CHAPTER 5

MORE FOR LESS AND COMPARATIVE ADVANTAGE

1. Introduction

The first purpose of this paper is to present extensions of the more for less theorems in Charnes, Duffuaa, Ryan 1980, 1987 and Ryan 1998, 2000a to potentially Pareto improving exchanges of commodities and/or of property rights between two individuals. This is done in Section 2. Then in Section 3 these theorems, together with associated Kuhn Tucker complementary slackness conditions, are used to derive optimising conditions for cases in which gifts or exchanges of commodities correspond to mutually preferable optima. In Section 4 analogues of the Heckscher-Ohlin and Stolper-Samuelson results are considered and extended to cases in which exchanges of factors may be preferred at least in part to exchanges of commodities. This generalised approach is consistent with the fact that, if all labour and raw materials are mobile at a cost, given the opportunity to do so, individuals may choose to move themselves and their resources to another country rather than exchange their resources for the produced products of that other country. In Section 5, the models in earlier sections are specialised to cases with linear input output technologies in order to contrast these earlier results with predictions of work by ten Raa and Chakraborty 1991 for a more restricted class of linear cases. ten Raa and Chakraborty's principal conclusion, based on input output data for India and Europe, was that producers in India, if given the opportunity to do so, would use the same technologies as those in Europe for the production of their outputs. We will argue that this highly counter-intuitive conclusion may depend crucially on two assumptions that they make. These assumptions are that resources (e.g. of skilled labour) are not mobile between blocs and, that technology in one country is immediately accessible to another.

2. Some more for less results

The more for less paradox was first reported in the context of the distribution model of linear programming starting with work by Charnes and Kingman 1971 and Scwarcz 1971 and continuing with papers by Ryan 1980 (inter alia extending the paradox to more for nothing cases) and Charnes, Duffuaa and Ryan 1980. More recent papers in this line include those of Arsham 1992 and Gupta and Puri 1995 and Ryan 1998, 2000a. But, as first reported in Charnes, Dufuaa and Ryan 1987 the more for less phenomenon and associated resolutions of it applies to generally specified linear programming cases too.

It has long been clear that the more for less phenomenon invites economic applications and interpretations. Indeed in Ryan 1980 and Ryan 1998, 2000a, 2000b more for less and more for nothing analyses and interpretations have been related specifically to spatial competition, to regulation of spatial economic activity, and to economies of scale and scope. [As far as we are aware, these are the only published papers providing economic applications and interpret-ations of the more for less paradox. Others, including the papers cited above, focus on more narrowly mathematical and algorithmic properties of the more for less phenomenon with exclusive reference to the distribution model.]

Here we pursue another direction of (explicitly nonlinear) generalisations and applications of more for less and more for nothing phenomena to characterise conditions of potentially mutually advantageous gains from exchange between individuals or groups. With such potential applications in view assume that M is of a preemptively large magnitude and consider:

THEOREM 1

If a feasible solution exists to program (I) and if $M>c_{ri}^{rs}$, with $M>0$ all i,r,s then:

$$
\begin{aligned}
z = \text{Max}\ & \theta_1 f_1(y_{1i})+\theta_2 f_2(y_{2i})-M\Sigma x_{ri}^{rs} \\
\text{st}\quad & y_{1i} \le x_{1i} \\
& y_{2i} \le x_{2i} \\
& x_{1i} \le g_1(k_{1i},l_{1i}) + x_{2i}^{21} - x_{1i}^{12} \qquad \text{(I)} \\
& x_{2i} \le g_2(k_{2i},l_{2i}) + x_{1i}^{12} - x_{2i}^{21} \\
& \Sigma k_{1i} \le K_1^*, \qquad \Sigma k_{2i} \le K_2^* \\
& \Sigma l_{1i} \le L_1^* \qquad \Sigma l_{2i} \le L_2^*
\end{aligned}
$$

All variables nonnegative

$$
\begin{aligned}
\le z' = \text{Max}\ & \theta_1 f_1(y_{1i})+\theta_2 f_2(y_{2i})-\Sigma c_{ri}^{rs} x_{ri}^{rs} \\
& \text{st constraints of (I)} \qquad \text{(Ia)}
\end{aligned}
$$

PROOF

Any feasible solution to (I) is a feasible solution to (Ia) and conversely. But any optimal solution to (I) with $x_{ri}^{rs}=0$ all i,r,s is a feasible but not necessarily an optimal solution to (Ia). It follows that there may exist optimal solutions to (Ia) such that $z'>z$ or $z'=z$ with $x_{ri}^{rs}>0$ some i,r,s.

REMARK

The feasibility condition in Theorem 1 is very weak. Formally it would always be met by a zero consumption, zero production, zero exchange and zero factor utilisation solution to (I). In that sense Theorem 1 is very general. For example no specific assumptions concerning the form of $f_r(\ )$, $g_i(\ )$ are required.

AN APPLICATION

Consider $f_r(y_{ri})$ as corresponding to the preference relations of two individuals r=1,2 and K_r^*,L_r^* as their endowments of two commodities. Assume further that x_{ri}, x_{ri}^{rs} are respectively outputs of commodities i by processes $g_r(k_{ri},l_{ri})$ and transfers (if any) of commodities i from individual r to individual s. Then Theorem 1 has the interpretation that in general an economy with the potential for nonzero transfers may be preferable in the sense of $z'>z$ to one effectively prohibiting such transfers. (Note that an arbitrarily large value for c_{ri}^{rs} is consistent with nontradeability of i for technological reasons.)

Although, in the sense that (Ia) makes it possible that $z'>z$ vis a vis (I) so that the specification (Ia) is collectively preferable to the specification (I), it is not necessarily the case that optimal evaluations $f_r(y_{ri})^{**}$ to (Ia) will have the Pareto improving property $f_r(y_{ri})^{**} \ge f_r(y_{ri})^*$, where $f_r(y_{ri})^*$ are consistent with an optimal solution to (I).

A result which ensures that transfer economies consistent with the conditions of Theorem 1 *guarantee* at least weak Pareto improvements is obtained by modifying Theorem 1 to:

THEOREM 2

If an optimal solution $f_1(y_{1i})^*,f_2(y_{2i})^*$ exists to program (II), then:

$$
\begin{aligned}
z = \text{Max}\ & \theta_1 f_1(y_{1i})+\theta_2 f_2(y_{2i})-M\Sigma x_{ri}^{rs} \\
& \text{st constraints of (I)} \qquad \text{(II)} \\
\le z' = \text{Max}\ & \theta_1 f_1(y_{1i})+\theta_2 f_2(y_{2i})-\Sigma c_{ri}^{rs} x_{ri}^{rs} \\
& \text{st constraints of (I) } \textit{and;} \qquad \text{(IIa)} \\
& f_1(y_{1i})\ge f_1(y_{1i})^*, f_2(y_{2i})\ge f_2(y_{2i})^*
\end{aligned}
$$

PROOF

Similar to Theorem 1. (As in Theorem 1 the feasibility requirement for (II) is extremely weak. In particular no restrictions are placed on the forms of $f_r(\ )$, $g_i(\ )$.)

AN APPLICATION

With interpretations as in the application under Theorem 1, Theorem 2 implies that an economy with the potential for nonzero transfers will be at least weakly *Pareto* preferable in the sense of $z'\ge z$ to one effectively prohibiting such transfers. That is: a transfer economy as in (IIa) is in general at least weakly Pareto preferable to a nontransfer economy as in (II).

One class of extensions of systems (II),(IIa) is that in which there may be transfers of resources as well as (or instead of) produced commodities, viz:

THEOREM 3

If an optimal solution $f_1(y_{1i})^*,f_2(y_{2i})^*$ exists to (III) and $c_{ri}^{rs} \ll M^{\alpha}, d_r^{rs} \ll M^{\beta}, e_r^{rs} \ll M^{\gamma}$, $\alpha,\beta,\gamma \ge 1$:

$$
\begin{aligned}
z = \text{Max}\ & \theta_1 f_1(y_{1i})+\theta_2 f_2(y_{2i})-M^{\alpha}\Sigma x_{ri}^{rs} - M^{\beta}\Sigma k_r^{rs} - M^{\gamma}\Sigma l_r^{rs} \\
\text{st}\quad & y_{ri} \le x_{ri} \\
& x_{ri} \le g_{ri}(k_{1i},l_{1i}) + x_{si}^{sr} - x_{ri}^{rs} \qquad \text{(III)} \\
& \Sigma k_{ri} \le K_r^* + k_r^{sr} - k_s^{rs} \\
& \Sigma l_{ri} \le L_r^* + l_r^{sr} - l_j^{rs}
\end{aligned}
$$

All variables nonnegative

$$
\begin{aligned}
\le z' = \text{Max}\ & \theta_1 f_1(y_{1i})+\theta_2 f_2(y_{2i})-\Sigma c_{ri}^{rs} x_{ri}^{rs} - \Sigma d_r^{rs} k_r^{rs} \Sigma e_r^{rs} l_r^{rs} \\
& \text{st constraints of (III)} \qquad \text{(IIIa)} \\
\text{and}\quad & f_1(y_{1i})\ge f_1(y_{1i})^*, f_2(y_{2i})\ge f_2(y_{2i})^*
\end{aligned}
$$

PROOF

Similar to Theorem 1.

SOME GENERAL RESULTS

Under the conditions of Theorem 3, (IIIa) will always be at least weakly Pareto preferred to (III). Further, (IIa) is equivalent to a special case of (IIIa) with c_i^{rs} d_r^{rs},e_r^{rs} arbitrarily large, so that optimally $x_{ri}^{rs}=k_r^{rs}=l_r^{rs}=0$ in (IIIa). Because Theorem 3 contains all of the conditions of Theorem 2 as a special case, an optimal solution via (IIIa) will also be at least weakly Pareto preferred to (IIa) which in turn is at least weakly Pareto preferable to (II).

COMMENTS

The preceding models and results include cases in which technologies are either the same or different in both economies i.e. $g_{ri}(\,)=g_{si}()$ or $g_{ri}(\,)\neq g_{si}()$.

Further results would extend the preceding models to include explicit transfers of technology and/or to make preferences also depend on endowments. The latter type of extension together with generally applicable Kuhn-Tucker optimality conditions are the subjects of the next section.

3. Exchange related potentials, tariffs and prices

Theorem 4 and programs (IV),(IVa) extend Theorem 3 and programs (III),(IIIa) to cases in which preferences depend on endowments as well as on consumption:

THEOREM 4

If an optimal solution $f_r(y_{ri},k_{ri},l_{ri})^*$ exists to (IV) then:

$$z_I = \text{Max. } \theta_1 f_1(y_{1i}, k_{1i}, l_{1i}) + \theta_2 f_2(y_{2i}\, k_{2i}, l_{2i}) - M^{\alpha}\Sigma x_{ri}^{rs} - M^{\beta}\Sigma k_r^{rs} - M^{\gamma}\Sigma l_r^{rs}$$

st constraints of (III), viz:

$$\begin{array}{lll} \phi_{ri} & y_{ri} \leq x_{ri} & \\ \psi_{ri} & x_{ri} \leq g_{ri}(k_{ri},l_{ri}) + x_{si}^{sr} - x_{ri}^{rs} & \\ \mu_r & \Sigma k_{ri} \leq K_r^* + k_r^{sr} - k_r^{rs} & \text{(IV)} \\ \omega_r & \Sigma l_{ri} \leq L_r^* + l_r^{sr} - l_j^{rs} & \end{array}$$

All variables nonnegative

$$\leq z_I = \text{Max } \theta_1 f_1(y_{1i}, k_{ri}, l_{1i}) + \theta_2 f_2(y_{2i}\, k_{2i}, l_{2i}) - \Sigma c_{ri}^{rs} x_{ri}^{rs} - \Sigma d_r^{rs} k_r^{rs} - \Sigma e_r^{rs} l_r^{rs}$$

st constraints of (IV) (IVa)

and λ_r $\quad f_r(y_{ri}, k_{ri},l_{ri}) \geq f_r(y_{ri}, k_{ri},l_{ri})^*$

PROOF

Similar to Theorem 1.

REMARK

Individuals' preferences may depend, too, on others' endowments or on others' work effort. But we will not detail these extensions of Theorem 4 here.

If f( ), g( ) are concave the Kuhn-Tucker conditions are sufficient for optimal solutions to any of (I)...(IVa). In particular, associating the indicated dual variables with its constraints, the Kuhn Tucker conditions for (IVa) are:

$$\begin{array}{lll} y_{ri} & \phi_{ri} \geq (\theta_r - \lambda_r) f_r'(y_{ri}, k_{ri}, l_{ri}) & (3.1) \\ x_{ri} & -\phi_{ri} + \psi_{ri} \geq 0 & (3.2) \\ x_{ri}^{rs} & \psi_{ri} - \psi_{si} \geq -c_{ri}^{rs} & (3.3) \\ k_{ri} & \mu_r - \psi_{ri} g_{ri}'(k_{ri}, l_{ri}) \geq (\theta_r - \lambda_r) f_r'(y_{ri}, k_{ri}, l_{ri}) & (3.4)\text{(IVa)}' \\ l_{ri} & \omega_r - \psi_{ri} g_{ri}'(k_{ri}, l_{ri}) \geq (\theta_r - \lambda_r) f_r'(y_{ri}, k_{ri}, l_{ri}) & (3.5) \\ k_r^{rs} & \mu_r - \mu_s \geq -d_r^{rs} & (3.6) \\ l_r^{rs} & \omega_r - \omega_s \geq -e_r^{rs} & (3.7) \end{array}$$

For each of (3.1)..(3.7), by complementary slackness positive values for the primal variables imply that the corresponding Kuhn Tucker condition holds as an equality at an optimum to (IVa). At such an optimum these conditions take on interpretations as follows:

- First and foremost, from (3.1), if an optimal solution to (IVa) is (Pareto) preferred to the reference state in (IV), then change or exchange is unanimously at least weakly Pareto preferred to no change or exchange. Further: in that case: <u>*either*</u> $\lambda_r=0$ (strict preference) so that optimally the demand price ϕ_{ri} for region r and commodity i is given by $\phi_{ri}=\theta_r f_r'(y_{ri},k_{ri},l_{ri})$, <u>*or*</u> $f_r'(y_{ri},k_{ri},l_{ri})=0$, i.e there is indifference to (further) exchange so that optimally the demand price ϕ_{ri} for region r and commodity i is optimally zero.
- Next; conditions (3.2) require that commodity i be produced for local consumption in region r, if at all, then up to the point where the demand price ϕ_{ri} is sufficient to meet the marginal supply price ψ_{ri}.
- Conditions (3.3) require that commodity i be produced for export to region r, if at all, then to the point where the offered supply price ψ_{ri} in region r is sufficient to meet the marginal supply price ψ_{si} (i.e. production cost) in region s plus the marginal transfer cost c_{ri}^{rs}. [Conditions (3.2) and (3.3) are together consistent with *absolute advantage.* From them it follows that, if local production cost is higher

than foreign production plus transport costs, that type of commodity would be wholly imported.]

- If optimally $\theta_r f_r'(y_{ri}, k_{ri}, l_{ri})=0$ in (3.4) and (3.5) those conditions require that factors k and l be used for the production of commodity i in region r, if at all, then to the point where their marginal value μ_r (resp ω_r) equals the value imputed to their product at the margin, or up to the point where $\mu_r=\psi_{ri}g_{ri}'(k_{ri},..)$ (resp $\omega_r=\psi_{ri}g_{ri}'(..,l_{ri})$). If optimally $\theta_r f_r'(y_{ri},k_{ri},l_{ri})\neq 0$ then an implicit factor ownership related *tax or subsidy* may be introduced to secure an overall optimum accordingly.
- Finally, (3.6) and (3.7) require that factors, when potentially mobile, optimally move from region r to region s, if at all, then only to the point where the marginal return to them in the destination region μ_s (resp ω_s) is sufficient to compensate for the marginal reward to them in the origin region μ_r (resp ω_r) plus the net tax or subsidy adjusted relocation costs d_r^{rs} (resp e_r^{rs}).

By Theorem 3 solutions via (IIIa) will always be at least weakly Pareto preferred to solutions via (III). Further, a striking feature of this and other results is that in each of these theorems Pareto preferable states may correspond to *unbalanced* exchange optima. Underlining this: Theorems 1-3 do not necessarily imply two way exchanges. For example they are consistent with cases in which all labour and/or capital transfers from one country to another. Further: even if it is assumed that in some sense Pareto preferable states correspond to two way exchanges with given factor and commodity prices, in general it will *not* follow that at an optimum what is transferred from s to r is equal in value at those prices to what is transferred to r from s. This in turn implies that at an optimum inputs and outputs will generally both be associated with *relatively unbalanced* budgets.

If balanced budgets are to be required as a precondition for a socially Pareto preferable state then such conditions would need to be introduced explicitly. This can be done by appending a budget balance relation with appropriately chosen prices to (IVa) with potential deviations from it, $b_r^+ - b_r^-$, and refining Theorem 4 accordingly:

THEOREM 5

If an optimal solution $f_r(y_{ri},k_{ri},l_{ri})^*$ exists to (V) then:

$$z_1 = \text{Max } \theta_1 f_1(y_{1i}, k_{1i}, l_{1i}) + \theta_2 f_2(y_{2i}\, k_{2i}, l_{2i}) - \Sigma c_{ri}^{rs} x_{ri}^{rs} - \Sigma d_r^{rs} k_r^{rs} - \Sigma e_r^{rs} l_r^{rs} - 0\Sigma(b_r^+ + b_r^-) \quad (V)$$

st constraints of (IV)

$$\textit{and } \Sigma c_{ri}^{rs} x_{ri}^{rs} + \Sigma d_r^{rs} k_r^{rs} + \Sigma e_r^{rs} l_r^{rs} + b_r^+ - b_r^- = 0$$

$$\geq z_1 = \text{Max } \theta_1 f_1(y_{1i}, k_{1i}, l_{1i}) + \theta_2 f_2(y_{2i}\, k_{2i}, l_{2i}) - \Sigma c_i^{rs} y_i^{rs} - \Sigma d_r^{rs} k_r^{rs} - \Sigma e_r^{r} l_r^{rs} - M\Sigma(b_r^+ + b_r^-) \quad (Va)$$

st constraints of (IVa)

$$\textit{and } \Sigma c_{ri}^{rs} x_{ri}^{rs} + \Sigma d_r^{rs} k_r^{rs} + \Sigma e_r^{rs} l_r^{rs} + b_r^+ - b_r^- = 0$$

PROOF

Similar to Theorem 1.

COROLLARY

Since in general (V) will yield at least as high an optimum as (Va) and in that sense will be at least weakly Pareto preferred to (Va), it follows that in that sense unbalanced budgets will in general be at least weakly Pareto preferred to balanced budgets. Equivalently: in the absence of the preemptive budget balancing conditions in (Va), potentially unbalanced budgets, as in (V), (which is equivalent to (IVa)), will be at least weakly Pareto preferred to balanced budgets.

4. Further properties of the Kuhn Tucker conditions

It has already been stressed that one consequence of Theorems 1-5 is that, both for factors and for products, in general changes or exchanges of commodities and/or of factors in a manner consistent with the conditions of those theorems will be weakly Pareto preferable to no changes or exchanges in the sense of the conditions of those Theorems. The Kuhn-Tucker conditions (IVa)' yield an alternative way of making these points. If parameters $c_{ri}^{rs}, d_j^{rs}, e_j^{rs}$ are arbitrarily high in conditions (3.3), (3.6) and (3.7) respectively, then the Kuhn-Tucker conditions in which those variables appear will become strict inequalities and, by complementary slackness, the corres-ponding variables x_{ri}^{rs}, k_j^{rs} and l_j^{rs} will optimally become zero. Conversely, if technological and/or regulatory adjustments become attainable such that those parameters are reduced from effectively arbitrarily large values, consequent technological or regulatory changes may lead to corresponding Pareto improvements as if via the conditions of the less restricted of the pairs of programs in Theorems 1 through 4.

From another perspective, the Kuhn-Tucker conditions (IVa)' yield analogues of the Heckscher-Ohlin and Stolper-Samuelson results. Before deriving these we briefly consider a model which explicitly introduces transport related production processes both for shipments of produced commodities and for shipments of factors on the production side, as well as transport related externalities on the consumption side, in an extended specification of Theorem 4 as follows:

THEOREM 6

If an optimal solution $f_r(y_{ri},k_{ri},l_{jr}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})^*$ exists to (VI) then:

$$z_1 = \text{Max } \Sigma\theta_r f_r(y_{ri}, k_{ri},l_{ri}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs}) - M^\alpha\Sigma x_{ri}^{rs} - M^\beta\Sigma k_r^{rs} - M^\gamma\Sigma l_r^{rs}$$

ϕ_{ri} st: $y_{ri} \le x_{ri}$

ψ_{ri} $x_{ri} \le g_{ri}(k_{ri},l_{ri}) + x_{si}^{sr} - x_{ri}^{rs}$ (VI)

ψ_{ri}^{rs} $x_{ri}^{rs} \le g_{ri}^{rs}(k_{ri}^{rs},l_{ri}^{rs})$

μ_r $\Sigma k_{ri} + \Sigma k_{ri}^{rs} \le K_r^* + k_r^{sr} - k_r^{rs}$

ω_r $\Sigma l_{ri} + \Sigma l_{ri}^{rs} \le L_r^* + l_r^{sr} - l_j^{rs}$

All variables nonnegative

$$\le z_1 = \text{Max } \Sigma\theta_r f_r(y_{ri}, k_{ri},l_{ri}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs}) - \Sigma c_{ri}^{rs}x_{ri}^{rs} - \Sigma d_r^{rs}k_r^{rs} - \Sigma e_r^{rs}l_r^{rs}$$

s.t. constraints of (VI) (VIa)

and λ $f_r(y_{ri},k_{ri},l_{ri},k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs}) \ge f_r(y_{ri},k_{ri},l_{jr}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})^*$

PROOF

Similar to Theorem 1.

Assuming $f_r(\ )$ and $g_{ri}(\ )$ and $g_{ri}^{rs}(\ )$ are concave and associating the indicated dual variables with the constraints of (VIa) yields Kuhn-Tucker conditions as follows:

y_{ri} $\phi_{ri} \ge (\theta_r-\lambda_r)f_r'(y_{ri},..)$ (4.1)

x_{ri} $-\phi_{ri} + \psi_{ri} \ge 0$ (4.2)

x_i^{rs} $\psi_{ri} - \psi_{si} \ge -\psi_{ri}^{rs} - c_{ri}^{rs}$ (4.3)

k_{ri} $\mu_r - \psi_{ri}g_{ri}'(k_{ri},..) \ge (\theta_r-\lambda_r)f_r'(...k_{ri},..)$ (4.4)(VIa)'

l_{ri} $\omega_r - \psi_{ri}g_{ri}'(..,l_{ri}) \ge (\theta_r-\lambda_r)f_r'(..l_{ri}...)$ (4.5)

k_{ri}^{rs} $\mu_r - \psi_{ri}^{rs}g_{ri}^{rs\prime}(k_{ri}^{rs},..) \ge (\theta_r-\lambda_r)f_r'(..k_{ri}^{rs},.)$ (4.6.)

l_{ri}^{rs} $\omega_r - \psi_{ri}^{rs}g_{ri}^{rs\prime}(..,l_{ri}^{rs}) \ge (\theta_r-\lambda_r)f_r'(..l_{rs}^{rs}...)$ (4.7)

k_r^{rs} $\mu_r - \mu_s \ge -d_r^{rs}$ (4.8)

l_r^{rs} $\omega_r - \omega_s \ge -e_r^{rs}$ (4.9)

At an optimum conditions (VIa)' have interpretations analogous to those of (IVa)', except that there are now additional transport related labour and capital productivity relationships (4.6) and (4.7) and that in the interregional transport conditions (4.3) there are now additional terms ψ_{ri}^{rs} endogenising resource related costs of interregional transfers of commodities.

A further extension incorporating outputs and interregional transfers of commodities x_{ri}, x_{ri}^{rs}, x_{ri}^{sr} into individuals' preference relations would lead to the endogenous determination of tax or subsidy terms analogous to c_i^{rs} in conditions (4.3). In any case (4.3) implies that at an optimum to (VI) commodities i will be transported between regions r and s, if at all, then only to the point where the sum of transport costs and evaluations of preference related externalities, if any, equals the price differential for commodity i between regions r and s. [Note the analogy between $\psi_{ri}^{rs}+c_{ri}^{rs}$ in (4.3) of (VIa)' and c_{ri}^{rs} (3.3) of (IVa)'. In this respect (IV) is equivalent to (VI) if c_{ri}^{rs} in (IV) is interp-reted as equivalent to tariffs, if any, plus transport costs.) Similar externality related implications and interpretations apply to (4.6) and (4.7) and so to the transport cost related expressions (4.8) and (4.9) for capital and labour. (Incidentally technologically non transportable capital is equivalent to capital for which the potential transportation cost would be arbitrarily large.]

With interpretations as in the previous paragraph Theorem 6 implies that Pareto improvements relative to higher tariff and transport cost alternatives may be brought about by changes in technology to reduce transportation costs and/or changes in preferences equivalent to a preference for an effective reduction in interregional tariffs. Formally:

THEOREM 7 ***(Weak pareto improvements via changes in preferences.)***

Define $\theta_r f_r(y_{ri},k_{ri},l_{ri},k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})$ and $\theta_r h_r(y_{ri}, k_{ri},l_{ri},k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})$ as two alternative weighted preference relations for individuals r and let $\theta_r f_r(y_{ri},k_{ri},l_{ri},k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})^{**}$ be consistent with an optimal solution to (VIa). Then, if a feasible solution exists to program (VII) the resulting transfers will be at least weakly Pareto preferred to those of (VIa) and Theorem 6.

$$z_1 = \text{Max } \Sigma\theta_r h_r(y_{ri}, k_{ri},l_{ri}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs}) - \Sigma c_{ri}^{rs}x_{ri}^{rs} - \Sigma d_r^{rs}k_r^{rs} - \Sigma e_r^{rs}l_r^{rs}$$

st: $y_{ri} \le x_{ri}$

$x_{ri} \le g_{ri}(k_{ri},l_{ri}) + x_{si}^{sr} - x_{ri}^{rs}$ (VII)

$x_{ri}^{rs} \le g_{ri}^{rs}(k_{ri}^{rs},l_{ri}^{rs})$

$$\Sigma k_{ri} + \Sigma k_{ri}^{rs} \leq K_r^* + k_r^{sr} - k_r^{rs}$$
$$\Sigma l_{ri} + \Sigma l_{ri}^{rs} \leq L_r^* + l_r^{sr} - l_j^{rs}$$
$$h_r(y_{ri}, k_{ri}, l_{ri}, k_r^{rs}, l_r^{rs}, k_{ri}^{rs}, l_{ri}^{rs}) \geq$$
$$f_r(y_{ri}, k_{ri}, l_{jr}, k_r^{rs}, l_r^{rs}, k_{ri}^{rs}, l_{ri}^{rs})^{**}$$

All variables nonnegative

PROOF

By construction an optimal solution to (VIa) consistent with $\theta_r f_r(y_{ri},k_{ri},l_{ri}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})^{**}$is a feasible solution to (VII). And, if a feasible solution is attainable to (VII), among other things the weak Pareto preference condition $h_r(y_{ri}, k_{ri},l_{ri},k_r^{rs}, l_r^{rs},k_{ri}^{rs},l_{ri}^{rs}) \geq f_r(y_{ri}, k_{ri},l_{jr}, k_r^{rs},l_r^{rs},k_{ri}^{rs},l_{ri}^{rs})^{**}$ must obtain.

COROLLARY (*Weak Pareto preference and potential for freer exchange)*

As one class of implications a move from $f_r(\ )$ to $g_r(\ )$ may correspond to an increased preference (a reduced reluctance) for movement of factors vis a vis movements of commodities.

REMARKS

This is a generalised transfer result. In common with Theorems 1-6 this Theorem does not require that transfers be two way or that taxes or tariffs be zero at an optimum. A fortiori it does not require that exchanges (if any) be balanced. Again in common with Theorems 1-6, Theorem 7 requires no special assumptions concerning the forms of relations f( ) and g( ). In particular Theorem 6 is potentially consistent with increasing returns to scale in production in the relevant ranges of output in both regions.

If conditions (4.3) of (IVa)' are specialised to cases in which transport related externalities and transportation specific production activities and costs are as if each optimally zero they become equivalent to:

$$x_{ri}^{s}>0 \Rightarrow \quad \psi_{ri} - \psi_{si} = 0 \qquad (4.10)$$

Conditions (4.10) in turn are equivalent to those of a *(commodity) price equalisation theorem.* Here there are two classes of cases satisfying such a result via (VI) and (VIa)', the first in which $y_{ri}>0$, $y_{si}>0$, $x_{ri}^{s}>0$ so that from (4.2) $\phi_{ri}=\psi_{ri}$, $\phi_{si}=\psi_{si}$, $\psi_{ri}=\psi_{si}$, and a second class of cases for which demand for a final commodity is zero in one country and positive in the other so, for example $y_{ri}=0$, $y_{si}>0$, $x_{ri}^{s}>0$ and $\phi_{ri}=0<\psi_{ri}$ and $\phi_{si}=\psi_{si}$, $\psi_{ri}=\psi_{si}$.

Analogous *(factor) price equalisation* results hold via conditions (4.8) and (4.9) when transport cost terms are such that $d_r^{rs}=e_r^{rs}=0$.

Next; assume that potentially available technologies in region r and s are the same and that transportation costs (plus net tariffs) between the regions ($\psi_{ri}^{rs}+c_{ri}^{rs}$) are strictly *positive*. If there is an exchange of a commodity i from region r for a commodity j from region s, conditions (4.11) and (4.12) must apply:

From (5.3) $\quad x_i^{rs}>0 \Rightarrow \psi_{ri}<\psi_{si} \qquad (4.11)$

and $\quad x_j^{sr}>0 \Rightarrow \psi_{rj}>\psi_{sj} \qquad (4.12)$

If both commodities are optimally produced in both regions and if commodities are exchanged and shipping related expenses are positive then, in the absence of production factor related externalities (i.e. if the right hand terms of conditions (4.4),(4.5) are zero), it follows that the marginal rates of commodity substitution must be different in the two regions. If $g_{ri}(\ ) \equiv g_{si}(\ )$ with fixed coefficients the commodity using relatively more of the relatively more abundant factor will tend to be reduced in relatively greater quantities and so tend to be exported from each region in a manner consistent with the standard Heckscher-Ohlin result. [The argument is a marginal one and can be made by considering a two commodity two factor case in which initially a different factor is fully employed in each country and noting that, if exchange can be Pareto improving, such a solution will generally be suboptimal. (That is a Pareto improvement will then be attainable by substituting toward the relatively slack resource in each country and equilibrating demands by means of relatively greater imports of the relatively reduced output.)].

With reference to (VI) and Theorem 6 it follows that in general sufficient conditions for the optimality of Heckscher-Ohlin result in addition to those of different relative endowments of two factors in the two regions and the availability of identical production technologies are that there are no production related or factor related, or factor transportation related, externalities in either region and that commodities rather than factors are relocated at an optimum.

But (VIa) and Theorem 6 comprehend large classes of other types of optima too. As examples: with reference to (VIa) and Theorem 6, optima may obtain at variance with the conclusions of the Heckscher-Ohlin result if production related, or factor related, or factor transport related externalities are such that neither region exports, or indeed such as to determine flows from the region with relative scarcity of a factor to that with a relative abundance. [It would not *inevitably* follow that, if labour were available to members of a society, they would prefer to use relatively more labour intensive production techniques, even if capital were relatively scarce at those levels of output.] A more obvious class of exceptions is that in which, in response to interregional differentials in factor productivities, factors (rather than their products) are optimally relocated from region to region.

Now consider a movement from the conditions of (VI) in Theorem 6 under which initially $c_{ir}^{rs}, d_j^{rs}, e_j^{rs}$ are all arbitrarily large (i.e. in the absence of both of product exchanges and factor mobility) to a variant of those of (VIa) of Theorem 6 with c_{ir}^{rs} sufficiently reduced and yet d_j^{rs}, e_j^{rs} remaining arbitrarily large (corresponding to potential product exchanges and no factor mobility). In that case Theorem 6 is open to the interpretation that two regions opening to exchange under conditions which may be consistent with the Heckscher-Ohlin result (as in the preceding paragraph) may do so in a manner consistent, too, with the predictions of the Stolper-Samuelson Theorem, namely in such a way that production of relatively labour intensive commodities - and thence the relative price of labour - is reduced in the relatively labour scarce region and increased in the relatively capital abundant region. In that way, other things equal, initially different marginal rates of technical substitution between factors would tend to converge in mutually preferable (Pareto improving) directions.

Not only does Theorem 6 contain the Stolper-Samuelson result as a special case but that theorem also includes classes of potential counterexamples to that result.

An obvious class of counterexamples are those in which optimal accommodations to differences in labour (and/or capital) productivities take the form of labour (resp. capital) *migrations*. But Theorem 6 and its extension to Theorem 7 also include other classes of increasing marginal returns to factors and increasing returns to factors related and preference change related counterexamples both to the Heckscher-Ohlin results and to the Stolper-Samuelson results which can stem from the comprehensive character of the specification of the production relations g( ), as well as the general specifications of preference relations f( ) and h( ). However we will not pursue such extended classes of counterexamples here.

The important points here are that Theorem 7 admits counterexamples as well as standard Heckscher-Ohlin and Stolper-Samuelson results and the latter are special cases of Theorem 7. Another class of special cases is that corresponding to conditions under which relations f( ) and g( ) in (VIa) of Theorem are linear. That class is the subject of the next Section.

5. Specialisation to a linear production economy

Consider the following system, which in effect replaces preference relations $\theta f_r(\ )$ in the objective and final constraints in (IIa) of Theorem 3 by linear goal oriented relations, and the production relations $g_{ri}(\ )$ by linear input output systems, so that output z_{ri} from a product and region specific subset of processes $i\varepsilon I_{ir}$ is used as inputs to processes j in region r, $\Sigma a^r_{ij} z_{rj}$, and for final consumption or export, x_{ri}, as follows:

$$\begin{aligned}
\text{Max } & \Sigma\theta_r p_{ri} y_{ri} - \Sigma t_{ri}^{+} y_{ri}^{+} - \Sigma t_{ri}^{-} y_{ri}^{-} - \Sigma c_{ri}^{rs} x_{ri}^{rs} \\
& - \Sigma d_r^{rs} k_r^{rs} - \Sigma e_r^{rs} l_r^{rs} \\
\text{st } \quad & y_{ri} \leq x_{ri} + x_{si}^{sr} - x_{ri}^{rs} \\
& y_{ri} + y_{ri}^{+} - y_{ri}^{-} = y_{ri}^{*} \\
& \Sigma a^r_{ij} z_{rj} + x_{ri} \leq \sum_{i\varepsilon I_{ri}} z_{ri} \qquad \text{(VII)} \\
& \Sigma k_{ri} z_{ri} \leq K_r^{*} + k_r^{rs} - k_r^{ss} \\
& \Sigma l_{ri} z_{ri} \leq L_r^{*} + l_r^{rs} - l_r^{sr} \\
& \Sigma p_{ri} y_{ri} \geq \Sigma p_{ri}^{*} y_{ri}^{*} \qquad r=1,2
\end{aligned}$$

All variables nonnegative

REMARK

The inclusion of the possibility that output i may be supplied from a subset of available processes $i\varepsilon I_{ri}$ in region r will in due course allow direct comparisons between this model, which is directly related to the preceding Pareto improvement and exchange imbalance and factor transfer related Theorems 1-7, and models and empirical work by ten Raa and

Chakraborty 1991 for a two bloc example, which is significantly more restrictive in scope.

THEOREM 8

An optimal solution to (VII) with some $c_i^{rs}, d_r^{rs}, e_r^{rs}$ nonpremptively large will be at least weakly Pareto preferred to an optimal solution to (VII) with all d_r^{rs}, e_r^{rs} preemptively large.

PROOF

If $\Sigma p_{ri}^{*} y_{ri}^{*}$ r=1,2 represent initially attainable states and $\Sigma p_{ri}^{*} y_{ri}^{*}$ r=1,2 represent potentially preferred alternative states for individuals r=1,2, and if t_{ri}^{+}, t_{ri}^{-} all r,i, then $\Sigma p_{ri} y_{ri} \geq \Sigma p_{ri} y_{ri}^{*}$ r=1,2. It immediately follows that, according to these criteria, an optimal solution to (VII) is at least weakly Pareto preferred to the alternatives $\Sigma p_{ri} y_{ri}^{*}$ r=1,2.

REMARKS

From Theorem 8 (or equivalently from an appropriate specialisation of Theorem 6), in general exchange of resources instead of, or in addition to, commodities will be at least weakly Pareto preferable to no exchange of resources instead of, or in addition to, commodities. Also, in common with Theorems 1-6, Theorem 8 does not require that the exchange values of commodities be balanced. In common with Theorems 1-7, Theorem 8 is consistent, too, with a revealed (at least weak) Pareto preference for tax and/or subsidy regulated optima over non tax or subsidy related optima. This will be clearer from the Kuhn Tucker conditions associated with (VII) at an optimum.

Associating the indicated dual variables with the constraints of (VII) the analogues of the Kuhn Tucker conditions (VIa)' are:

y_{ri}	$\phi_{ri} \geq (\theta_r - \lambda_r) p_{ri} + \tau_{ri}$		(5.1)
y_{ri}^{+}, y_{ri}^{+}	$-t_{2i}^{+} \leq \tau_{ri} \leq t_{2i}^{-}$		(5.2)
x_{ri}	$-\phi_{ri} + \psi_{ri} \geq 0$	$i \varepsilon I_{ri}$	(5.3)
x_i^{rs}	$\psi_{ri} - \psi_{si} \geq -c_i^{rs}$		(5.4) (VIIa)'
z_{ri}	$\mu_r k_{ri} + \omega_r l_{ri} + \Sigma \psi_{rj} a^r_{ij} - \psi_{ri} \geq 0$		(5.5)
k_r^{rs}	$\mu_r - \mu_s \geq -d_r^{rs}$		(5.6)
l_r^{rs}	$\omega_r - \omega_s \geq -e_r^{rs}$		(5.7)

In a manner wholly analogous to the interpretation of conditions (4.1) of (VIa)' in Section 5, (5.1) requires that for a strictly Pareto improving optimum, $\lambda_r=0$, and that, if $y_{ri}>0$, the local net supply price $(\phi_{ri}-\tau_{ri})$ will equal the demand price p_{ri}, where, from (5.2) and complementary slackness, $y_{2i}^{+}>0 \Rightarrow \tau_{ri}=t_{2i}^{+}$ and $y_{2i}^{+}>0 \Rightarrow \tau_{ri}=t_{2i}^{-}$, so that τ_{ri} takes on contingent interpretations respectively as a (nonnegative) consumption related tax or subsidy at any weakly Pareto preferred optimum to (VII). [Production related and/or commodity or factor transport related goals, taxes and/or subsidies could be added in a similar way to (VII) and the corresponding dual relations. For a more comprehensive goal programming approach which incorporates these and other features, including endogenised transport processes and endogenised processes of capital formation in a multiproduct and multiperiod multiregional analysis see Ryan 1992.]

From (5.3) the optimal rule is to supply output of type i in region r, if at all, then from the lowest cost subset of the available set of production processes $i \varepsilon I_{ri}$, where, from conditions (5.5), the marginal cost ψ_{ri} of producing output of type i in region r is made up of the marginal cost of intermediate inputs to its production $\Sigma \psi_{rj} a^r_{ij}$ plus the marginal costs of labour and capital required for its production.

From (5.4) and (5.5) the marginal supply cost ψ_{ri} for commodity i in region r will be the lower of the import cost and the lowest attainable local production cost. Finally, from (5.6),(5.7), the marginal cost of supply of labour and capital will, in cases where these can be imported, be equal to the marginal supply cost in another region plus the net marginal interregional transfer cost d_r^{rs} (resp e_r^{rs}).

In addition to conditions (5.1)...(5.7), in this linearized case it follows by the dual theorem of linear programming that at an optimum to (VII):

$$\text{Max } \Sigma\theta_1 p_{1i} y_{1i} - \Sigma t_{1i}^{+} y_{1i}^{+} - \Sigma t_{1i}^{-} y_{1i}^{-} + \Sigma\theta_2 p_{2i} y_{2i} - \Sigma t_{2i}^{+} y_{2i}^{+} - \Sigma t_{2i}^{-} y_{2i}^{-} - \Sigma c_i^{rs} y_i^{rs} - \Sigma d_r^{rs} k_r^{rs} - \Sigma e_r^{rs} l_r^{rs}$$
$$= \text{Min}\Sigma\, \Sigma p_{ri}^{*} y_{ri}^{*} + \Sigma \mu_r K_r^{*} + \Sigma \omega_r L_r^{*} \qquad (5.8)$$

After some rearrangement (5.8) takes on the interpretation that, at an optimum, net expenditure on consumption commodities $\Sigma\theta_r p_{ri} y_{ri}$ will be equal to the revenue to labour, to capital and to transport services, plus any net expenditure on consumption related taxes and/or subsidies.

If d_r^{rs}, e_r^{rs} are assumed to be arbitrarily large (i.e factors are assumed to be immobile) and the sets $i \varepsilon I_{ri}$ and $i \varepsilon I_{si}$ are assumed to be identical (i.e. the available technologies are assumed to be the same in regions r,s), then results analogous to the Stolper-Samuelson and Heckscher-Ohlin results

derived in association with (VIa)' in Section follow immediately. But clearly other classes of results, including results for which factors may be mobile or available technologies not identical, are also available via (VII) for this linear and easily computable class of cases.
One way of emphasising the rich variety of results thus available is to compare predictions of (VII) with those of a more restrictive model due to ten Raa and Chakraborty 1991. ten Raa and Chakraborty's model is as follows:

$$
\begin{aligned}
&\text{Max } \pi(y+\mathit{y}) \\
\text{st}\quad & Ax+clx+y\leq x \\
& \mathit{Ax+clx+y\leq x} \qquad \text{(VIII)} \\
& kx\leq K,\ \ lx\leq L \\
& \mathit{kx\leq K,\ \ lx\leq L} \\
& y+\mathit{y}\geq 0 \\
& y_N \mathit{y_N}\geq 0,\ x,\mathit{x}\geq 0
\end{aligned}
$$

In (VIII) y, *y* are net exports of final consumption commodities for two economies (one italicised and the other not). Each economy has an input output technology where, according to the second and third constraints of (VIII), output x is used as inputs to production of output x plus inputs to production of labour clx. The third and fourth constraints refer to available resources of labour and capital in each economy and the final constraints require that produced outputs be nonnegative, that imports from one economy equal exports from the other and, finally, that nontradeables be positively produced in each country.

Using this model and its dual, together with a variety of empirical data for a European bloc and for India, ten Raa and Chakraborty 1991 (TRC) conclude that Europe has an absolute advantage over India in every sector of production, in the sense that, if freely available to them, Indian producers would always prefer European to Indian production technologies.

Now contrast this model and predictions with those attainable via (VII) and its dual. Three differences are immediate and striking. First, except for the implicit association of arbitrarily large potential transportation costs with non-tradeables, TRC associate no transport costs with commodities. (Nor are there any transport related production processes in their model.) Second; in the TRC model, even though by assumption European technology is freely available to India, and vice versa, factors of production, namely labour and capital, are by assumption *not* transportable between blocs. (Equivalently it is implicitly assumed by TRC that potential costs of trans-porting labour and capital between blocs would be arbitrarily large.) Thirdly; there is no explicitly welfare related justification for the adoption in the TRC model of the *net* value of final demand as a criterion of collective preference, as distinct from a Pareto improving criterion.

With the exception of Theorem 1 and the associated models, all of the models and results of our paper embody each of these features. However, for the purposes of comparison with the TRC paper we focus here just on program (VII) and its predictions:

If product price equals supply cost in each country and if the same technology is available in each country, then differences in commodity prices will essentially depend only on factor price differences between the two countries.

However, not only is free access to others' technology incredible in fact, since patents and royalties and management, as well as labour related output and earnings differentials, would in fact make a difference to the nature of available technologies, but those facts themselves call attention to the potential significance of interregional transfers of labour and/or of capital stemming among other things from skill related earnings differentials. Model (VII) incorporates this feature, as does empirical evidence on emigration and immigration rates for India vis a vis the EU. Perhaps more significantly, there is clear evidence of successful attempts by the members of the EU to introduce administrative restrictions on levels of immigration from India on the ground that such individuals would otherwise migrate in significantly larger numbers for self interested economic reasons.

This latter point, which relates to the potentially adversarial nature of individuals' membership of a labour force and/or of the wider society of which that labour force forms part, brings us to the Pareto improvement related shortcoming of the TRC approach.

In distinction from the approach by TRC, the goal programming formulation in (VII) explicitly accommodates the possibility that movements of commodities and/or of factors between economies may *not* in fact lead to at least weak Pareto improvements for the members of *both* economies. More generally, program (VII) and the even more comprehensively specified systems (II)..(VI) can be used in association with Theorems 2 through 8 to evaluate the Pareto improving possibilities, if any, of relaxing restrictions on commodity mobility and/or on factor mobility by means of changes in national or multinational regulatory regimes. Finally, an important message stemming from all of Theorems 2 through 8 is that, it will *not* follow in general that Pareto improving market arrangements will be consistent with conditions which mimic balanced budget or tariff-free and/or free labour and capital mobility conditions associated with wholly and only competitively organised markets for commodities and for factors.

6. Conclusion

In this paper we have presented very generally applicable more for less theorems with reference to transfers of commodities and of factors between individuals or regions in a production economy. It has been shown how, under extremely weak assumptions concerning the form of individual preferences and of individual production relationships, in general regulated transfers of commodities and/or of factors can be at least weakly Pareto preferable to unregulated transfers of commodities alone, and that balanced net values of transfers between individuals will in general be at least weakly Pareto dominated by unbalanced value transfers between individuals. An important policy implication of the paper is that, even where the marginal production and consumption conditions of standard models of interregional competition are attainable, appropriate transfers of resources (e.g. of labour resources by emigration and immigration) may be at least weakly Pareto preferable to exchanges of produced commodities alone.

References

Arsham H, 1992, "Post Optimality Analysis of the Transportation Problem", *Journal of the Operational Research Society,* 43, 2, 121-129.

Charnes A, S.Duffuaa and M.J.Ryan, 1980, "Degeneracy and the More for Less Paradox", *Journal of Information and Optimisation Sciences,* 1,1, 52-56.

Charnes A, S.Duffuaa and M.J.Ryan, 1987, "The More for Less Paradox in Linear Programming", *European Journal of Operational Research*, 31, 194-197.

Charnes A. and D.Klingman, 1971, "The More for Less Paradox in the Distribution Model", *Cahiers du Centre d'Etudes de Recherche Operationelle*, 13, 1, 11-22.

Gupta A, and M.C.Puri, 1995, "More (Same)-for-Less Paradox in the Minimal Cost Network Flow Problem" *Optimisation,* 33, 167-177.

Ryan M.J, 1980, "More on the More for Less Paradox in the Distribution Model" *Lecture Notes in Economics and Mathematical Systems 174: Extremal Methods and Systems Analysis,* 275-303.

Ryan M.J., 1992, *Contradiction, Self Contradiction and Collective Choice: New Directions for Characteristics and Commodities Analysis,* Avebury.

Ryan M.J., 1994, "Constrained Gaming Approaches to Production Decisionmaking Under Uncertainty", *European Journal of Operational Research*, 77, 70-81.

Ryan M.J., 1995, "Constrained Games, Intervening Duality and the Nature of Probability", *European Journal of Operational Research*, 86, 2, 366-373.

Ryan M.J., 1998, "Multiple Criteria and the Framing of Decisions", *Journal of Infor-mation and Optimization Sciences,* 19, 1, 25-42.

Ryan M.J., 2000a, "The Distribution Model, the More for Less (Nothing) Paradox and Economies of Scale and Scope", *European Journal of Operational Research,* 121,1, 92-104.

Ryan M.J., 2000b, "Economies of Scale and Scope, Contestability, Windfall Profits and Regulatory Risk", *Manchester School,* 68,6, 701-722.

Szwarc W, 1971, "The Transportation Paradox", *Naval Research Logistics Quarterly,* 18,185-202.

ten Raa, T. and D.Chakraborty 1991, "Indian Comp-arative Advantage vis a vis Europe as Revealed by Linear Programming of the two Economies", *Economic Systems Research,* 3, 2, 1991.