Parabolic distribution

$$\sigma^2 = \int_{\mu-a}^{\mu+a}(x-\mu)^2 f(x)\,dx$$

$$\sigma^2 = \int_{\mu-a}^{\mu+a}(x-\mu)^2 \frac{3}{4a}\left[1-\left(\frac{x-\mu}{a}\right)^2\right]dx$$

$$\left(\frac{x-\mu}{a}\right)=t$$

$$\frac{dx}{a}=dt$$

$$\sigma^2 = \int_{-1}^{+1}a^2 t^2 \frac{3}{4a}\left(1-t^2\right)a\,dt$$

$$\frac{4}{3}\left(\frac{\sigma}{a}\right)^2 = \int_{-1}^{+1}t^2\left(1-t^2\right)dt =$$

$$=\left[\frac{t^3}{3}-\frac{t^5}{5}\right]_{-1}^{+1}=2\left(\frac{1}{3}-\frac{1}{5}\right)=\frac{4}{15}$$

$$\left(\frac{\sigma}{a}\right)^2 = \frac{3}{15}=\frac{1}{5}$$

$$\boxed{\sigma = \frac{a}{\sqrt{5}}}$$

❖