (\text{Xa}) \\
& \quad y_{ik}, x_{ij}, x_j^{ri}, x_j^{ir} \geq 0 \\
& \quad U_1(\cdot) \geq U_1^*(\cdot), U_2(\cdot) \geq U_2^*(\cdot)
\end{aligned}$$

## PROOF

Analogous to that for Theorem 3.

## REMARKS

- Theorems 3C and 3D can be related variously to gifts or exchanges between nations as well as between individuals  $i, r$  and/or to economies of scale and of scope relating to exchanges stemming respectively from increased production and from previously unattainable connections between markets.
- Neither in Theorem 3C nor in Theorem 3D need exchanges be optimally "balanced". In particular optimal solutions may correspond to gifts from  $i$  to  $r$  without reciprocation from  $r$  to  $i$ . It follows that conditions  $x_j^{ir} > 0$ ,  $i \neq r$  some  $j$  may optimally obtain simultaneously with  $x_j^{ri} = 0$  all  $j$ .
- The exchange related systems (IX), (IXa) and (X), (Xa) can be used to model conditions of *duress* (via preemptive magnitudes  $M$  in (IX), (X)), as well as goal related extensions of (Xa) with potentially socially determined penalties or inducements (taxes or subsidies)  $c_{ij}^+$ ,  $c_{ij}^-$ .

## 6. Conclusion

In this chapter I have focused on the establishment of more for less and more for nothing results and exchange related interpretations of them. Subsequent chapters will use these and other exchange and regulation related results to focus more narrowly on specific types of applications. These further applications might include applications to trade theory and comparative advantage and applications yielding new kinds of potentially Pareto improving yet regulated processes of exchange between individuals.

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