Aztec Refinery

Problem 1.7 (& 3.7)

Bronson & Naadimuthu, 2001, Schaum — pp 8 & 51

$$z = 12(x_1 + x_2) + 14(x_3 + x_4) - 8(x_1 + x_3) - 15(x_2 + x_4) = \qquad \{1\}$$
$$= 4x_1 - 3x_2 + 6x_3 - x_4$$

$$x_1 + x_2 \le 100000 \qquad \{2\}$$

$$x_3 + x_4 \le 20000 \qquad \{3\}$$

$$x_1 + x_2 \ge 50000 \qquad \{4\}$$

$$x_3 + x_4 \ge 5000 \text{ (corrected)} \qquad \{5\}$$

$$x_1 + x_3 \le 40000 \qquad \{6\}$$

$$x_2 + x_4 \le 60000 \qquad \{7\}$$

$$87\frac{x_1}{x_1 + x_2} + 98\frac{x_2}{x_1 + x_2} \ge 88 \qquad 87\frac{x_3}{x_3 + x_4} + 98\frac{x_3}{x_3 + x_4} \ge 93$$

$$87x_1 + 98x_2 \ge 88(x_1 + x_2) \qquad 87x_3 + 98x_4 \ge 93(x_3 + x_4)$$

$$-x_1 + 10x_2 \ge 0 \qquad \{8\}$$

$$-6x_3 + 5x_4 \ge 0 \qquad \{9\}$$

$$25\frac{x_1}{x_1 + x_2} + 15\frac{x_2}{x_1 + x_2} \le 23 \qquad 25\frac{x_3}{x_3 + x_4} + 15\frac{x_3}{x_3 + x_4} \le 23$$

$$25x_1 + 15x_2 \le 23(x_1 + x_2) \qquad 25\frac{x_3}{x_3 + x_4} + 15\frac{x_3}{x_3 + x_4} \le 23$$

$$2x_1 - 8x_2 \le 0 \qquad \{10\}$$

$$2x_3 - 8x_4 \le 0 \qquad \{11\}$$

Finally,

$$[\max]\, z = 4x_1 - 3x_2 + 6x_3 - x_4 \qquad \{12\}$$

subject to

$$x_1 + x_2 \le 100000$$
$$x_3 + x_4 \le 20000$$
$$x_1 + x_2 \ge 50000$$
$$x_3 + x_4 \ge 5000$$
$$x_1 + x_3 \le 40000$$
$$x_2 + x_4 \le 60000$$
$$x_1 - 10x_2 \le 0$$
$$6x_3 - 5x_4 \le 0$$
$$2x_1 - 8x_2 \le 0$$
$$2x_3 - 8x_4 \le 0$$
$$\mathbf{x} \ge 0$$

Problem in *lp* format (suitable to CPLEX and Lindo) and in canonical (matrix) format:

Problem in *lp* format

```
max
4x1 -3x2 +6x3 -x4
subject to
x1 + x2 < 100000
x3 + x4 < 20000
x1 + x2 > 50000
x3 + x4 > 5000
x1 + x3 < 40000
x2 + x4 < 60000
x1 - 10x2 < 0
6x3 -5x4 < 0
2x1 -8x2 < 0
2x3 -8x4 < 0
end
```

Problem in canonical (matrix) format

```
+1 :max|min (+1|-1)
4 -3 +6 -1 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 +100
0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 +20
1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 +40
0 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0 +60
1 -10 0 0 0 0 0 0 1 0 0 0 0 0 0 0 +0
0 0 6 -5 0 0 0 0 0 1 0 0 0 0 0 0 +0
2 -8 0 0 0 0 0 0 0 0 1 0 0 0 0 0 +0
0 0 2 -8 0 0 0 0 0 0 0 1 0 0 0 0 +0
1 1 0 0 0 0 0 0 0 0 0 0 -1 0 1 0 +50
0 0 1 1 0 0 0 0 0 0 0 0 0 -1 0 1 +5
15 16 artificials
1e+3 :bigM
5 6 7 8 9 10 11 12 15 16 :initial basis
```

Solution: $z^* = 125\,000$

$\mathbf{x}$ = (37 727.(27), 12 272.(72), 2 272.(72), 2 272.(72)) or

$\mathbf{x}$ = (40 000, 10 000, 0, 5 000) (more ?)