Machine Shop Scheduling

Our next task is to show that problems of this sort can occur in ``practical" situations; that problems of interest in the ``real world" lead to this type of *constrained optimisation* problem.

A machine shop makes two products called (rather unimaginatively) A and B. Product A can be made with two options -- as A_1 and A_2, while product B is available in options B_1, B_2 and B_3. The machine shop makes the two products using an appropriate combination of three machines, which can be used in any order. The production contract requires that 60 units of item A and 85 units of item B be produced per week, although they can be produced in any of the various options. The objective of the exercise is to determine the product mix that is most profitable. The situation is summed up in Table 1.1.

Table 1.1: Machine shop costs.

Product	Option	Unit production time on machine number			Unit Profit
		1	2	3	
A	1	0.5	-	0.2	2
	2	-	0.4	0.2	2.5
B	1	0.4	0.3	-	5
	2	0.4	-	0.3	4
	3	-	0.6	0.3	4
Hours per week that machines are available		38	31	34	

In order to write down in detail what is required, we need to introduce suitable variables.

> *Choosing variables is often the hardest part of the whole process. One way is to think what you need to know in order to solve the problem -- give the orders or instruct the foreman. Such variables are often known as* ***decision variables*** *because knowing their values enables a decision to be made. In this case, the decision is ``how many of each option of each product do we make each week?"*

It is thus natural to introduce the following variables. Let x_1 be the number of units of product A_1 to be produced per week, x_2 be the number of units of product A_2 to be produced per week, x_3 be the number of units of product B_1 to be produced per week, x_4 be the number of units of product B_2 to be produced per week and x_5 be the number of units of product B_3 to be produced per week.

The profit from such a product mix is given by

$$P = 2x_1 + 2.5x_2 + 5x_3 + 4x_4 + 4x_5,$$

and this is the function (of x_1, x_2,..., x_5) that we wish to maximise. The constraints are of three sorts:

$$\begin{aligned} x_1 + x_2 &= 60, \\ x_3 + x_4 + x_5 &= 85, \end{aligned} \qquad \text{(Required production)}$$

$$\begin{aligned} 0.5x_1 + 0.4x_3 + 0.4x_4 &\leq 38, \\ 0.4x_2 + 0.3x_3 + 0.6x_5 &\leq 31, \\ 0.2x_1 + 0.2x_2 + 0.3x_4 + 0.3x_5 &\leq 34, \end{aligned} \qquad \text{(Machine time)}$$

$$x_i \geq 0 \quad \text{for each } i. \qquad \text{(Reality)}$$

Solving this constrained optimisation problem then gives the values of x_1, x_2,..., x_5 which give the most profit for this particular contract.

Remark 1..1 Much of the remainder of the course is devoted to solving such problems. When you can, and have enough facility with MAPLE, come back to this problem. You should find that the problem is feasible and that the maximum profit is 520 units.

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