Questions 1 (Hints and solutions start here.)

Q 1..1 A paper mill produces rolls of paper each of which is 18 ft. wide. These are then cut into various widths as required by customers. A roll can be cut into narrower rolls as many times as necessary.

a) A customer requires rolls in widths of 9 ft., 7ft. and 5 ft. In how may different ways can the 18 ft. rolls be cut to yield (one or more of) these sizes? In each case, say how much waste is created.

b) An order is received for 10 rolls 9 ft. wide, 20 rolls 7 ft. wide and 50 rolls 5 ft. wide. Formulate a linear programming problem to fill the order using the minimum number of rolls. Do *not* solve this problem, but say what difficulties you see in putting the solution into practice.

c) Now suppose that surplus rolls less than 5 ft. wide are sold for $\pounds k$ per ft., so that a roll of width $w < 5$ is sold for $\pounds kw$. Suppose also that a standard roll costs $\pounds P$ to produce, and that each cut costs $\pounds C$. Formulate a linear programming problem to satisfy the requirements at minimum net cost.

Q 1..2 A factory makes three types of small decorative garden sculpture, known as Bashful, Dozy and Happy. Their manufacture is done using three different machines, called A, B and C, which can be used in any order. The number of hours needed on each of the three machines to make each sculpture is given in the following table

	A	B	C	Profit/unit
Bashful	2	1	2	£2
Dozy	1	0	3	£4
Happy	0	3	2	£3

which also gives the net profit made on each of the sculptures. Machine A is available for 43 hours per week, machine B for 37 and machine C for 42 hours per week.

The problem is to decide how many of each type should be made each week in order to maximise the profit on the operation. Formulate the problem as a linear programming problem. [You are *not* asked to solve the problem.]

Q 1..3 A pharmaceutical company is creating a tablet for a new drug. Each tablet is to contain a binder, a disintegrant and a filler in addition to the active drug ingredient, which is to be 14% of the weight of each tablet. Chemical and physical considerations mean that the weight of the disintegrant should not exceed 25% of the combined weights of the binder and the active ingredient, and that there should be at most 10 times as much filler as binder. The disintegrant costs £15, the binder £50 and filler £2 per kilogram.

The problem is to decide how to formulate the tablet in order to minimise its cost. Express the problem as a linear programming problem. [You are *not* asked to solve the problem.]

Q 1..4 A manufacturer makes a range of three types of car, A, B and C in two factories; an engine plant E and a body factory F. The nett profit on a car of type A is £1100, on type B it is £1200 and on type C it is £1450. There are 10120 labour units (a labour unit is approximately one ``man hour") available in E and 11000 labour units in factory F each month and the number of labour units, needed to build the various products is given in Table 1.2.

Table 1.2: Productions times

Car type	Engine	Body
A	8	8
B	8	9
C	9	11

Formulate the problem of maximising the profit subject to these constraints as a linear programming problem. [You are *not* asked to solve it.]

One purported solution of this problem shows no cars of type B are made. Is this plausible? Describe briefly three ways in which this mathematical model of car production might be modified to be more realistic.

Q 1..5 A Natural Food store makes up three types of muesli, which it sells as ``Crunchy", Healthy" and ``Rich". Each type is made by mixing different proportions of cereals, nuts and dried fruit. Existing advertising material means that the restrictions and selling prices given in Table 1.3 are fixed.

Table 1.3: Required muesli ingredients and selling prices.

Type	Cereal	Dried Fruit	Nuts	Selling price per kilo (£)
Crunchy			At least 60%	1.60
Healthy	At least 60%		At most 20%	1.20
Rich	At most 20%	At least 60%		2.00

The suppliers can deliver at most 100 kilos of cereal at £ 0.80 per kilo, 80 kilos of dried fruit at £1.50 per kilo and 60 kilos of nuts at £1.00 per kilo each week. The store is able to sell all the muesli that it mixes. Formulate the problem of finding the most profitable mixing scheme as a linear programming problem. You are *not* asked to solve the problem.

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