9.3 For the data of Fig. 9-2, determine a shipping schedule that meets all demands at a minimum total cost.

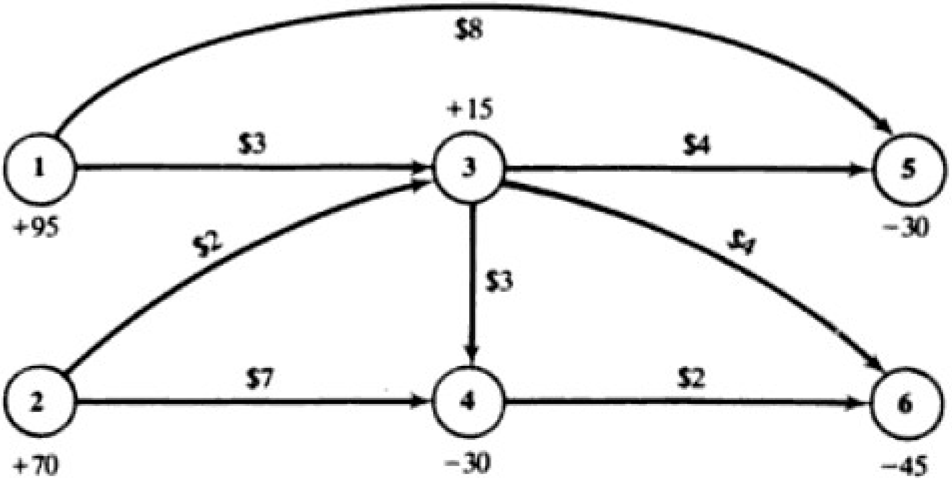

Fig. 9-2

Locations 1 and 2 are sources, while locations 5 and 6 are destinations. Location 3 is both a source and a junction, whereas location 4 serves both as a destination and a junction. Because total supply is 180 units but total demand is only 105 units, location 7 is created as a dummy destination with a demand of 180 − 105 = 75 units. Since every junction is made both a source and a destination, by adding 180 units to both its supply and its demand, the transportation tableau will involve sources 1, 2, 3, 4, and destinations 3, 4, 5, 6, 7. Besides the costs given in Fig. 9-2, we have zero as the cost from a junction (as a source) to itself (as a destination), zero as the cost from any source to the dummy, and an excessive amount ($10 000) as the cost over any nonexistent link (e.g., 1 → 6).

Tableau 3 is the optimal transportation tableau. Location 3 receives 20 units from location 1 and 70 units from location 2, whereupon it redistributes these units along with its own initial supply of 15 units to locations 4, 5, and 6. After all demands have been satisfied, location 1 will remain with 75 units, indicated

Destinations

	3	4	5	6	(dummy) 7	Supply	u_i
1	3 **20**	10000 (9994)	8 (1)	10000 (9993)	0 **75**	95	3
2	2 **70**	7 (2)	10000 (9994)	10000 (9994)	0 (1)	70	2
3	0 **90**	3 **30**	4 **30**	4 **45**	0 (3)	195	0
4	10000 (10003)	0 **180**	10000 (9999)	2 (1)	0 (6)	180	−3
Demand	180	210	30	45	75		
v_j	0	3	4	4	−3		

Tableau 3

in Tableau 3 by the allocation from location 1 to the dummy. The allocations $x^*_{33} = 90$ and $x^*_{44} = 180$ are book entries signifying the numbers of units that do not pass through junctions 3 and 4, respectively.

9.4 Solve Problem 1.13 by the Hungarian method.

Table 1-1 of Problem 1.13 is expanded to make the number of events equal to the number of swimmers; the result is Tableau 4A. As usual, costs (times) associated with the dummies, events 5 and 6, are taken to be zero. The rationale here is that events 5 and 6 do not exist, so they can be completed in zero time; swimmers assigned to these events will be the ones not entered in the four-swimmer relay.

The Hungarian method is initiated by subtracting 0 from every row of Tableau 4A and then subtracting 65, 69, 63, 55, 0, and 0 from columns 1 through 6, respectively; this generates Tableau 4B. Since this matrix does not contain a zero-cost feasible solution, we cover the existing zeros by as few horizontal and vertical lines as possible. One such covering is that shown in Tableau 4B; another, equally good, is obtained by replacing the line through row 3 by a line through column 4. The smallest uncovered element is 1, appearing in the (2, 2) position. Subtracting 1 from every uncovered element in Tableau 4B and adding 1 to every element covered by two lines—the (1, 5), (1, 6), (3, 5), (3, 6), (5, 5), and (5, 6) elements—we arrive at Tableau 4C.

Tableau 4C also does not contain a feasible zero-cost assignment. Repeating Step 3 of the Hungarian method, we determine that 1 is again the smallest uncovered element. Subtracting it from each uncovered