24.5 A small bank has two tellers, who are equally efficient and who are each capable of handling an average of 60 customer transactions per hour, with the actual service times exponentially distributed. Customers arrive at the bank according to a Poisson process, at a mean rate of 100 per hour. Determine (*a*) the probability that there are more than three customers in the bank at the same time, (*b*) the probability that a given teller is idle, and (*c*) the probability that a customer spends more than 3 min in the bank.

This is an M/M/2 system, with $\lambda = 100$ and $\mu = 60$. Since

$$\rho = \frac{100}{2(60)} = \frac{5}{6} < 1$$

steady-state conditions will prevail eventually. Using (*24.5*), we calculate

$$\frac{1}{p_0} = \frac{2^2(5/6)^3}{2!\,[1-(5/6)]} + \sum_{n=0}^{2} \frac{(5/3)^n}{n!} = \frac{125}{18} + \frac{1}{0!}\left(\frac{5}{3}\right)^0 + \frac{1}{1!}\left(\frac{5}{3}\right)^1 + \frac{1}{2!}\left(\frac{5}{3}\right)^2 = 11$$

or $p_0 = 1/11 = 0.0909$. The remaining steady-state probabilities are then determined from (*24.6*) as

$$p_1 = \frac{(5/3)^1}{1!}\left(\frac{1}{11}\right) = 0.1515$$

$$p_2 = \frac{(5/3)^2}{2!}\left(\frac{1}{11}\right) = 0.1263$$

$$p_3 = \frac{2^2(5/6)^3}{2!}\left(\frac{1}{11}\right) = 0.1052$$

$$p_4 = \rho p_3 = \frac{5}{6}(0.1052) = 0.0877$$

and so on.

(*a*)
$$1 - (p_0 + p_1 + p_2 + p_3) = 1 - (0.0909 + 0.1515 + 0.1263 + 0.1052) = 0.5261$$

(*b*) A given teller is idle if there are no customers in the bank or if there is one customer in the bank and that customer is being served by the other teller.

$$p_0 + \tfrac{1}{2}p_1 = 0.0909 + \tfrac{1}{2}(0.1515) = 0.1667$$

(*c*) Using (*24.8*), we find the probability that a customer will spend more than 3 min, or 1/20 h, in the bank to be

$$W\left(\frac{1}{20}\right) = e^{-60(1/20)}\left\{1 + \frac{(5/3)^2(1/11)[1 - e^{-60(1/20)(2-1-(5/3))}]}{2![1-(5/6)][2-1-(5/3)]}\right\} = 0.4113$$

24.6 A state department of transportation has three safety investigation teams who are on call continuously and whose job it is to analyze road conditions in the vicinity of each fatal accident on a state road. The teams are equally efficient; each takes on the average 2 days to investigate and report on an accident, with the actual time apparently exponentially distributed. The number of fatal accidents on state roads appears to follow a Poisson process, at a mean rate of 300 per year. Determine L, L_q, W, and W_q for this process and give meaning to each of these quantities.

This is an M/M/3 process, with $\lambda = 300$ accidents per year, $\mu = 365/2 = 182.5$ reports per team per year, and

$$\rho = \frac{300}{3(182.5)} = \frac{40}{73}$$

To evaluate L_q by *(24.7)*, we must first determine p_0. From *(24.5)*,

$$\frac{1}{p_0} = \frac{3^3(40/73)^4}{3![1-(40/73)]} + \sum_{n=0}^{3} \frac{1}{n!}\left(\frac{300}{182.5}\right)^n$$

$$= 0.89737 + \frac{1}{0!}\left(\frac{300}{182.5}\right)^0 + \frac{1}{1!}\left(\frac{300}{182.5}\right)^1 + \frac{1}{2!}\left(\frac{300}{182.5}\right)^2 + \frac{1}{3!}\left(\frac{300}{182.5}\right)^3 = 5.63263$$

Hence $p_0 = 1/5.63263 = 0.177537$. Then,

$$L_q = \frac{3^3(40/73)^4(0.177537)}{3!\,[1-(40/73)]^2} = 0.3524$$

On the average, the department has a backlog of 0.3524 accidents.

Using *(23.6)*, with $\bar{\lambda} = \lambda = 300$, we have

$$W_q = \frac{1}{300}(0.3524) = 0.001175 \text{ year} = 0.429 \text{ day}$$

There elapses, on the average, slightly less than $\frac{1}{2}$ day between a fatal accident and the start of its investigation.

It follows from *(23.4)* that

$$W = 0.001175 + \frac{1}{182.5} = 0.006654 \text{ year} = 2.429 \text{ days}$$

On the average, it takes slightly less than $2\frac{1}{2}$ days for the department to complete its work once a fatal accident has occurred.

Finally, we determine from *(23.5)* that

$$L = 300(0.006654) = 1.996 \text{ accidents}$$

On the average, the department has nearly two cases under its jurisdiction, awaiting final action.