# Goal Programming

**Note:** See problem 13.13 for the problem statement. We assume that part-time (fractional) workers are allowed.

**Example 1: Preemptive Goal Programming**

The problem is currently stated as a preemptive goal program. In a preemptive GP, we have one LP/ILP/MILP for each priority level. If you are working on priority level $k$, the LP/ILP/MILP looks like:

$$\text{min sum of weighted deviations for level } k \text{ goals}$$

subject to

$$\begin{gathered}\text{ALL functional constraints}\\ \text{ALL goal constraints}\\ \text{ALL nonfunctional constraints}\\ OF_1 = v_1^*\\ OF_2 = v_2^*\\ \vdots\\ OF_{k-1} = v_{k-1}^*\end{gathered}$$

where $OF_j$ is the objective function for the $j$-th priority level and $v_j^*$ is the optimal *value* of the objective function for the $j$-th priority level ($1 \le j \le k-1$).

Let
$x_1$ = number employees assigned to phones
$x_2$ = number employees assigned to door-to-door

The **functional constraints** (i.e. the constraints that MUST be satisfied) are

$$x_1 + x_2 \le 35 \qquad \text{there are 35 employees total}$$

The **goals** may be written as constraints as follows:

1. Level 1

   - Achieve (at least) \$20K in expected weekly sales

$$20(0.06)(400)x_1 + 20(0.20)(150)x_2 \ge 20000$$

- Spend no more than \$10K in weekly salaries

$$240x_1 + 300x_2 \leq 10000$$

2. Level 2

   - Reach (at least) 6000 potential customers per week

$$400x_1 + 150x_2 \geq 6000$$

3. Level 3

   - Assign at least 10 employees to phones

$$x_1 \geq 10$$

   - Assign at least 10 employees to door-to-door

$$x_2 \geq 10$$

Based on the above, we can see that the *detrimental deviations* are

| *Goal* | *Deviation* | *Reason* |
|---|---|---|
| 1 | $U_1$ | implies that sales were *under* \$20,000 |
| 2 | $E_2$ | implies that salaries *exceeded* \$10,000 |
| 3 | $U_3$ | implies that the potential customers reached was *under* 6,000 |
| 4 | $U_4$ | implies that the number employees assigned to phones was *under* 10 |
| 5 | $U_5$ | implies that the number employees assigned to door-to-door was *under* 10 |

Priority Level 1 Program

There are two goals in priority level one. These goals are equally important so they can have the same weight.

$$\min U_1 + E_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 35 \\ 480x_1 + 600x_2 + U_1 - E_1 &= 20000 \\ 240x_1 + 300x_2 + U_2 - E_2 &= 10000 \\ 400x_1 + 150x_2 + U_3 - E_3 &= 6000 \\ x_1 + U_4 - E_4 &= 10 \\ x_2 + U_5 - E_5 &= 10 \\ x, U, E &\geq 0 \end{aligned}$$

Priority Level 2 Program

There is one goal in priority level two. The priority one objective is now included as a constraint, highlighted below.

$$\min U_3$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 35 \\ 480x_1 + 600x_2 + U_1 - E_1 &= 20000 \\ 240x_1 + 300x_2 + U_2 - E_2 &= 10000 \\ 400x_1 + 150x_2 + U_3 - E_3 &= 6000 \\ x_1 + U_4 - E_4 &= 10 \\ x_2 + U_5 - E_5 &= 10 \\ \mathbf{U_1 + E_1} &= \mathbf{v_1^*} \\ x, U, E &\geq 0 \end{aligned}$$

Priority Level 3 Program

There are two goals in priority level three. These goals are equally important so they can have the same weight. The priority one and two objectives are now included as constraints, highlighted below.

$$\min U_4 + U_5$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 35 \\ 480x_1 + 600x_2 + U_1 - E_1 &= 20000 \\ 240x_1 + 300x_2 + U_2 - E_2 &= 10000 \\ 400x_1 + 150x_2 + U_3 - E_3 &= 6000 \\ x_1 + U_4 - E_4 &= 10 \\ x_2 + U_5 - E_5 &= 10 \\ \mathbf{U_1 + E_1} &= \mathbf{v_1^*} \\ \mathbf{U_3} &= \mathbf{v_2^*} \\ x, U, E &\geq 0 \end{aligned}$$

The Preemptive GP Solution

If you are using the template, you can just enter in the functional constraints and the 5 goals, specifying their priority levels then run solver. But, if you are not using the templates, you'll need to solve the 3 LPs IN THE ORDER they are listed above. When we do this, we obtain the following:

| Level | $x_1$ | $x_2$ | Alternate Optima | OF Value |
|---|---|---|---|---|
| 1 | 3.57 | 30.48 | Yes | 0 |
| 2 | 3.57 | 30.48 | Yes | 0 |
| 3 | 8.33 | 26.67 | No | 1.67 |

Based upon the solution to the level 3 program, we can state the following:

| Goal | Achieved | Amt Under | Amt Over |
|---|---|---|---|
| 1 | Yes | 0 | 0 |
| 2 | Yes | 0 | 0 |
| 3 | Yes | 0 | 1333.33 |
| 4 | No | 1.67 | 0 |
| 5 | Yes | 0 | 16.67 |

**Example 2: Non-Preemptive Goal Programming**

A non-preemptive goal program looks like

min sum of weighted deviations for ALL goals

subject to

ALL functional constraints
ALL goal constraints
ALL nonfunctional constraints

Suppose we weren't given the priority levels for the KarKleen problem. Suppose instead we were given the following information:

- The sales goal and the salary goal are equally important.
- The employee assignment (phone or door-to-door) goals are equally important.
- The sales goal is twice as important as the potential customers goal.
- The potential customers goal is three times as important as either employee assignment goal.

We can use the above to develop *relative weights*. Let $w_i$ be the weight for goal $i$, $(1 \leq i \leq 5)$.

- Since the employee assignment goals are the least important (according to the above) let $w_4 = w_5 = 1$.

- Since the potential customers goal is three times as important as either employee assignment goal, we can let $w_3 = 3$.
- Since the sales goal is twice as important as the potential customers goal, we can let $w_1 = 2w_3 = 2(3) = 6$.
- Since the sales goal and the salary goal are equally important, we can let $w_2 = w_1 = 6$.

There non-preemptive goal program would be

$$\min 6U_1 + 6E_2 + 3U_3 + U_4 + U_5$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 35 \\ 480x_1 + 600x_2 + U_1 - E_1 &= 20000 \\ 240x_1 + 300x_2 + U_2 - E_2 &= 10000 \\ 400x_1 + 150x_2 + U_3 - E_3 &= 6000 \\ x_1 + U_4 - E_4 &= 10 \\ x_2 + U_5 - E_5 &= 10 \\ x, U, E &\geq 0 \end{aligned}$$

It turns out that the optimum solution is the same as that for the non-preemptive goal programming. But this need not always be the case.