CHEN, Der-San, Robert G. BATSON, Yu DANG,
"Applied Integer Programming: modeling and solution", Wiley, 2010

Chapter 3 — Transformation using 0-1 variables

3.6 Transform Nonsimultaneous Constraints

3.6.1 Either/or constraints

Two disjunctive regions: x is outside the interval (3, 10)

$$x \le 3 \quad \vee \quad x \ge 10$$

This becomes, exclusively,

$$\begin{cases} x - 3 \le 0 \\ -x + 10 \le 0 \end{cases}$$

Let M be a big (enough) number and y binary.

$$\begin{cases} x - 3 \le M\ y \\ -x + 10 \le M(1 - y) \end{cases}$$

Verify. If $y = 1$, the 2.nd constraint (only) applies:

$$\begin{cases} (x - 3 \le M \approx \infty) \\ -x + 10 \le 0 \end{cases}$$

If $y = 0$, the 1.st constraint (only) applies:

$$\begin{cases} x - 3 \le 0 \\ (-x + 10 \le M \approx \infty) \end{cases}$$

3.6.2 *p* out of *m* constraints must hold

This case is a direct generalization of the previous one, where it was $m = 2$ and $p = 1$.

$$f_i(\mathbf{x}) \le b_i \qquad i = 1..m$$

$$f_i(\mathbf{x}) - b_i \le 0 \qquad i = 1..m$$

With vector **y** (i.e., y_i, $i = 1..m$) binary,

$$f_i(\mathbf{x}) - b_i \le M\ y_i \qquad i = 1..m$$

$$\sum_{i=1}^{m} y_i = m - p$$

3.6.3 Disjunctive constraint sets

3.6.4 Negation of a constraint

(Obvious.)

3.6.5 If/then constraints

$$\text{If } f_1(\mathbf{x}) - b_1 \le 0 \quad \text{then} \quad f_2(\mathbf{x}) - b_2 \le 0$$

is equivalent to

$$\text{Either} - f_1(\mathbf{x}) + b_1 \le 0 \quad \text{or} \quad f_2(\mathbf{x}) - b_2 \le 0$$

By “either/or”,

$$\begin{cases} -f_1(\mathbf{x}) + b_1 \le M\,y \\ f_2(\mathbf{x}) - b_2 \le M(1-y) \end{cases}$$

Solved (related) problems in the book

Problem text page	Solution page	Problem No.
48	425	2.3
49	427	2.4
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50	428	2.7
50	428	2.11
51	429	2.13
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73	430	3.4
74	431	3.6
75	432	3.10
75	433	3.11
75	434	3.12

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