23.3 The men's department of a large store employs one tailor for customer fittings. The number of customers requiring fittings appears to follow a Poisson distribution with mean arrival rate 24 per hour. Customers are fitted on a first-come, first-served basis, and they are always willing to wait for the tailor's service, because alterations are free. The time it takes to fit a customer appears to be exponentially distributed, with a mean of 2 min. (*a*) What is the average number of customers in the fitting room? (*b*) How much time should a customer expect to spend in the fitting room? (*c*) What percentage of the time is the tailor idle? (*d*) What is the probability that a customer will wait more than 10 min for the tailor's service?

This is an M/M/1 system, with $\lambda = 24\ \text{h}^{-1}$,

$$\mu = \tfrac{1}{2}\ \text{min}^{-1} = 30\ \text{h}^{-1}$$

and $\rho = 24/30 = 0.8$.

(*a*) From (*23.7*),

$$L = \frac{0.8}{1-0.8} = 4 \text{ customers}$$

(*b*) From (*23.9*),

$$W = \frac{1}{30-24} = \tfrac{1}{6}\ \text{h} = 10\ \text{min}$$

The result also follows from (*23.5*):

$$W = \frac{1}{\lambda} L = \frac{1}{24}(4) = \tfrac{1}{6}\ \text{h}$$

(*c*) The tailor is idle if and only if there is no customer in the fitting room. The probability of this event is given by (*23.3*) as

$$p_0 = \rho^0(1-\rho) = 1(1-0.8) = 0.2$$

The tailor is idle 20 percent of the time.

(*d*) From (*23.12*), with $t = 10\ \text{min} = \tfrac{1}{6}\ \text{h} = W$,

$$W_q(\tfrac{1}{6}) = (0.8)e^{-1} = 0.2943$$

23.4 For the system of Problem 23.3, determine (*a*) the average wait for the tailor's service experienced by all customers, (*b*) the average wait for the tailor's service experienced by those customers who have to wait at all.

Supplementary Problems

23.14 The take-out counter at an ice cream parlor is serviced by one attendant. Customers arrive according to a Poisson process, at a mean arrival rate of 30 per hour. They are served on a FIFO basis, and, because of the quality of the ice cream, they are willing to wait if necessary. The service time per customer appears to be exponentially distributed, with a mean of $1\frac{1}{2}$ min. Determine (*a*) the average number of customers waiting for service, (*b*) the amount of time a customer should expect to wait for service, (*c*) the probability that a customer will have to spend more than 15 min in the queue, and (*d*) the probability that the server is idle.

23.15 A barber runs a one-man shop. He does not make appointments but attends customers on a first-come, first-served basis. Because of the barber's reputation, customers are willing to wait for service once they arrive; arrivals follow a Poisson pattern, with a mean arrival rate of two per hour. The barber's service time appears to be exponentially distributed, with a mean of 20 min. Determine (*a*) the expected number of customers in the shop, (*b*) the expected number of customers waiting for service, (*c*) the average time a customer spends in the shop, and (*d*) the probability that a customer will spend more than the average amount of time in the shop.

23.16 The arrival pattern of cars to a single-lane, drive-in window at a bank appears to be a Poisson process, with a mean rate of one per minute. Service times by the teller appear to be exponentially distributed, with a mean of 45 s. Assuming that an arriving car will wait as long as necessary, determine (*a*) the expected number of cars waiting for service, (*b*) the average time a car waits for service, (*c*) the average time a car spends in the system, and (*d*) the probability that there will be cars waiting in the street if bank grounds can hold a maximum of five automobiles.

23.17 Aircraft request permission to land at a single-runway airport on an average of one every 5 min; the actual distribution appears to be Poisson. Planes are landed on a first-come, first-served basis, with those not able to land immediately due to traffic congestion put in a holding pattern. The time required by the traffic controller to land a plane varies with the experience of the pilot; it is exponentially distributed, with a mean of 3 min. Determine (*a*) the average number of planes in a holding pattern, (*b*) the average number of planes that have requested permission to land but are still in motion, (*c*) the probability that an arriving plane will be on the ground in less than 10 min after first requesting permission to land, and (*d*) the probability that there are more than three planes in a holding pattern.

23.18 A typist receives work according to a Poisson process, at an average rate of four jobs per hour. Jobs are typed on a first-come, first-served basis, with the average job requiring 12 min of the typist's time; the actual time per job appears to be exponentially distributed about this mean. Determine (*a*) the probability that an arriving job will be completed in under 45 min, (*b*) the probability that all jobs will have been completed by the typist at the end of the business day, and (*c*) the probability that a job will take less than 12 min to complete once the typist begins it.

CHAPTER 23

23.14 (*a*) 2.25, (*b*) 4.5 min, (*c*) 0.062, (*d*) 0.25.

23.15 (*a*) 2, (*b*) 1.33, (*c*) 1 h, (*d*) 0.368.

23.16 (*a*) 2.25, (*b*) 2.25 min, (*c*) 3 min, (*d*) 0.178.

23.17 (*a*) 0.9, (*b*) 1.5, (*c*) 0.7364, (*d*) 0.07776.

23.18 (*a*) 0.528, (*b*) 0.2, (*c*) 0.632.

23.19 \$16.80.

23.20 Yes, with expected daily savings of \$105.

23.21 110 ft^2.

23.22 None on L or L_q; W is reduced by 1/2.

23.23 $\rho^{n-2}(1-\rho)$.

23.24 $(1-\rho)^{-1}$.

23.26 The expected rate of transitions into state n is $\lambda p_{n-1} + \mu p_{n+1}$ (or μp_1, if $n = 0$); the expected rate transitions out of state n is $\lambda p_n + \mu p_n$ (or λp_0, if $n = 0$). Equating these and dividing through by μ gi (*1*) and (*2*) of Problem 23.7.

23.27

$$F(z) = \frac{p_0}{1 - \rho z}$$

23.28 By Theorem 21.1, the departure stream is a Poisson process *while the server is busy*. This is the cas fraction ρ of the time; hence, the expected number of departures in a unit time interval is

$$\rho\mu + (1-\rho)(0) = \lambda$$