CHAPTER 7

INTERVENING DUALITY AND BARGAINING WITH A FARMER-LANDOWNER EXAMPLE

1. Introduction

In chapter 6 I developed the idea of intervening duality with a context of a matching pennies game in which the two players were represented as dual to the duals of an intervening coin. In this chapter I show how intervening duality can be used to model and evaluate more general kinds of opportunities for mutually advantageous gain from exchange under conditions of uncertainty.

The main focus of the chapter is on bargaining to determine the magnitudes of contingent rentals and payoffs within an intervening duality framework for a game between profit oriented landowners and land users. For this class of examples as if agreed production plans and weather forecasts play an intervening role analogous to that of a relatively neutral coin in a coin tossing game between two persons. (While landowners and land users are each separately engaged in weather forecast related constrained games against nature, both are also engaged in a profit oriented bargaining game against each other.)

The main result is the demonstration that intervening duality provides a natural way of modelling processes potentially generating mutually advantageous exchanges. In this case each of two parties may agree respectively on the elements of a relatively neutral intervening set of contingent production plans and contingent resource evaluations and weather forecasts so as to potentiate gains not only relative to those plans but in that way relative to each other *through* those plans.

The structure of the chapter is as follows: After the related concepts of constrained games and intervening duality have been introduced in Sections 2 and 3, in Sections 4 and 5 I go on to develop an intervening duality framework with particular reference to weather related risk and bargaining concerning the magnitude of agricultural rents. In this analysis each player will be playing games not just relative to “nature” as the determinant of the profitability of a farm, but relative to each other. Value will accrue to each of the two parties to a farm related production plan, among other things through the weather forecasts of the farmer and more generally production based forecasts by the owner. A constant sum numerical example in Section 6 and nonconstant sum extensions to more general bimatrix and non-preemptively framed cases in Sections 7 and 8 will help to illustrate these points.

In these ways the emphasis is on individuals’ choice of frame for an as if agreed intervening dual structure and their potentiation of gains relative to elements of it by means of principles and processes of contradiction stemming from individual differences of preferences and beliefs. In contrast to work on cross constrained gaming approaches (Charnes et al 1990, 1993), standard common knowledge based Bayesian approaches (see Aumann 1987), and evolutionary game theoretic approaches (e.g. Mailath 1992, Friedman 1996) which focus on equilibrium within a given initial frame, here initial frames and related intervening dual structures are *not* necessarily accepted or believed by the parties concerned. At best they are valued only for the opportunities that an as if agreed intervening structure can provide for gains stemming from individually determined differences of preferences and beliefs relative to them. I will return to this point in the context of choice related variations of frames in farm owner-farm renter bargaining conditions in Section 7 and also in the conclusion.

2. Constrained games

As I also noted in chapter 6 the constrained game idea is due to Charnes (Charnes 1953), and has been developed and used in various ways, see Owen 1982, Banker 1984, but particularly by authors in agricultural economics including Hazell 1970, Kawaguchi and Toyama 1972 and McInerney 1976. In all of these papers the focus is on variants of probabilistically constrained two person games. More recently Charnes et al 1990, 1993 have used the term “cross constrained game” with reference to generalisations of the Nash equilibrium solution for certain classes of n person games. Surprisingly none of these authors has used constrained games for perhaps the most obvious classes of decision theoretic and agricultural applications, namely “fair” coin tossing and weather forecast related applications. Both of these classes can usefully be explored by this approach. To see how consider a farm planning model. (Coin based applications are considered in the next Section):

> Assume that a farmer with land L must plan crops j for next season. Assume too that the area of crop j is x_j and unit profitabilities, contingent on potentially forthcoming weather conditions k, are π_{jk}, and that the farmer makes a weather forecast as if preemptively $q_k=q_k^*$. If a maximin profit objective is appropriate and with M^2 interpreted as arbitrarily large, the farmer can be represented as engaged in a (weather forecast) *constrained game* against nature to find an optimal solution to the first of programmes (I),(I)', with state conditioned expected payoffs ρ, μ:
> [Other classes of constrained game formulations for this application include those imputing a strict or weak probability ranking to nature. For more on that kind of example see Ryan 1994.]

$$
\begin{aligned}
&\text{Max } \rho \;+\sum_k R_k q_k^* \\
&\text{st } \sum_j \pi_{jk} x_j - R_k \geq \rho \qquad\qquad \text{(I)} \\
&\qquad \sum_j x_j \leq L \\
&x_j \geq 0,\;\; -M^2 \leq R_k \leq M^2
\end{aligned}
$$

$$
\begin{aligned}
&\text{Min } \mu L - M^2 \sum_k (q_k^+ + q_k^-) \\
&\text{st } \sum_k \pi_{kj} q_k \leq \mu \qquad\qquad \text{(I)'} \\
&\qquad q_k + q_k^+ - q_k^- = q_k \\
&\qquad \sum_k q_k \geq 1 \\
&\qquad q_k, q_k^+, q_k^- \geq 0
\end{aligned}
$$

Since a feasible solution to (I) always exists (consider $x_j=0$ all j), by the dual theorem optimal solutions always exist both for (I) and for (I)'. At an optimum interpretations of R_k in (I) include variously ex ante and ex post interpretations as marginal returns to weather forecasting information of type k. (Notice that either class of interpretations for R_k is consistent with a *benevolent* role for nature. By the principle of optimality an optimal solution to (I)' including elements of a weather forecast $q_k=q_k^*$ will be at least as great as optima not including an explicit weather forecast.)

The constrained game idea is a powerful one with a wide range of applications, including applications to production scheduling (see Ryan 1994) and to the representation and resolution of Allais’ paradoxes (see developments on chapter 10 below), but here I want to focus on a narrower range of agricultural applications and associated framing and intervening duality related bargaining issues. To do this I first revisit relevant framing and intervening duality ideas with the context of the matching pennies applications I used in chapter 6.

3. Constrained games, framing and intervening duality

		PLAYER 1	
		Head	**Tail**
	Head	**1**	**-1**
PLAYER 2			
	Tail	**1**	**-1**

Table 1

With payoffs as in Table 1 and assuming for simplicity that maximin-minimax assumptions are appropriate, and with q*,p* representing proportions of time committed to that particular set of potential outcomes by the two players, the simple matching pennies game can be represented as one class of solutions to (II) and (II)' below.

With p*=q* the arbitrarily large quantities M, M^2 respectively impute heads and tails probabilities e.g. $q_1^*=1/2$, $q_2^*=1/2$ to the coin, and exclude the possibility of any other coin related outcomes p_j^+, q_k^+ or of other non coin related outcomes p_j^-,q_k^-. In these ways these magnitudes explicitly *frame* the game to outcomes $j\in J$ and $k\in K$ (here heads and tails) for the two players and particular prior probabilities for nature.

$$\begin{array}{llr} \text{Max } \rho q^* + \sum_k R_k q_k^* - Mp^+ - Mp^- & & \\ \text{st} \quad \sum_j \pi_{jk} p_j - R_k \geq \rho & & \text{(II)} \\ \quad \sum_{j\varepsilon J} p_j + p^+ - p^- = p^* & & \\ p_j, p^+, p^- \geq 0, \ -M \leq \rho \leq M & & \end{array}$$

$$\begin{array}{llr} \text{Min } \mu p^* + Mq^+ + Mq^- + M^2 \sum_k (q_k^+ + q_k^-) & & \\ \text{st} \quad \sum_k \pi_{kj} q_k \leq \mu & & \text{(II)'} \\ \quad \sum_{k\varepsilon K} q_k + q^+ - q^- = q^* & & \\ \quad q_k + q_k^+ - q_k^- = q_k^* & & \\ q_k, q^+, q^-, q_k^+, q_k^- \geq 0, \ -M \leq \mu \leq M & & \end{array}$$

While this extension of the standard two person constant sum formulation may seem appropriate for a player with prior knowledge or beliefs q_k^* playing a matching pennies game against nature, the literature on coin tossing games (see Shubik 1982, Wang 1988, Hart 1992) considers another type of interpretation as one person playing against another. In the first type "nature" is associated with a specific programme, but in the second it is not. In that sense the latter kind of problem is incompletely specified. This immediately suggests the intervening duality idea. According to it the two persons would be represented as playing against each other by acting as if dual to the duals of a (thereby intervening) coin. With (III),(IIIa)' representing the game of the second player against nature, this gives a structure with (II),(III) representing the players and (IIa)',(IIIa)' the intervening coin.

By construction, if $S_j'=0$, (IIa)' becomes equivalent to (II)' and dual to (II). Similarly, if $R_k'=0$, (IIIa)' becomes equivalent to a system (III)' dual to (III). In a matching pennies context this is consistent with two players agreeing to a common heads-tails related frame with: i) $j\varepsilon J=$ {heads, tails}, $k\varepsilon K=${heads,tails}; ii) common prior probabilities $p_j^*=1/2p^*$, $q_k^*=1/2q^*$ and; iii) as if indifference (via $R_k'=0,S_j'=0$) to coin related outcomes relative to the system and yet; iv) via outcomes $S_j\neq 0,R_k\neq 0$, *not* necessarily indifferent to outcomes relative to self.

$$\begin{array}{llr} \text{Max } \rho q^* + \sum_k R_k q_k^* - Mp^+ - Mp^- & & \\ \text{st} \quad \sum_j \pi_{jk} p_j - R_k \geq \rho & & \text{(II)} \\ \quad \sum_j p_j + p^+ - p^- = p^* & & \\ p_j, p^+, p^- \geq 0, \ -M \leq \rho \leq M \ \ -M^2 \leq R_k \leq M^2 & & \end{array}$$

$$\begin{array}{llr} \text{Min } \mu p^* + \sum_j S_j' p_j^* + Mq^+ + Mq^- + M^2 \sum_k (q_k^+ + q_k^-) & & \\ \text{st} \quad \sum_k \pi_{kj} q_k - S_j' \leq \mu & & \text{(IIa)'} \\ \quad q_k + q_k^+ - q_k^- = q_k^* & & \\ \quad \sum_k q_k + q^+ - q^- = q^* & & \\ q_k, q^+, q^-, q_k^+, q_k^- \geq 0, \ -M \leq \mu \leq M \ \ -M^2 \leq S_j' \leq M^2 & & \end{array}$$

$$\begin{array}{llr} \text{Min } \mu p^* + \sum_j S_j p_j^* + Mq^+ + Mq^- & & \\ \text{st} \quad \sum_k \pi_{kj} q_k - S_j \leq \mu & & \text{(III)} \\ \quad \sum_k q_k + q^+ - q^- = q^* & & \\ q_k, q^+, q^- \geq 0, \ -M \leq \mu \leq M \ \ -M^2 \leq S_j \leq M^2 & & \end{array}$$

$$\begin{array}{llr} \text{Max } \rho q^* + \sum_k R_k' q_k'^* - Mp^+ - Mp^- - M^2 q_k^+ - M^2 q_k^- & & \\ \text{st} \quad \sum_j \pi_{jk} p_j - R_k' \geq \rho & & \text{(IIIa)'} \\ \quad p_j + p_j^+ - p_j^- = p_j^* & & \\ \quad \sum_j p_j + p^+ - p^- = p^* & & \\ p_j, p^+, p^-, p_j^+, p_j^- \geq 0, \ -M \leq \rho \leq M \ \ -M^2 \leq R_k' \leq M^2 & & \end{array}$$

As one class of solutions (IIIa)',(IIIa)' are dual to each other if $q_k^*=p_j^*=1/2$ and $R_k'=0$, $S_j'=0$. More subtly, optimality of (IIIa)' with values $\rho=0$, $p_1=p_2=1/2$, $R_k'=0$ is consistent with optimality of the relatively dual programme (III) with values $\mu=0$, $q_1=q_2=1/2, S_j=0$. Similarly optimality of (IIa)' with values $\mu=0, q_1=q_2=1/2$, $S_j'=0$ is consistent with optimality of the relatively dual system (II) with values $\rho=0$, $p_1=p_2=1/2$, $R_k=0$.

Together these various optima are consistent with a coin tossing story according to which two individuals act as if indifferent (via conditions $R_k'=0, S_j'=0$) to the informational value of outcomes of a single toss of a fair coin *both* relative to the system *and,* via conditions as if optimally $R_k=0$ and $S_j=0$ as if indifferent to those outcomes relative to themselves. But, if both players were in any case indifferent to the informational value of a toss of a coin, why would they choose to toss it?

Rather than pursue this question in detail notice that alternative optima exist for (II) and (III) with $R_k \neq 0, S_1 \neq 0$. In particular, with $\mu=0, p_1=p_2=1/2$, $R_1=1, R_2=-1$ the intervening dual structure (II),(IIa)',(III),(IIIa)', is open to interpretation as if two individuals with relatively *oppositely* evaluated preferences and/or beliefs concerning heads and tails outcomes relative to themselves nevertheless agree to frame an as if agreed "fair" structure (II)',(III)' for an intervening dual heads-tails game, but now with the potential, via evaluations $R_k \neq 0$ and $S_j \neq 0$ to generate net gains ex post.

Parenthetically, not only do (II)' and (III)' potentially yield alternative optima, respectively relative to (II) and (III), with $R_k'=0$, $q_k=1/2$, $k \varepsilon K$ and $S_j'=0$, $p_j=1/2$, $j \varepsilon J$. But with these values both systems are linearly dependent and so degenerate. Both conditional degeneracy and alternative optima may be removed in such a way as to be consistent with individuals' subjective preference for particular outcomes relative to self over relatively neutral outcomes relative to a wider system by the introduction of appropriately weighted perturbations. (See Charnes 1951.) For example, conditions $q_k^*=1/2$ in (IVa)* might be perturbed to $q_1^*=1/2+\varepsilon_1, q_2^*=1/2-\varepsilon_2$, $\varepsilon_1 \neq \varepsilon_2$ to give:

$$
\begin{array}{ll}
\text{Max}\,\rho + R_1(1/2+\varepsilon_1) - R_1(1/2+\varepsilon_1) - Mp^+ - Mp^- & \\
\text{st} \quad 1p_1 - 1p_2 - R_1 \geq \rho & \\
\quad -1p_1 + 1p_2 - R_2 \geq \rho & \\
 & \text{(II)*} \\
\quad p_1 + p_2 + p^+ - p^- = 1 & \\
p_j, p^+, p^- \geq 0, \; -M \leq \rho \leq M \quad -M^2 \leq R_k \leq M^2 &
\end{array}
$$

$$
\begin{array}{ll}
\text{Min}\,\mu + \Sigma S_j' 1/2 + Mq^+ + Mq^- + M^2 \Sigma(q_k^+ + q_k^-) & \\
\text{st} \quad 1q_1 - 1q_2 - S_j' \leq \mu & \\
\quad -1q_1 + 1q_2 - S_j' \leq \mu & \\
q_1 + q_1^+ - q_1^- = 1/2 + \varepsilon_1 & \text{(IIa)*'} \\
q_2 + q_2^+ - q_2^- = 1/2 - \varepsilon_2 & \\
q_1 + q_2 + q^+ - q^- = 1 & \\
q_k, q^+, q^-, q_k^+, q_k^- \geq 0, \; -M \leq \mu \leq M \quad -M^2 \leq S_j \leq M^2 &
\end{array}
$$

With $p_1=1, p_2=0, R_1=1, R_2=-1$ this is consistent with conditions as if, via the objective of (II)*, one potential outcome (e.g. "heads") will be revealed as if strictly preferred to the other (e.g. "tails") by the first individual. Analogous perturbations and corresponding conditions $q_1=1, q_2=0, S_1=1, S_2=-1$ would lead to relatively opposite "heads" and "tails" related conclusions with reference to a system (III)* and the second individual. In these ways such dually related perturbations and potentials are consistent with interpretations in relation in relation to relative *bias* of outcomes in general, and in relation to bias of coins in particular.

Matching pennies applications are of interest in themselves, but they have been introduced here primarily because the framing and intervening duality ideas associated with them generalise to production scheduling cases and more particularly to crop related weather forecast constrained agricultural bargaining applications.

4. Farm related interpretations of constrained games

Agricultural decisionmakimg applications and interpretations of kinds considered in relation to (I),(I)' suggests interpretations for (II),(II)' according to which p_j, p_j^+ and p_j^- respectively represent proportions of available land L given over to crops j, land left fallow (or rented out to other farmers), and land rented in from other farmers, with q_k^* representing elements of a weather forecast. In short (II),(II)' are open to interpretations corresponding to a class of weather constrained games against nature analogous to an individual playing a coin tossing game against nature.

Although such interpretations may be appropriate to a class of owner managed farm applications, in the real world large numbers of farmers are tenants, not owners, and are simultaneously engaged in weather related crop planning games against nature and weather related land rent determining bargaining with one or more landowners. An intervening duality framework analogous to (II),(IIa)', (III),(IIIa)' seems particularly appropriate for analyses of such cases.

5. Intervening duality specifications for farm planning

Assume that a farm renter makes production plans by forming a weather forecast and then solving a problem of the form of (II),(II)' with p_j relating to the proportion of land devoted to crops $j \varepsilon J$ etc, as in the preceding section. One outcome of such a process would be the imputation of a marginal value μ to his/her land contingent on that plan and consistent with the minimisation of the marginal opportunity cost of its use - i.e. with the efficient utilisation of that level of availability of land to the renter.

Now consider an intervening duality structure with farmer and owner related problems (IV),(IVa)' and (V),(Va)' analogous to (II),(IIa)' and (III),(IIIa)'. If, other things equal, the farm owner's objective is to maximise rental income, this leads to:

$$\begin{aligned}
&\text{Max } \rho q^* + \sum_k R_k q_k^* - Mp^+ - Mp^- \\
&\text{st} \quad \sum_j \pi_{jk} x_j - R_k \geq \rho \qquad \text{(IV)} \\
&\qquad \sum_j x_j + x^+ - x^- = L \\
&x_j, x^+, x^- \geq 0, \ -M \leq \rho \leq M \ \ -M^2 \leq R_k \leq M^2
\end{aligned}$$

$$\begin{aligned}
&\text{Max } \mu L - \sum_k S_k x_k^* + Mq^+ + Mq^- \\
&\text{st} \quad \sum_k \pi_{jk} q_k + S_j \geq \mu \qquad \text{(V)} \\
&\qquad \sum_k q_k + q^+ - q^- = q^* \\
&q_k, q^+, q^{--} \geq 0, \ -M \leq \rho \leq M \ \ {}_-M^2 \leq S_j \leq M^2
\end{aligned}$$

$$\begin{aligned}
&\text{Min } \mu L + \sum_j S_j' x_j^* + Mq^+ + Mq^- + M^2 \sum_k (q_k^+ + q_k^-) \\
&\text{st} \quad \sum_k \pi_{kj} q_{k} - S_j' \leq \mu \qquad \text{(IVa)'} \\
&\qquad q_k + q_k^+ - q_k^- = q_k^* \\
&\qquad \sum_k q_k + q^+ - q^- = q^* \\
&q_k, q^+, q^-, q_k^+, q_k^- \geq 0, \ -M \leq \mu \leq M \ \ -M^2 \leq S_j' \leq M^2
\end{aligned}$$

$$\begin{aligned}
&\text{Min } \rho q^* - \sum_k R_k' q_k^* + Mx^+ + Mx^- + M^2 \sum_j (x_j^+ + x_j^-) \\
&\text{st} \quad \sum_j \pi_{kj} x_j + R_k' \leq \rho \qquad \text{(Va)'} \\
&\qquad x_j + x_j^+ - x_j^- = x_j^* \\
&\qquad \sum_j x_j + x^+ - x^- = L \\
&x_j, x^+, x^-, x_j^+, x_j^- \geq 0, \ -M \leq \mu \leq M \ \ -M^2 \leq R_k' \leq M^2
\end{aligned}$$

Assuming that all other input expenses are subsumed into π_{jk} via $\pi_{jk} =_{def}$(contingent revenue jk) - (variable input expense jk), (IV),(1Va)' are consistent with a farm renter's game against nature, the value μ then being the maximum rate of payment of rent at the margin consistent with normal profit for the farm renter. Similarly (V),(Va)' are consistent with an owner's game against nature with μ being the maximum rate of payment of rent at the margin consistent with the association of a prior production plan x_j^* and weather forecast q_k with that land by the owner.

Clearly in general solutions to the dual pairs (IV),(IVa)' and (V),(Va)' would be inconsistent with each other. But there is one exceptional class of cases for which q_k^* is optimal for (IV) and p_j^* is optimal for (V). These conditions in turn correspond to elements of a bargaining related intervening duality structure as follows:

Within frames as if preemptively determined via arbitrarily large weights M, M^2 a prospective renter could form a weather forecast and determine an optimal production plan via (IV),(Va)' (as above). Then the farm owner, given that plan, and as if via (Va)', could agree not just to the rentals but to the weather forecasts q_k^* implicit in it via a correspondingly optimal solution to (V).

In these ways both parties may act as if to teach each other and to learn from each other from relatively external conditions and thence to agree on weather forecast-ing and planning information and on profits and costs. That is: as if all such relatively external information becomes common information.

Further, if these predictions are correct, prior information q_k^*, x_j^* becomes as if perfectly predictive of posterior information q_k, x_j with the consequence that optimally $R'=0, S'=0$. Alternatively in these circumstances as if posterior information relative to one party becomes as if perfectly predictive of prior information relative to the other, and conversely so that for such cases each party would act as if optimally to *validate* the other's plans.

But to get such equivalence in general R_k and S_j (if nonzero) will be relatively opposite in sign. This in turn is consistent with relatively opposed profit motives at the margin between land owner and land user - since at an optimum at the margin for one party land is a cost whereas for the other it is a reward.

6. A numerical example

Consider a crop 1:crop 2: rain:no rain example with L=100 and contingent payoffs and forecast probabilities as in Table 2 and (IV)*,(IVa)*', (V)*,(Va)*':

	FORECAST PROBABILITIES		CONTINGENT PAYOFFS	
	CASE A	*CASE B*	*CROP 1*	*CROP 2*
RAIN	**no forecast**	**0.25**	**50**	**30**
NO RAIN	**no forecast**	**0.75**	**20**	**40**

Table 2

$$\text{Max } \rho q^* + \sum_k R_k q_k^* - Mp^+ - Mp^-$$

$$\begin{aligned} \text{st} \quad & 50x_1 + 30x_2 - R_1 \geq \rho \qquad \text{(IV)*} \\ & 20x_1 + 40x_2 - R_2 \geq \rho \\ & \sum_j x_j + x^+ - x^- = L \\ & x_j, x^+, x^- \geq 0, \ -M \leq \rho \leq M \ \ -M^2 \leq R_k \leq M^2 \end{aligned}$$

$$\text{Min } \mu L + \sum_j S_j' x_j^* + Mq^+ + Mq^- + M^2 \sum_k (q_k^+ + q_k^-)$$

$$\begin{aligned} \text{st} \quad & 50q_1 + 20q_2 - S_1' \leq \mu \qquad \text{(IVa)*'} \\ & 30q_1 + 40q_2 - S_2' \leq \mu \\ & q_k + q_k^+ - q_k^- = q_k^* \\ & \sum_k q_k + q^+ - q^- = q^* \\ & q_k, q^+, q^-, q_k^+, q_k^- \geq 0, \ -M \leq \mu \leq M \ \ -M^2 \leq S_j' \leq M^2 \end{aligned}$$

$$\text{Max } \mu L - \sum_k S_j x_j^* + Mq^+ + Mq^-$$

$$\begin{aligned} \text{st} \quad & 50q_1 + 20q_2 + S_1 \geq \mu \qquad \text{(V)*} \\ & 30q_1 + 40q_2 + S_2 \geq \mu \\ & \sum_k q_k + q^+ - q^- = q^* \\ & q_k, q^+, q^{--} \geq 0, \ -M \leq \mu \leq M \ \ -M^2 \leq S_j \leq M^2 \end{aligned}$$

$$\text{Min } \rho q^* - \sum_k R_k' q_k^* + Mx^+ + Mx^- + M^2 \sum_j (x_j^+ + x_j^-)$$

$$\begin{aligned} \text{st} \quad & 50x_1 + 30x_2 + R_1' \leq \rho \qquad \text{(Va)*'} \\ & 20x_1 + 40x_2 + R_2' \leq \rho \\ & x_j + x_j^+ - x_j^- = x_j^* \\ & \sum_j x_j + x^+ - x^- = L \\ & x_j, x^+, x^-, x_j^+, x_j^- \geq 0, \ -M \leq \rho \leq M \ \ -M^2 \leq R_k' \leq M^2 \end{aligned}$$

CASE A (No explicit weather forecast.)

With data as in Table 2 and $S_j'=0$, start with the farm user versus nature pair (IV)*,(IVa)*' in the intervening duality structure (IV)*,(IVa)*',(V)*, (Va)*' and Bayes-Laplace-like prior conditions $q_k^*=1/2$ in (IVa)*'. The optimum for (IVa)*' then has $q_1=q_2=1/2$, $\mu L=3500$, with $S_1'=S_2'=0$. The corresponding optimum for (IV)* has $x_1=25$, $x_2=75$ with $\rho q^*=3500$ and $R_1=R_2=0$.

Next set $x_1^*=25$, $x_2^*=75$ in (Va)*'. The optimum is then $x_1=25$, $x_2=75$ with $\rho q^*=3500$ and $R_1=R_2=0$ and the correspondingly dual optimum to (V)* is $q_1=q_2=1/2$, $\mu L=3500$, with $S_1=S_2=0$. These values in turn are as if perfectly predictive of $q_k^*=1/2$ in the initial system (IVa)*'. This is consistent with the statement above, viz:

Further, if these predictions are correct, prior information q_k^*, x_j^* becomes as if perfectly predictive of posterior information q_k, x_j with the consequence that optimally $R'=0, S'=0$. Alternatively in these circumstances as if posterior information relative to one party becomes as if perfectly predictive of prior information relative to the other, and conversely so that for such cases each party would act as if optimally to validate the other's plans.

CASE B (Explicit weather forecast).

With data as in Table 2 start with the farm user versus nature pair (IV)*,(IVa)*' and conditions $q_1^*=0.25$, $q_2^*=0.75$ in (IVa)*'. The optimum for (IVa)*' then has $q_1=0.25$, $q_2=0.75$, $\mu L=3750$, with $S_1'=S_2'=0$. The corresponding optimum for (IV)* then has $x_1=0$, $x_2=100$ with $\rho q^*=4000$ $R_1=-1000$, $R_2=0$.

Next set $x_1^*=100$, $x_2^*=0$, $q_1^*=0.25$ $q_2^*=0.75$ in (Va)*' The optimum for that system is then $x_1=100$, $x_2=0$ with $\rho q^*=4000$ and $R_1'=1000$, $R_2'=0$. The corresponding optimum to (V)* conditional on the farm owner accepting the weather forecast $q_1=0.25$, $q_2=0.75$ is then $\mu L=3750$, with $S_1'=0$, $S_2'=0$. (If the owner does not accept the farm user's prior weather forecast and acts as if ignoring the term $\Sigma R_k' q_k^*$ in (Va)* ' the optimal solution to (V)* has $\mu L=4250$, with $q_1=0.75$, $q_2=0.25$ and $S_1'=0$, $S_2'=5$.)

In this way the prospective farm user with maximin anticipated return $\rho q^*=4000$ via (IV)* conditional on the weather forecast $q_1^*=0.25$, $q_2^*=0.75$ in (IVa)*', is optimistic relative to the return $\mu L=3750$ in (IVa)*' (and optimistic *a fortiori* relative to the weather related probabilities $q_1^*=0.5$, $q_2^*=0.5$, as in Case A.) Put another way, the prospective farm user's optimal plan stemming from the dual systems (IVa)*,(IVa)*' is such that the expected return $\rho q^*=4000$ can be interpreted as made up of a potentially guaranteed imputation $\mu L=3750$ to a land owner plus a weather contingent premium $\Sigma R_k q_k^*=250$.

Similarly the owner with a maximin expected return relative to self of $\mu L=3750$ in (V)* contingent on acceptance of the farm user's weather forecast is again apparently optimistic the 3500 which would be anticipated under Case A. (The owner would be acting as if optimistic *a fortiori* relative to the no forecast Case A if they did not accept the user's forecast and offered the alternative $\mu L=4250$, with $q_1=0.75$, $q_2=0.25$ and $S_1'=0$, $S_2'=5$.)

These various evaluations are relatively optimistic and pessimistic via measures S_1', S_2' and R_1, R_2 respectively in a manner analogous to the coin tossing case. Also analogously to the coin tossing case there is potentially room for a mutually advantageous exchange - in this case of land at a rental of 3750 in exchange for cash - if the land owner accepts the land user's initial forecast. Alternatively of course the process could continue, for example by the farm owner responding to the (from their viewpoint) relatively pessimistic forecast $q_1=0.25$, $q_2=0.75$ and seeking to modify/ perturb the prior forecasts in (IVa)*' to accord more closely with the relatively unconstrained forecast $q_1=0.75$, $q_2=0.25$. (In that case essentially subjective information relative to the second party becomes as if objective relative to the first, potentially to the mutually advantageous bargaining advantages of both.)

Summarising: whereas there is a sense in which the nil prior information solution is consistent with - indeed is as if perfectly predictive of - optimality for (IVa)*' and thence for (IV)* nevertheless in the earlier sequence, in which both parties acted, via $S_j'=S_j=0$, as if indifferent at the margin to weather related information, each one's planned weather forecast and land based rental evaluations is as if perfectly predictive of the other's. In that sense, for each of the two players (owner and user) plans and weather forecasts are as if unanimously agreed, not just relative to the system, but relative to themselves. However other optima exist (as in Case B) where, via potentials $R_k\neq 0, S_j\neq 0$ and thence relatively oppositely oriented measures of non zero gains/losses relative to self at the margin, there is room for individuals to *disagree* both relative to values R' and S' relative to a wider system and, through that system, relative to their relatively abstract selves. (Again there is an analogy with coin tossing cases. Unless there are potential oppositions of marginal evaluations μ, ρ, why seek to gain from negotiations to let land? Equivalently, if evaluations of the opportunity set open to the land in question were known to be identical for both potential lessor and for potential lessee, there would be no rational purpose in (continuing) a rent related process of bargaining between them.)

7. Strategic equivalence and bargaining

Consider transformations $\pi_{kj}^{s\prime}=_{def}\theta^s\pi_{kj}+c_s$, $0<\theta^s$, $c_s>0$ with s=1,2,3,4 corresponding to (IV),(IVa)', (V),(Va)'. If $[\rho,x_j,R_k]$, $[\mu,q_k,S_j']$ and $[\mu,q_k,S_j]$, $[\rho,x_j,R_k']$ are respectively optimal for those systems those solutions remain feasible (but not necessarily optimal) with these transformed values. If also $\underline{R_k}=_{def}\theta^s R_k$, $\underline{S_j}'=_{def}\theta^s S_j'$, $\underline{R_k}'=_{def}\theta^s R_k'$, $\underline{S_j}=_{def}\theta^s S_j$ then, because of the preemptive nature of the framing weights M, M^2, optimal values $[\rho,x_j,R_k]$, $[\mu,q_k,S_j']$ and $[\mu,q_k,S_j]$, $[\rho,x_j,R_k']$ remain optimal in (IV),(IVa)',(V),(Va)'. Under those conditions the valuations π_{kj} and the linear transformations $\pi_{kj}^{s\prime}=_{def}\theta^s\pi_{kj}+c_s$ of them are *strategically equivalent*. Four classes of special cases are:

First, if $\theta^1=\theta^2$ and $c_1=c_2$ and $\underline{R_k}=_{def}\theta^s R_k$,$\underline{S_j}'=_{def}\theta^s S_j'$ the dual pair (IV), (IVa)' with $\pi_{kj}^{s\prime}=_{def}\theta^s\pi_{kj}+c_s$ is strategically equivalent to (IV), (IVa)' with $\pi_{kj}'=\pi_{kj}$ in a manner which will be familiar in the context of the standard two person constant sum game. (Though by contrast to the standard constant sum case, here there is explicit *framing* via the explicit specification of ranges of outcomes via preemptive penalties on x^+,x^-, q^+,q^-, and of prior probabilities

q_k* again via preemptive penalties, in this case on magnitudes q_k^+, q_k^-.).

Secondly, in a manner analogous to the previous case, if $\theta^3=\theta^4$ and $c_3=c_4$ and $\underline{S_j}={}_{def}\theta^s S_j$, $\underline{R_k}'={}_{def}\theta^s R_k'$, the dual pair (V), (Va)' with $\pi_{kj}{}^{s\prime}={}_{def}\theta^s\pi_{kj}+c_s$ is strategically equivalent to (V), (Va)' with $\pi_{kj}'=\pi_{kj}$.

Thirdly, starting with four distinct sets of values $[\theta^s, c_s]$, each of the two preceding classes of cases itself comprehends two distinct classes of subcases according to which solutions e.g. with $[\theta^1, c_1]$ became strategically equivalent to solutions with $[\theta^2, c_2]$, or conversely.

Fourthly, the preceding classes of cases together illustrate the more general point that conditions of strategic equivalence may exist according to which: i)for every pair s_1, s_2, $\theta^{s1}\neq\theta^{s2}$, $c_{s1}\neq c_{s2}$ yet ; ii)for every such pair problems (IV), (IVa)',(V) and (Va)' with payoffs $\pi_{kj}{}^{s1\prime}={}_{def}\theta^{s1}\pi_{kj}+c_{s1}$, $0<\theta^{s1}$, $c_{s1}>0$ are strateg-ically equivalent to equivalently framed games with payoffs $\pi_{kj}{}^{s2\prime}={}_{def}\theta^{s2}\pi_{kj}+c_{s2}$, $0<\theta^{s2}$, $c_{s2}>0$.

Here "equivalently framed" is understood to imply not only that preemptive framing weights M, M^2 apply, but that values $[\underline{R_k}={}_{def}\theta^{s1}R_k$, $\underline{S_j}'={}_{def}\theta^{s1}S_j'$, $\underline{R_k}'={}_{def}\theta^{s1}R_k'$, $\underline{S_j}={}_{def}\theta^{s1}S_j]$ and $[\underline{R_k}={}_{def}\theta^{s2}R_k$, $\underline{S_j}'={}_{def}\theta^{s2}S_j'$, $\underline{R_k}'={}_{def}\theta^{s2}$, $\underline{S_j}={}_{def}\theta^{s2}S_j]$ are respectively optimal in (IV), (IVa)', (V) and (Va)'.

This variety of strategically equivalent cases is consistent with the fact that in practice each set of payoffs may be determined differently according to the players and according to the system in a correspondingly wide variety of different ways. For instance π_{kj} in Table 2 may correspond to contingent profit contributions and μ to an imputed rental via (IVa)' for a farm user relative to a farm owner. But the farm owner may attract a lump sum payment based on ownership of land L and a tax on rental income such that a transformation $\pi_{kj}{}^{s3\prime}={}_{def}\theta^{s3}\pi_{kj}+c_{s3}$ with $0<\theta^{s3}<1$, $c_{s3}>0$ becomes the appropriate measure of relative gain/loss for the owner in (V). Then, even disregard-ing the quantities R_k, S_j', R_k', S_j, which in any case have nonconstant sum related implications, the intervening duality system corresponds to a particular kind of nonconstant sum bimatrix game related structure.

Thus, given an invariant frame via appropriately preemptive weights M,M^2, it is possible to determine a wide variety of (possibly interrelating) strategically equivalent specific-ations. But in practice it may be inappropriate to treat the frame as if preemptively specified. In the next Section I consider more general cases, first in which conditions relating to relatively *interior* probabilities are non preemptive and secondly cases in which conditions on relatively *exterior* probabilities are non preemptive. In short in the real world frames are mutable. In coin tossing cases individuals could choose to modify frames by choosing to devote more or less time (e.g. no time) to coin toss related activities, whereas a farm owner and/or a farm user could change a land related frame by renting out or renting in land.

8. Frame related variations in bargaining outcomes

So far I have considered preemptive frame related specifications and outcomes. Now consider potentially non preemptive generalisations of (IV), (IVa)', (V), (Va)' as in (VI),(VI)',(VII),(VII)' below.

Clearly if penalties c^+, c^-, d^+, d^-, e_k^+, e_k^-, c_j^+, c_j^- are arbitrarily large this intervening duality specification is equivalent to (IV), (IVa)', (V), (Va)'. But it also comprehends nonpreemptive alternatives. Consider a land related example.

With reference to land, if c^+ in (VI),(VI)' were sufficiently reduced (became sufficiently negative) the farm user might optimally respond by selecting $x^+>0$ in (VI) and planting relatively less land to crops. In turn this would correspond to a planned variation $x_j^+>0$ some j in (VII)'. Assuming that c^+ and crop specific magnitudes c_j^+ were sufficiently small, this would vary the optimal solutions via $\rho=-c^+$ in (VII)' and $S_j=-c_j^+$ in (VII) and may generate variations relative to the initial forecast q_k* in (VI)'.

A numerical illustration with reference to data in Table 2 will help here. Recall that the optimal solution to the preemptive specification (IVa)*' had $\mu L=3750$. But with $c^+=-40<c^-$ (i.e. non-preemptive) and the other parameters as in (IV)*,(IVa)*', (V)*,(Va)*' the optimal solution to (VI)' becomes $\mu L=4000$ with $S_1'=S_2'=0$. This is consistent in turn with a risk (and user related profit) free optimal solution to (VI) with $-c^+x^+=4000$ and $x_j=0$, $R_k=40$, $x^+=100$. In this case the opportunity cost of *any* kind of weather is 40 i.e. with reference to this particular plan, the farm user would be indifferent with respect to changes in forecasts for particular weather types. (This is unsurprising since in this case the optimal plan involves no output of crops.)

$$\text{Max } \rho q^* \ + \sum_k R_k q_k^* - c^+x^+ - c^-x^-$$

$$\text{st} \quad \sum_j \pi_{jk} x_j - R_k \geq \rho \qquad \text{(VI)}$$

$$\sum_j x_j + x^+ - x^- = L$$

$$x_j, x^+, x^- \geq 0, \ -d^- \leq \rho \leq d^+ \ \ -e_k^- \leq R_k \leq e_k^+$$

$$\text{Max } \mu L - \sum_k S_j x_j^* + d^+q^+ + d^-q^-$$

$$\text{st} \quad \sum_k \pi_{jk} q_k + S_j \geq \mu \qquad \text{(VII)}$$

$$\sum_k q_k + q^+ - q^- = q^*$$

$$q_k, q^+, q^- \geq 0, \ -c^- \leq \mu \leq c^+ \ -c_j^- \leq S_j \leq c_j^+$$

$$\text{Min } \mu L + \sum_j S_j' x_j^* + d^+q^+ + d^-q^- + \sum_k (e_k^+ q_k^+ + e_k^- q_k^-)$$

$$\text{st} \quad \sum_k \pi_{kj} q_k - S_j' \leq \mu \qquad \text{(VI)'}$$

$$q_k + q_k^+ - q_k^- = q_k^*$$

$$\sum_k q_k + q^+ - q^- = q^*$$

$$q_k, q^+, q^-, q_k^+, q_k^- \geq 0, \ -c^+ \leq \mu \leq c^- \quad -c_j^+ \leq S_j' \leq c_j^-$$

$$\text{Min } \rho q^* - \sum_k R_k' q_k^* + c^+x^+ + c^-x^- + \sum_j (c_j^+ x_j^+ + c_j^- x_j^-)$$

$$\text{st} \quad \sum_j \pi_{kj} x_j + R_k' \leq \rho \qquad \text{(VII)'}$$

$$x_j + x_j^+ - x_j^- = x_j^*$$

$$\sum_j x_j + x^+ - x^- = L$$

$$x_j, x^+, x^-, x_j^+, x_j^- \geq 0, \ -d^- \leq \rho \leq d^+ \ \ -e_k^- \leq R_k' \leq e_k^+$$

Given the zero production plan with $x_j^*=0$ via (VII)' and if the owner accepts the weather forecast q_k^* the correspondingly optimal solutions to (VII),(VII)' give $\mu L=4000$ with $q_1=0.25$, $q_2=0.75$, $S_1=7.5$, $S_2=2.5$ and $c^+x^+=4000$ with $x_j=0$, $x^+=100$, $R_k=0$. (The context underlines the interpretation of the quantities S_j as marginal evaluators of opportunity costs, in this case of *not* producing particular crops j.)

In this way an intervening duality system provides a means of modelling the workings of a *setaside scheme* inducing a farm owner and user to agree to the setting aside of land on the grounds that the associated risk free payments to the owner would exceed respectively minimax and maximin opportunity cost related - rewards to alternative uses of the land.

Still in the farming context, if c^- in (VI), (VII)' was sufficiently reduced, the unit cost of additional supplies of land from a relatively external rental market would become equal to the farm user's own risk related assessment μ of the internal opportunity cost of land. In that case the user - or in the setaside case, the owner - might optimally plan to rent not just all of this particular owner's land but more at that rate from that relatively outside source. (Technically in that case (VI) and (VII)' would become unbounded but that could easily be remedied by appending an appropriate upper bound on the quantities x^- to each of those systems.)
With reference to a weather forecast related interpretation of (VII)' it has already been seen how, via a sequence of changes relative to programmes (VI)', (VI), (VII)', (VII), (VI)'.., changes in returns to alternative uses of land c^+, c^- might prompt changes relative to weather forecasts q_k^*. As an extension evidently such a sequence might *start* with a changed weather forecast in (VI)' - however induced - leading e.g. via $q_k^+>0$ and, by complementary slackness $R_k=e_k^+$ in (VI), to a change in predicted maximin weather related returns ρq^* and thence to a change relative to the initially predicted crop planting plan in (VII)' and minimax payoff μ in (VII).

In general, considering c^+ and c^- parametrically, these quantities may be used to potentiate *switches* from a relatively interior choice between relatively uncertain (crop related) alternative and a relatively certain (fallow) related alternative and a relatively exterior alternative of renting out all of the available land. (For cases in which the relatively interior and uncertain alternative was a coin toss, analogous extensions might relate to rewards to a riskless alternative use of leisure time and to marginal rewards to work time respectively.)

9. Conclusion and extensions

In this chapter I have considered in detail just one class of land and weather constrained owner-user related bargaining applications of the intervening duality idea. Also, analyses and developments here have focussed on known production processes and associated costs and contingent revenues. One direction for extensions would be to draw on work in Ryan 1994 and include multiply resource constrained production decisionmaking problems. Another would be toward more explicitly intertemporal specifications and criteria.

I close by emphasising the potential in the intervening duality systems which have been developed here for modelling teaching and learning. The opportunity for developments and interpretations of that kind is clear once it is recognised that the intervening duality approach comprehends the potential for one individual to convey new information to another about the very existence, as well as magnitudes and contingent payoffs for particular outcomes (e.g. a particular crop). It does this in ways in which individuals may interact with the explicit intention of gaining from others by expanding their opportunity sets. This feature alone makes this approach fundamentally different from others, including the cross constrained (see Charnes et al 1990,1993, Li 1996). evolutionary (see Mailath 1992, Friedman 1996), or Bayesian game theoretic approaches (see Aumann 1987), or of the method of production decisionmaking with recourse of (White 1992). Those approaches respectively emphasise strategic dominance, long run equilibrium, experiment based validation /learning, all together with the desirability of securing (optimal) long run decisionmaking processes *within an initially given frame/set of potential outcomes*. By contrast one of the novel and powerful features of the intervening duality approach is that it incorporates opportunities for individuals to gain by learning from each other and/or by reframing programmes making up an intervening structure, as well as potentially gaining by exploiting opportunities for mutually advantageous exchanges within such frames.

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