CHAPTER 11

THE DISTRIBUTION PROBLEM, THE MORE FOR LESS (NOTHING) PARADOX AND ECONOMIES OF SCALE AND SCOPE

1. Introduction

If an optimal solution to the distribution problem exhibits the more for less or more for nothing paradox, any subsequent solution which fully exploits those conditions is necessarily degenerate and decomposable. This result, which was first conjectured in Ryan 1980 and proved formally in Charnes, Duffuaa Ryan 1980, has implications both for economies of scale and for economies of scope. It is the purpose of this paper to show how these two types of interpretations might arise and, by using homogeneous product and heterogeneous labour market related examples, how they might usefully be exploited.

The chapter is organised as follows. The next section presents generally applicable more for less and more for nothing results before specializing them to the distribution problem. Then in Sections 3 and 4 distribution structures are related to issues and interpretations in relation to degeneracy and decomposability and in relation to variously spatially competitive and non competitive markets. In Section 5 I introduce new goal programme related definitions of economies of scale and scope. Finally in Sections 6 and 7 I turn to more specific examples using these definitions, the first with reference to a homogeneous commodity and interregional exchange and the second with reference to redundancy and retraining costs and labour markets.

2. Two general more for less (nothing) results

THEOREM 1

If a feasible solution exists for programme (I) then:

$$\begin{aligned} &\text{Max } \Sigma f(x_j) - M\Sigma x_j^+ - M\Sigma x_j^- = z \le z' = \\ &\qquad \text{st } \Sigma a_{ij}x_j + x_j^+ - x_j^- = b_i \qquad \text{(I)} \\ &\qquad\qquad x_j, x_j^+, x_j^- \ge 0 \\ &\qquad\qquad\quad \le \\ &\text{Max } \Sigma f(x_j) - \Sigma c_j^+ x_j^+ - \Sigma c_j^- x_j^- \\ &\qquad \text{st constraints of (I)} \qquad \text{(Ia)} \end{aligned}$$

PROOF

Any feasible solution to (I) is a feasible solution to (Ia), but not conversely. Thus any optimal solution to (I) is a feasible but not necessarily an optimal solution to (Ia). It follows that there may exist optimal solutions to (Ia) such that z'>z or z'=z with $x_j^+, x_j^- > 0$ some x_j^+, x_j^-.

For example it need not always be optimal for a preference or profit maximizing farmer to choose crop production plans in such a way that they exactly exhaust each and all of his/her resources of land, machinery and time. (This is consistent with the fact that crop production involves seasonally intensive activities.)

One class of special cases are those in which both $f(x_j)$ are linear:

THEOREM 1*

If a feasible solution exists for programme (I*) then:

$$\begin{aligned} &\text{Max } \Sigma f_j x_j - M\Sigma x_j^+ \\ &\text{st } \Sigma a_{ij}x_j + x_j^+ = b_i \qquad \text{(I*)} \\ &\qquad x_j, x_j^+ \ge 0 \\ &\qquad\quad \le \\ &\text{Max } \Sigma f_j x_j \\ &\text{st constraints of (I*)} \qquad \text{(Ia*)} \end{aligned}$$

Analogous to Theorem 1 are a class of minimization cases as follows:

THEOREM 2

If a feasible solution exists for programme (II) then:

$$\begin{aligned} &\text{Min } \Sigma c(x_j) + M\Sigma x_j^+ + M\Sigma x_j^- \\ &\text{st } \Sigma a_{ij}x_j + x_j^+ - x_j^- = b_i \qquad \text{(II)} \\ &\qquad x_j, x_j^+, x_j^- \ge 0 \\ &\qquad\quad \ge \\ &\text{Min } \Sigma c(x_j) + \Sigma c_j^+ x_j^+ + \Sigma c_j^- x_j^- \\ &\text{st constraints of (II)} \qquad \text{(IIa)} \end{aligned}$$

PROOF

Any feasible solution to (II) is a feasible solution to (IIa), but not conversely. Thus any optimal solution to (II) is a feasible but not necessarily an optimal solution to (IIa).

It follows that there may exist optimal solutions to (IIa) such that z'>z or z'=z with x_j^+, x_j^- >0 some x_j^+, x_j^-. [Evidently, if $c(x_j) =_{def} -f(x_j)$, then theorems 1 and 2 are equivalent.]

Theorem 2 is the main result in Charnes, Duffuaa, Ryan 1987 (though the proof here is more succinct). With the context of the well known diet problem, in which x_j would be foods, b_i minimum dietary requirements and c_j unit costs, this theorem states the apparently paradoxical fact that in certain circumstances a diet *exceeding* minimum daily requirements of nutrients may be cheaper than one exactly meeting all of those requirements. (For more on this example and results concerning the degeneracy of more for less and more for nothing cases see Charnes, Duffuaa, Ryan 1987.)

Another class of special cases of Theorem 2 are those which conditionally correspond to the distribution problem, viz:

THEOREM 2*

If a feasible solution exists for programme (II)* then:

$$\text{Minimise } \sum_i\sum_j c_{ij}x_{ij} + \sum_i Mx_i^+ + \sum_i Mx_i^- + \sum_j My_j^+ + \sum_j My_j^-$$

$$\left.\begin{array}{l} \sum_j x_{ij} + x_i^+ - x_i^- = a_i \\ \sum_i x_{ij} + y_j^+ - y_j^- = b_j \\ \sum_i a_i = \sum_j b \\ x_{ij}, x_i^+, x_i^-, y_j^+, y_j^- \geq 0 \end{array}\right\} \quad \text{(II)*}$$

$$\geq$$

$$\text{Minimise } \sum_i\sum_j c_{ij}x_{ij} + \sum_i c_i^+x_i^+ + \sum_i c_i^-x_i^- + \sum_j d_j^+y_j^+ + \sum_j d_j^- y_j^-$$

st constraints of (II)*

PROOF

As for Theorem 2. (To exclude trivial cases c_{ij}>0 will be assumed.)

In the original form stated by Charnes and Klingman 1971 for more for less cases and by Ryan 1986 for more for nothing cases, the more for less (nothing) paradox in the distribution model was stated as follows. (The notation degenerate* recognizes that the requirement $\Sigma a_i = \Sigma b_j$ implies that any distribution problem in the standard form will be linearly dependent and thence degenerate with at most m+n-1 positive shipments in an optimal basis.):

> Given a non degenerate* optimal solution to the distribution problem with m+n-1 positive shipments it is possible to ship more total product at less (equal) total cost while shipping at least as much from each origin and to each destination if and only if $R_i+K_j<0$ (resp $R_i+K_j=0$)for some non basic route ij.

Clearly a feasible solution exists for (II) with $x_i^+, x_i^-, y_j^+, y_j^-$ =0 all i,j. Programme (II) is then equivalent to the distribution problem in its standard form (se Charnes and Cooper 1961, Shogan 1986).

Theorem 2A is a generalization of the more for less (nothing) paradox in the distribution model since it admits more for less and more for nothing cases with y_j^-, x_i^->0 some i,j as two classes of special cases. (Other cases would include those with y_j^+, x_i^+ optimally positive some j in (IIa).)

In Ryan 1980, I developed both linear and nonlinear applications and examples. I showed that if more for less or more for nothing conditions in the distribution model are fully exploited, a connected set of markets optimally decomposes into spatially disjoint sets of submarkets.

The general degeneracy-decomposability result was later proved formally in Charnes, Duffuaa and Ryan 1980. But it can be demonstrated in other ways too. Here I consider an approach in which the distribution problem is embedded within the larger goal programming structure implicit in (IIa). (Incidentally, while more work on the MFL/MFN paradox has subsequently been done by others, including Arshan 1992 and Gupta and Puri 1995, it focuses on partial post optimality analyses. It provides no market related economic interpret-ations. Nor does it use a goal programming approach.)

3. Duality, degeneracy decomposability and MFL/MFN

Associating dual variables R_i and K_j respectively with the origin and destination constraints of (IIa) its dual is:

$$\text{Maximize } \sum_i R_i a_i + \sum_j K_j b_j$$

$$\begin{aligned} R_i+K_j &\leq c_{ij} \qquad \text{(IIa)*'} \\ -c_i^- \leq R_i &\leq +c_i^+ \\ -d_j^- \leq K_j &\leq +d_j^+ \end{aligned}$$

If $c_i^-, c_i^+, d_i^-, d_i^+$ are sufficiently large and positive, optimal solutions to the potentially more for less (nothing) formulation (IIa)*' are equivalent to optimal solutions to the standard non more for less(nothing) formulation of the distribution problem. Conversely, if not sufficiently large and positive, $c_i^-, c_i^+, d_i^-, d_i^+$ may be of magnitudes such that, while (II)* is a feasible solution to (IIa)*, it is not optimal. In particular, if c_i^-, d_i^- are of appropriate magnitudes and if $x_i^- = y_j^- = \delta > 0$ for some non basic route ij in (IIa)*, an optimising MFL/MFN solution may be attained to (IIa)*' with $R_i=-c_i^-$, $K_j=d_i^-$ by complementary slackness. For a solution maximising the potential for MFL/MFN, δ will be set at its maximal level consistent with maintenance of the initial set of basic routes.

Degeneracy* follows immediately. A basic solution to (IIa)* would then have just m+n-2 positive shipments, there being m+n constraints with $x_i^-=y_j^->0$. Decomposability of such a solution follows from the fact that a basis non degenerate* in shipments x_{ij} for (IIa)* (minimally) spans that system so that, conversely, a degenerate* basis does not.

In the degenerate* case it is possible to set *two* distinct values R_i, K_j arbitrarily - e.g. $R_i=-c_i^-$, $K_j=d_i^-$. Nevertheless it is not necessary to set a pair of values R_i, K_j equal to relatively external magnitudes to obtain a MFL (MFN) result. All that is necessary are conditions consistent with $R_i+K_j \leq 0$ for some non basic route at an optimum. Various classes of special cases, including spatially competitive cases, will be considered in Section 4. First consider a numerical example.

4. A more for less (nothing) example

Consider an example in which supplies at two factories, demands at two markets and unit shipping costs are as indicated in Tableau 1.

	$K_1=4$	$K_2=10$	
$R_1=0$	4 $x_{11}=$ 10	10 $x_{12}=$ 10	$a_1=20$
$R_2=-7$	6 $x_{21}=$ 0	3 $x_{22}=$ 30	$a_2=30$
	$b_1=10$	$b_2=40$	

Tableau 1

$a_1 \rightarrow b_1$

$a_2 \rightarrow b_2$

Figure 1

Using the North West Corner Rule the initial basis is as in Tableau 1. Due to the degeneracy of (II)* at an optimum one dual variable can be selected arbitrarily. Setting $R_1=0$ the values of the other dual variables follow directly since, by complementary slackness, $R_i+K_j=c_{ij}$ for all basic routes ij. In this case this initial dual pair of solutions is feasible and thence optimal with a total shipping cost of 230.

Parenthetically, as I noted in Ryan 1980, for data organized routinely from top to bottom (North to South) and from left to right (West to East), a North West Corner Rule may be more efficient than other starting rules since naturally corresponding to the adjacencies inherent in a pre-existing pattern of shipments. (Try it with origins and destinations being Seattle and New York. A North West Rule starts with a shipment Seattle-Seattle, whereas a South West Rule would start with a cross country shipment New York-Seattle.)

The solution in Tableau 1 being nondegenerate* and $R_2+K_1=-3<0$ for the nonbasic route {2,1} exhibits the preconditions for MFL. (With $c_{22}=6$ and thence $R_2=-4$, $R_2+K_1=0$, they would be preconditions for MFN.)

Figure 1 will help to clarify how and why the more for less case arises in Tableau 1 by showing how a nondegenerate* basis with three positive shipments requires a "cross country" shipment, in this case $x_{12}=10$. Further, shipment costs in Tableau 1 are such that the sum of the unit costs of the two "local" shipments x_{11},x_{22} are less than the unit cost of the cross country shipment x_{12}. That is:

$$c_{11}-c_{12}-c_{22}= R_2+K_1=-3<0 \qquad (3.1)$$

It follows that:

$$\delta(c_{11}+c_{22})< \delta c_{12} \qquad (3.2)$$

That is if supply at origin 1 is increased by δ and demand at market 2 is increased by δ overall shipping cost can be reduced by correspondingly decreasing cross-country shipments by δ. Evidently the maximal feasible value of δ consistent with nondegeneracy in x_{12} in this case is $10-\varepsilon_{12}$ (with ε_{12} arbitrarily small) when overall transport costs are reduced by $3(10-\varepsilon_{12})$. If ε_{12}□0 then cross country shipments are reduced to zero and the initially connected pairs of factories and markets become disconnected - that is an initially connected basis in x_{ij} becomes degenerate* and decomposable.

Clearly more general more for less and more for nothing examples are available via Theorem 2. However, from the perspective of this paper the significant point is that in general costs may be reduced *both* by increasing the connectedness of markets - i.e. by increasing opportunities to generate economies of scope *and* by increasing the scale of operations of particular factories, and that, as here, optimizing tradeoffs may be attained between these two means of reducing overall costs.

The initial connectedness and subsequent disconnectedness of the pairs $\{O_1,D_1\}$ and $\{O_2, D_2\}$ respectively via an actual shipment $x_{12}>0$ and a potential shipment $\varepsilon_{12}>0$ immediately suggests interpretations in relation to potential competition and monopoly since under these conditions actual entry $x_{12}>0$ into market 2 from factory 1 become conditions of potential entry via $\varepsilon_{12}>0$ and non entry with $\varepsilon_{12}=0$. More subtly this example suggests interpretations in relation to economies of scale and scope, with economies of scope stemming from the connection of otherwise disconnected markets and economies of scale following from the expansion of the level of activity of one or more production plants. (In the example factory 2.)

5. The MFL(MFN) paradox and spatial competition

One condition of spatial competition is that if interregional transport costs are c_{ij} then for any connected pair of markets i,j origin prices p_i, and destination prices p^j are such that:

$$x_{ij}>0 \;\Rightarrow\; p^j-p_i=c_{ij} \qquad (3.3)$$

(Another and stronger condition would be that all markets be connected.)

Conditions (3.3) in turn suggest that, rather than starting with an arbitrary valuation $R_1=0$, that dual variable could be chosen as a *base price* $R_1=-c_1^-={}_{def}p_1$ thence generating prices consistent with conditions of spatial competition such that, for all basic routes:

$$x_{ij}>0 \;\Rightarrow\; p^j-p_i=R_i+K_j=c_{ij} \qquad (3.4)$$

and for all non basic routes:

$$x_{ij}>0 \;\Rightarrow\; p^j-p_i\leq R_i+K_j\leq c_{ij} \qquad (3.5)$$

It follows that competitive price regimes are potentially consistent both with conditions exhibiting and with conditions exploiting the MFL/MFN paradox. But, as I showed by means of an example in Ryan 1980, conditions of the MFL/MFN paradox are not necessarily consistent with conditions of spatial competition. This can be demonstrated more formally by considering a variant of (IIa)* with explicit incremental supply and demand goals as in (III). [Associating dual variables R_i, K_j, θ_i, φ_j with the constraints of (III) that system generates a dual as in (III)'.]:

$$\text{Minimise } \sum_j -p^j y_j^- + \sum_i\sum_j c_{ij}x_{ij} + \sum_i p_i x_i + M\sum_i\sum_j (x_i^+ + y_j^+) + \sum_i\sum_j c_i^{-+}x_i^{-+} + c_i^{--}x_i^{--} + d_j^{-+}y_j^{-+} + d_j^{--}y_j^{--})$$

$$\sum_j x_{ij} + x_i^+ - x_i^- = a_i$$

$$\sum_i x_{ij} + y_j^+ - y_j^- = b_j \qquad \text{(III)}$$

$$x_i^- + x_i^{-+} - x_i^{--} = x_i^*$$

$$y_j^- + y_j^{-+} - y_j^{--} = y^{j*}$$

$$\sum_i a_i = \sum_j b_j \qquad y^j, x_i, x_{ij}, x_i^+, x_i^-, y_j^+, y_j^-, x_i^{-+}, x_i^{--}, y_j^{-+}, y_j^{--} \geq 0$$

$$\text{Maximise} \sum_i R_i a_i + \sum_j K_j b_j + \sum_i \theta_i x_i^* + \sum_j \varphi_j y^{j*}$$

$$R_i + K_j \leq c_{ij}$$

$$-c_i^{--} \leq \theta_i \leq +c_i^{-+} \qquad \text{(III)'}$$

$$-d_j^{--} \leq \varphi_j \leq +d_j^{-+}$$

$$-R_i + \theta_i \leq p_i$$

$$-K_j + \varphi_j \leq -p^j$$

$$R_i, K_j \leq M$$

If in effect $R_i = -p_i$, $K_j = p^j$ at an optimum (III),(III)' are potentially consistent with spatial competition as defined above. But in general they are not. Indeed, by considering cases for which optimally $x_i^+, x_i^-, y_j^+, y_j^- > 0$, so that by complementary slackness $\theta_i \neq 0$ and/or $\varphi_j \neq 0$ some i,j in (III),(III)', those systems yield interpretations of θ_i and/or φ_j in relation to relative taxes and/or subsidies since:

$$x_i > 0 \Rightarrow -R_i + \theta_i = p_I \qquad (3.6)$$

$$y_i > 0 \Rightarrow -K_j + \varphi_j = -p^j \qquad (3.7)$$

$$\text{so } x_i \text{ and } y^j > 0 \Rightarrow R_i + K_j = p^j - p_i + \varphi_j + \theta_i \qquad (3.8)$$

If $\varphi_j = \theta_i = 0$ the latter conditions are consistent with spatial competition. But they are also consistent with relative demand and supply *taxes* if y^j (resp x_i) is above target and relative *subsidies* if below. (These interpretations are particular applications of general goal related tax/subsidy interpretations in Ryan 1992. Note that a relative tax and a relative subsidy may optimally apply to the same shipment.)

If MFL/MFN conditions are fully exploited, the resulting more for less optimum is degenerate* and decomposable. It is then consistent with *two* base prices and thence with *potential* spatial competition, if optimally $\varphi_j = \theta_i = 0$, or with tariffs/subsidies (e.g. import tariffs/ subsidies) otherwise. Such conditions could be consistent with various kinds of explicitly non-competitive systems, including second best related regulatory systems. Second best interpretations are especially germane here since they suggest the potential, which is in fact inherent in this approach, for interpretations in relation to economies of scale and scope.

6. MFL and MFN cases and economies of scale and scope

So far the emphasis has been on conditions under which it might become optimal to connect markets and/or to decompose a set of connected markets into sub-markets. Now consider *why* it might be optimal to seek to optimise within a multiple market structure in the first place. One reason is that there may be opportunities for mutually advantageous exchanges. In the spatial competition case above such exchanges would be of products for money. To the extent that opportunities to gain are increased by increases in the numbers of potential supplies and demanders involved, as well as by increases in the quantities of product which might be offered by suppliers, or required by demanders, there are opportunities for gains due to increases in *scope* (numbers of suppliers and/or demanders) as well as due to increases in *scale* (increases in quantity supplied by a given number of potential suppliers/users).

THEOREM 3 **(ECONOMIES OF SCOPE)**

Assume two alternative cost regimes $\{c_{ij},M\}$ and $\{c_{ij},c_{ij}'\}$ for potential shipments between sub-markets $i,j\varepsilon(I_1,J_1)$, $i,j\varepsilon(I_2,J_2)$, total availabilities a_i and requirements b_j being the same in each case. Then if a feasible solution exists for (IV):

$$\text{Min } \sum_j f(y_j^-)+\sum\sum_{I_1J_1} c_{ij}x_{ij}+M\sum_{I_2J_2} x_{ij}+\sum_i p_ix_i+M\sum\sum_{ij}(x_i^++y_j^+)+\sum\sum_{ij} c_i^{-+}x_i^{-+}+c_i^{--}x_i^{--}+d_j^{-+}y_j^{-+}+d_j^{--}y_j^{--})$$

subject to the constraints of (III) (IV)

$$=z \geq z'=$$

$$\text{Min } \sum_j f(yj^-)+\sum\sum_{I_1J_1} c_{ij}x_{ij}+\sum_{I_2J_2} c_{ij}'x_{ij}+\sum_i p_ix_i+M\sum\sum_{ij}(x_i^++y_j^+)+\sum\sum_{ij} c_i^{-+}x_i^{-+}+c_i^{--}x_i^{--}+d_j^{-+}y_j^{-+}+d_j^{--}y_j^{--})$$

subject to the constraints of (III) (IVa)

PROOF

Any feasible solution to (iv) is a feasible solution to (iva), but not conversely.thus any optimal solution to (iv) is a feasible but not necessarily an optimal solution to (iva). it follows that there may exist optimal solutions to (iva) such that z'<z or z'=z with $x_{ij}>0$ some x_{ij} some $i,j\varepsilon(i_2,j_2)$.

THEOREM 4 **(Economies of scale)**

Consider two distinct regulatory regimes, one associating prohibitive penalties m and the other non prohibitive penalties (p_i,p^j) with potentially marginal increases in sub-market demand and supply levels y_j^-,x_i^- in (iii). then, if a feasible solution exists for (v):

$$\text{Min } \sum_j My_j^-+\sum\sum_{I1J1} c_{ij}x_{ij}+M\sum_{I2J2} x_{ij}+\sum_i Mx_i^-+M\sum\sum_{ij}(x_i^++y_j^+)+\sum\sum_{ij} c_i^{-+}x_i^{-+}+c_i^{--}x_i^{--}+d_j^{-+}y_j^{-+}+d_j^{--}y_j^{--})$$

subject to the constraints of (III) (V)

$$\geq$$

$$\text{Min } \sum_j -p^jy_j^-)+\sum\sum_{I1J1} c_{ij}x_{ij}+M\sum_{I2J2} x_{ij} +\sum_i p_ix_i^-+M\sum\sum_{ij}(x_i^++y_j^+)+\sum\sum_{ij} c_i^{-+}x_i^{-+}+c_i^{--}x_i^{--}+d_j^{-+}y_j^{-+}+d_j^{--}y_j^{--})$$

subject to the constraints of (III) (Va)

PROOF

Any feasible solution to (V) is a feasible solution to (Va), but not conversely.Thus any optimal solution to (V) is a feasible but not necessarily an optimal solution to (Va). It follows that there may exist optimal solutions to (Va) such that z'<z or z'=z with $x_{ij}>0$ some x_{ij} some $i,j\varepsilon(I_2,J_2)$.

Clearly in general a potentially connected set of markets may exhibit economies of scale and then of scope or, conversely, of scope and then of scale. In each case the resulting configuration will conform to an optimal solution to an overall model of the form of (III) with the appropriate parameters. That is, with the appropriate parameters, (III) includes all of (IV),(IVa),(V) and (Va) as special cases.

7. A homogeneous product example

Consider an initial allocation as in Tableau 7.1:

	$K_1=9$	$K_2=6$	$K_3=3$	$K_4=5$	
$R_1=0$	[M]	[6] 13	[3] 3	[5] 4	20
$R_2=-2$	[M]	[4] A	[1] 14	[6]	14
$R_3=0$	[9] 11	[M]	[M]	[M]	11
	11	13	17	4	45

Tableau 7.1

Since the solution in Tableau 7.1 is both primal and dual feasible this market structure is consistent with optimal solutions to two disjoint distribution models of the standard type - and thence with optimal solutions to (IV) with preemptive weights M associated with shipments between factories 1 and 2 and market 1 and between factory 3 and markets 2,3,4. The overall shipping cost associated with this specific-ation is 99+121=220 units.

If the preemptive weights in Tableau 7.1 now become nonpreemptive the solution in that Tableau is no longer optimal. There are opportunities for economies of scope. (As in the movement from conditions of programme (IV) and to those of (IVa) in Theorem 3.) An optimal solution to the revised problem, with revised inter-market shipping costs indicated in bold is shown in Tableau 7.2:

	$K_1=1$	$K_2=6$	$K_3=3$	$K_4=2$	
$R_1=0$	[**1**] 11	[6] 9	[3]	[5]	20
$R_2=-2$	[**7**]	[4] 4	[1] 10	[6]	14
$R_3=2$	[9]	[**4**]	[**5**] 7	[**4**] 4	11
	11	13	17	4	45

Tableau 7.2

For this example the shipping cost reduction due to economies of scope gained by linking the sub-markets (O_3,D_1) and (O_1,O_2,D_2,D_3,D_4) is 78 units. (From 220 units to 142 units.) One interpretation of this would be, *inter alia,* that, while initially market 1 is supplied from factory 3, given the relatively freer opportunities for exchange in Tableau 2 that market becomes wholly supplied from factory 1.

Now notice that cell {2,1} is a more for less cell with $R_2 + K_1=-1<0$ and consider the same numerical example with reference to opportunities for gains to economies of scale. If the conditions of programme (V) in Theorem 4 correspond to an optimal solution to (III), (as in Tableau 7.2 above), and if supplies at O_2 are increased and demands at D_1 are also increased by a positive amount $\delta\leq 9$ in such a way that the initial basis remains unchanged, then shipment costs are actually reduced. If this more for less opportunity is fully exploited then $\delta=9$ and the set of markets decomposes into disjoint sub-markets as in Tableau 7.3 below.

The reader can verify that the solution in Tableau 7.3 is optimal and that in this case the resulting overall economies stemming from the increased scale of operation of factory 2 amount to -9 units.

Now reconsider the various stages in this example in more detail with reference to conditions of spatial monopoly and spatial competition. In the initial Tableau 7.1 market 1 is wholly supplied by plant 3 and prohibitively large weights M attach to potential entry from plants 1 and 2. That is, there is initially spatial monopoly in market 3 in the senses both of a sole provider and of no potential entry. By contrast the allocation in Tableau 7.2 is potentially consistent with spatial competition in both senses. There is more than one provider in market 3 and, with conditions $p^j-p_i=c_{ij}$ for all basic routes ij, the allocation in Tableau 7.2 is potentially consistent with the easy entry condition of spatial competition. (See also developments in Section 7.) With reference to the scale related more for less solution in Tableau 7.3, there is spatial competition at best only in the sense of easy entry into the relatively isolated pair (O_1,D_1) via the shipment ε_{12} on route {1,2}.

	$K_1=1$	$K_2=6$	$K_3=3$	$K_4=2$	
$R_1=0$	$\boxed{1}$ 20	$\boxed{6}$ ε	$\boxed{3}$	$\boxed{5}$	20
$R_2=-2$	$\boxed{7}$	$\boxed{4}$ 13	$\boxed{1}$ 10	$\boxed{6}$	23
$R_3=2$	$\boxed{9}$	$\boxed{4}$	$\boxed{5}$ 7	$\boxed{4}$ 4	11
	20	13	17	4	54

Tableau 7.3

More exactly in Tableau 7.2 there are conditions of spatial oligopoly since the number of suppliers in each region is small and varying in a perceptibly interdependent fashion with the market conditions.

In any case in the example of Tableaux 7.1-7.3 transport costs have been reduced *twice* - first due to market connection related economies of scope and then to more for less related economies of scale associated with simultaneous increases of 9 units in the supply at origin 2 and demand at destination 1.

8. A heterogeneous product example

An alternative interpretation of the example in Tableaux 7.1-7.3 might refer to labour markets and types of workers and initially two unconnected labour markets. For example they may be separated by prohibitive transportation costs and/or by initially prohibitive professional barriers to entry and/or retraining costs.

Now assume that opportunities for economies of scope are increased by reducing interregional transportation and/or professional access/retraining costs. One consequence is that a different pattern of optimally spatially and/or professionally differentiated employment may emerge, as in Tableau 7.2.

To develop ideas interpret Tableau 7.1 with reference to a proposed bank reorganization. Prior to merger there are 20 managers, 14 clerks 11 tellers. After the merger there will be 11 telebusiness workers, 13 managers, 17 clerical workers and 4 tellers. Costs in Tableau 7.1 then refer to potential retraining/relocation costs, where initially prohibitively large weights M might relate *inter alia* to union agreements prohibiting new computer related work practices for managers and clerical workers and requiring retraining of tellers as telebusiness workers.

If initially prohibitive transition costs are now reduced, for example. by renegotiating union agreements and/or conditions of service, economies of scope - and an overall cost reduction of 78 - stemming from the transition from the optimum in Tableau 7.1 to that in Tableau 7.2 - become attainable.

Next, having already attained these economies of scope, the parties to the reorganization may decide to expand the proposed scale of telebusiness related activities further. The transition from Tableau 7.2 to Tableau 7.3 indicates that, even though at present all of the proposed telebusiness related staff would be retrained tellers (i.e. *not* clerical staff) nevertheless, the cheapest way of getting up to an additional 9 telebusiness related workers would be to hire an additional 9 clerical staff and reorganise the retraining plan, as in Tableau 7.3. The advantage of this would be to allow an additional 9 clerical workers to be retrained for new kinds of managerial jobs rather than being retained as clerks as in Tableau 7.2.

Parenthetically, even though the numbers in this example are hypothetical it does have realistic features including the fact that, with increasingly sophisticated databases and communication software, hitherto exclusively old style managerial /white collar occupations may be (or become) some of the most easily replaced. Also, even if minimization of net retraining cost is the objective, it will not always be optimal to minimise the number of individuals being retrained. (In Tableaux 7.1, 7.2 and 7.3 respectively 18 of 45, 22 of 45 and 40 of 54 workers are retrained.)

More technically, economists commonly attribute the term economies of scale either to homogeneous production cases or to heterogeneous product cases with a fixed product mix and in those contexts restrict the term to cases for which, when all inputs are increased by a factor λ total costs increase by a factor less than λ. In more for less and more for nothing cases of types which have been considered in this paper total costs may actually *decrease* in absolute value, even when quantities of inputs and outputs of just one type of product are increased by a factor λ. If part (or all) of this overall cost reduction is attributable to the product bringing it about, the two types of definition can be reconciled. In that case in heterogeneous product cases, such as the spatially or professionally differentiated labour market cases which have just been considered, under the conditions of the more for less paradox an increase δ in inputs and outputs of at least one type of product will not simply lead to a proportionately lower increase in cost attributable to that product. That is, it will actually lead to a *reduction* in the absolute cost attributable to the increased output of that product.

In this context dual variables in Tableaux 7.1-7.3 are consistent with interpret-ations in relation to a differentiated labour market according to which, for example, all wages are related to a manager related *base wage* $R_1 =_{def} w_1$, all other wages being set via differentials $R_i + K_j = \Delta w_{ij}$ for $x_{ij} > 0$ at an optimum. It follows that these examples are potentially consistent with wholly private market and commuting/retraining cost related wage differentials.

But, just as in the earlier analyses, variants of model (IV)' were open to interpretations as relative taxes and subsidies, so might variants of the solutions in Tableaux 7.1-7.3 yield variants of (IV)' and interpretations with reference to elements of payroll taxes and initial hiring related subsidies. These might, at least in part, reflect scope and scale related labour market advantages of including an increased variety of potential transitions between types of workers and/or increased numbers of a particular type of worker respectively.

Corresponding interpretations in relation to regionally monopolistic or oligopolistic labour markets then follow in a manner analogous to the single commodity case considered in Section 6.

9. Conclusion

In this chapter I have introduced a new goal programming approach to the representation and resolution of the MFL and MFN paradoxes in the distribution model and to definitions of economies of scale and economies of scope in that context.

I close with two remarks. First: here, for simplicity the definitions of economies of scale and scope in Section 6 are given as if these concepts would apply on an all or nothing basis. But clearly they could apply on a partial basis. If any one (or more) of the quantities M in (IV) were reduced to a non pre-emptive magnitude c_{ij}' in (IVa) and/or if any one (or more) of the quantities M in (V) were reduced to a non pre-emptive magnitudes p^j, p_i in (Va) then relatively enhanced economies respectively of scope and of scale may become attainable.

Secondly: apart from potentially yielding interpretations in relation to economies of scale and scope, distribution problems exhibiting the MFL and/or MFN paradox also have other properties, some of which are yet to be fully explored. Among these is the fact that there may be a variety of potential MFL and MFN solutions each with distinct potentials for economies of scope and scale and consequently distinct decomposition patterns. To illustrate this reconsider Tableau 7.1. The potential for MFL evident in that Tableau, when fully exploited, generated Tableau 7.3. But Tableau 7.3 itself exhibits an as yet unexploited potential (via cell {4,2}) for attaining a MFN solution. If that potential is fully exploited an additional 7 units could be shipped from origin 2 to destination 4 at no additional overall cost. In that case, among other things, 11 units would be shipped wholly from origin 3 to market 4 and that origin-destination pair would become optimally isolated, giving a three way partition of origins and destinations and a correspondingly still more concentrated spatial market arrangement. (This example illustrates the more general point that, while nondegeneracy* of a distribution problem together with $R_i+K_j\leq 0$ for some nonbasic cell at an optimum may be a sufficient condition for MFL/MFN, as in the Charnes-Klingman theorem cited in Section 2, it is not always necessary.)

Finally a different more for less solution and subsequent more for nothing solution with different associated patterns of decompostion into disjoint subsets of origins and destinations would follow if the economies of scale and scope related applications in Sections 7 and 8 had started with the alternative optimum signalled by the “A” in cell {2,2} of tableau 7.1.

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