



Acceptance Sampling

Acceptance Sampling

- Accept/reject entire lot based on sample results
- Created by Dodge and Romig during WWII
- Not consistent with TQM of Zero Defects
- Does not estimate the quality of the lot



What is acceptance sampling?

Lot Acceptance Sampling

- A SQC technique, where a random sample is taken from a lot, and upon the results of appraising the sample, the lot will either be rejected or accepted
- A procedure for sentencing incoming batches or lots of items without doing 100% inspection
- The most widely used sampling plans are given by Military Standard (MIL-STD-105E)

What is acceptance sampling?

- Purposes
 - Determine the quality level of an incoming shipment or at the end of production
 - Judge whether quality level is within the level that has been predetermined
- **But! Acceptance sampling gives you no idea about the process that is producing those items!**

Types of sampling plans

- Sampling by attributes vs. sampling by variables
- Incoming vs. outgoing inspection
- Rectifying vs. non-rectifying inspection
 - What is done with nonconforming items found during inspection
 - Defectives may be replaced by good items
- Single, double, multiple and sequential plans

How acceptance sampling works

- Attributes (“go no-go” inspection)
 - Defectives-product acceptability across range
 - Defects-number of defects per unit
- Variable (continuous measurement)
 - Usually measured by mean and standard deviation

Why use acceptance sampling?

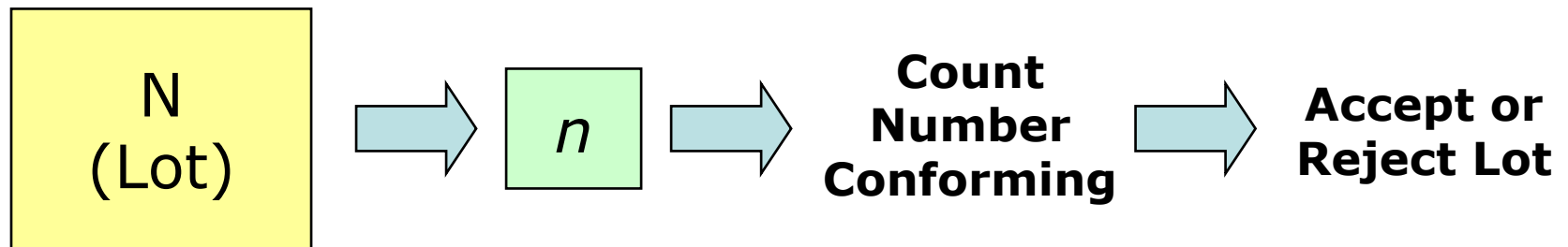
- Can do either 100% inspection, or inspect a sample of a few items taken from the lot
- Complete inspection
 - Inspecting each item produced to see if each item meets the level desired
 - Used when defective items would be very detrimental in some way

Why not 100% inspection?

Problems with 100% inspection

- Very expensive
- Can't use when product must be destroyed to test
- Handling by inspectors can induce defects
- Inspection must be very tedious so defective items do not slip through inspection

A Lot-by-Lot Sampling Plan

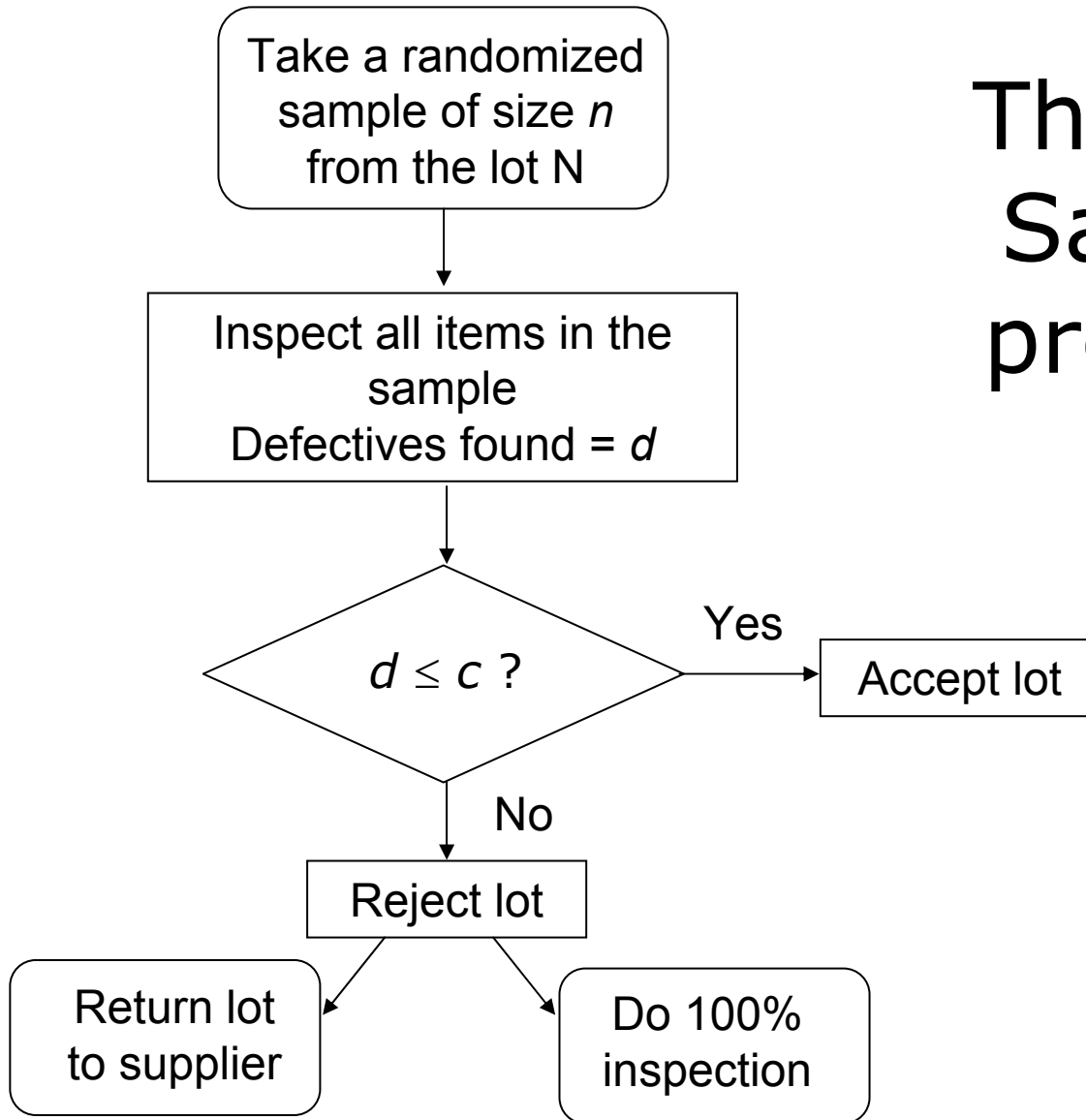


- Specify the plan (n, c) given N
- For a lot size N , determine
 - the sample size n , and
 - the acceptance number c .
- Reject lot if number of defects $> c$
- Specify course of action if lot is rejected

The Single Sampling Plan

- The most common and easiest plan to use but not most efficient in terms of average number of samples needed
- Single sampling plan
 - N = lot size
 - n = sample size (randomized)
 - c = acceptance number
 - d = number of defective items in sample
- Rule: If $d \leq c$, accept lot; else reject the lot

The Single Sampling procedure



Producer's & Consumer's Risks due to mistaken sentencing

- **TYPE I ERROR** = $P(\text{reject good lot})$
 α or Producer's risk
5% is common
- **TYPE II ERROR** = $P(\text{accept bad lot})$
 β or Consumer's risk
10% is typical value

Quality Definitions

- Acceptance quality level (AQL)

The smallest percentage of defectives that will make the lot definitely acceptable. A quality level that is the base line requirement of the customer
- RQL or Lot tolerance percent defective (LTPD)

Quality level that is unacceptable to the customer



How acceptance sampling works

- Remember
 - You are not measuring the quality of the lot, but, you are to sentence the lot to either reject or accept it
- Sampling involves risks:
 - Good product may be rejected
 - Bad product may be accepted
 - Because we inspect only a sample, not the whole lot!

Acceptance sampling contd.

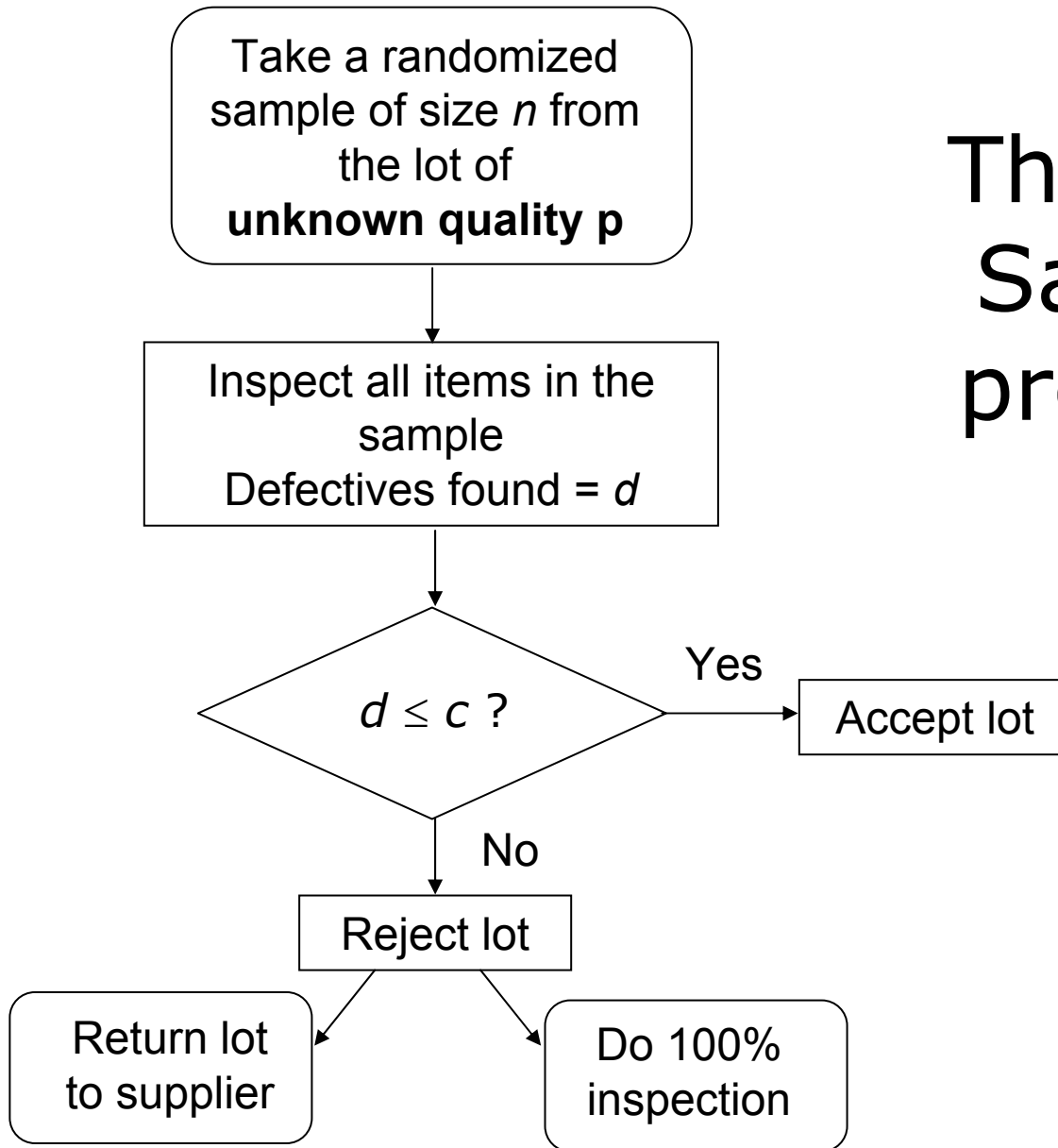
- Producer's risk
 - Risk associated with a lot of acceptable quality rejected
- Alpha α
 - = Prob (committing Type I error)
 - = P (rejecting lot at AQL quality level)
 - = producers risk

Acceptance sampling contd.

- Consumer's risk
 - Receive shipment, assume good quality, actually bad quality
- Beta β
 - = Prob (committing Type II error)
 - = Prob (accepting a lot at RQL quality level)
 - = consumers risk

The OC curve for a sampling plan quantifies these risks

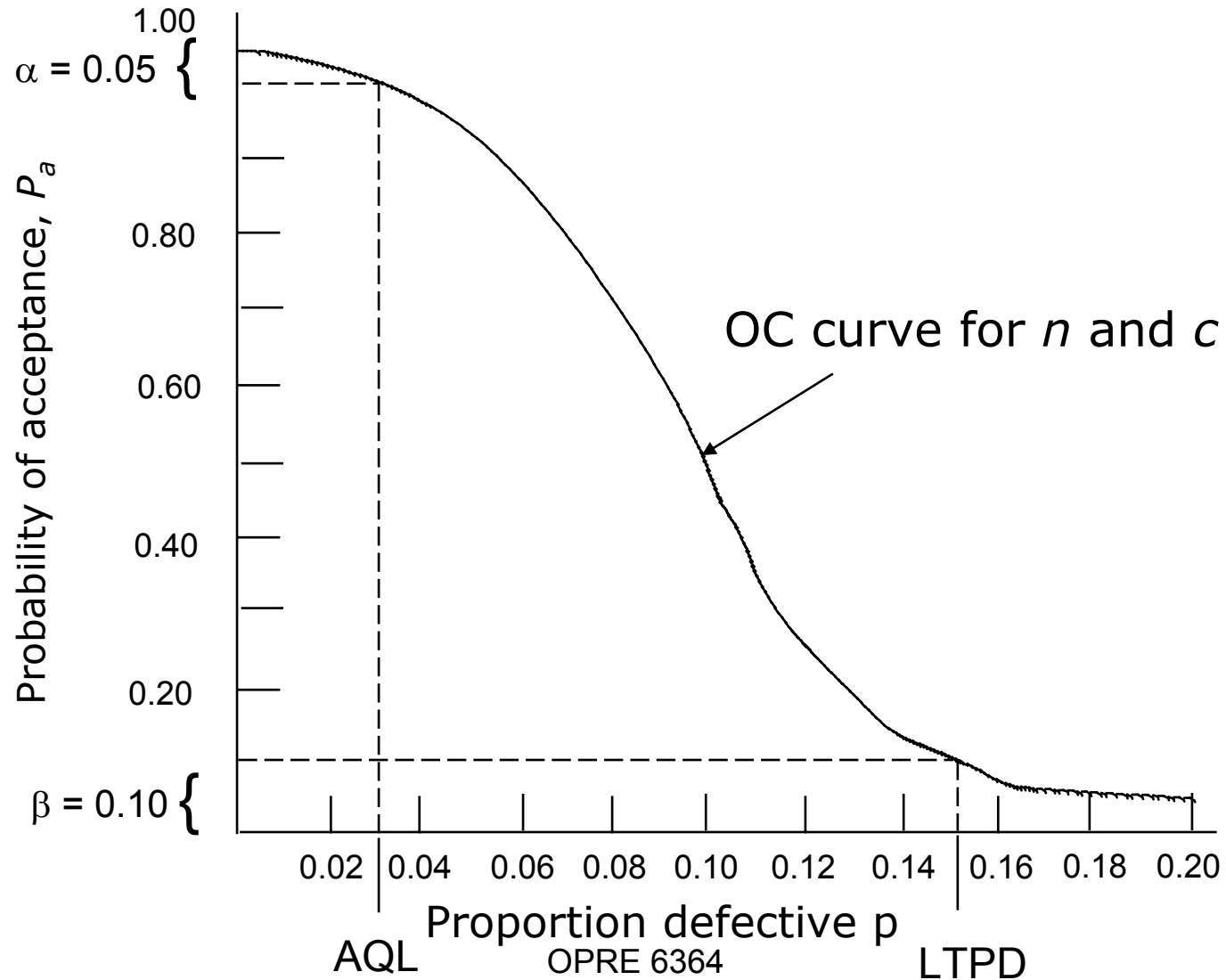
The Single Sampling procedure



Operating Characteristic (OC) Curve

- It is a graph of the % defective (p) in a lot or batch vs. the probability that the sampling plan will accept the lot
- Shows probability of lot acceptance P_a as function of lot quality level (p)
- It is based on the sampling plan
- Curve indicates discriminating power of the plan
- Aids in selection of plans that are effective in reducing risk
- Helps to keep the high cost of inspection down

Operating Characteristic Curve



Types of OC Curves

- Type A
 - Gives the probability of acceptance for an individual lot coming from finite production
- Type B
 - Give the probability of acceptance for lots coming from a continuous process or infinite size lot

OC Curve Calculation

The Ways of Calculating OC Curves

- Binomial distribution
- Hypergeometric distribution
 - $P_a = P(r \text{ defectives found in a sample of } n)$
- Poisson formula
 - $P(r) = ((np)^r e^{-np}) / r!$
- Larson nomogram

OC Curve Calculation by Poisson distribution

- A Poisson formula can be used
 - $P(r) = ((np)^r e^{-np}) / r! = \text{Prob}(\text{exactly } r \text{ defectives in } n)$
- Poisson is a limit
 - Limitations of using Poisson
 - $n \leq N/10$ total batch
 - Little faith in Poisson probability calculation when n is quite small and p quite large.
- For Poisson, $P_a = P(r \leq c)$

Constructing an OC Curve

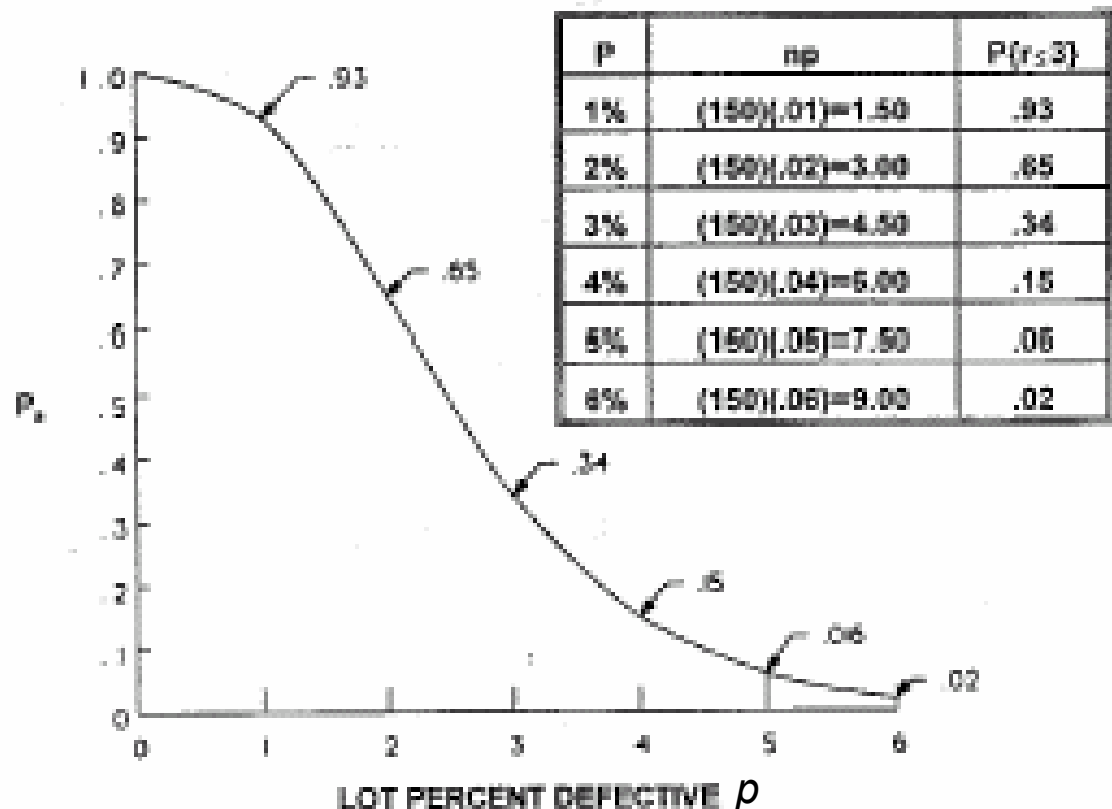
Poisson Distribution

$$P(r) = \frac{e^{-np} (np)^r}{r!}$$

= The probability of exactly r defective in a sample of n

For us, $P_a = P(r \leq c)$

Assume: $n = 150$, $c = 3$



OC Curve Calculation by Binomial Distribution

Note that we cannot always use the binomial distribution because

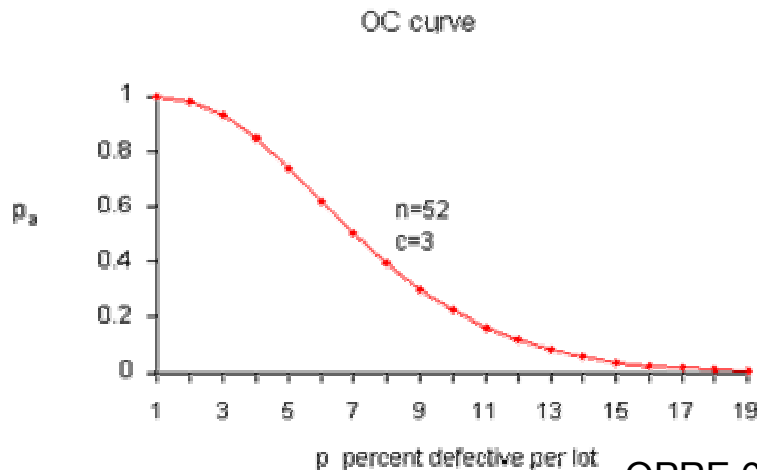
- Binomials are based on constant probabilities
 - N is not infinite
 - p changes as items are drawn from the lot

OC Curve by Binomial Formula

$$P_a = P\{d \leq c\} = \sum_{d=0}^c \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

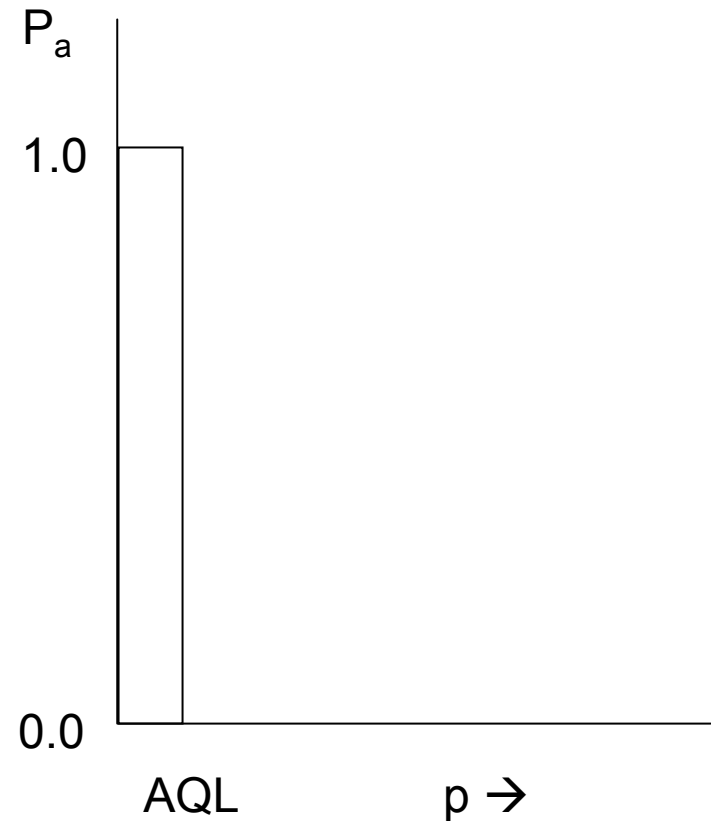
Using this formula with $n = 52$ and $c=3$ and $p = .01, .02, \dots, .12$ we find data values as shown on the right. This gives the plot shown below.

P_a	P_d
.998	.01
.980	.02
.930	.03
.845	.04
.739	.05
.620	.06
.502	.07
.394	.08
.300	.09
.223	.10
.162	.11
.115	.12



The Ideal OC Curve

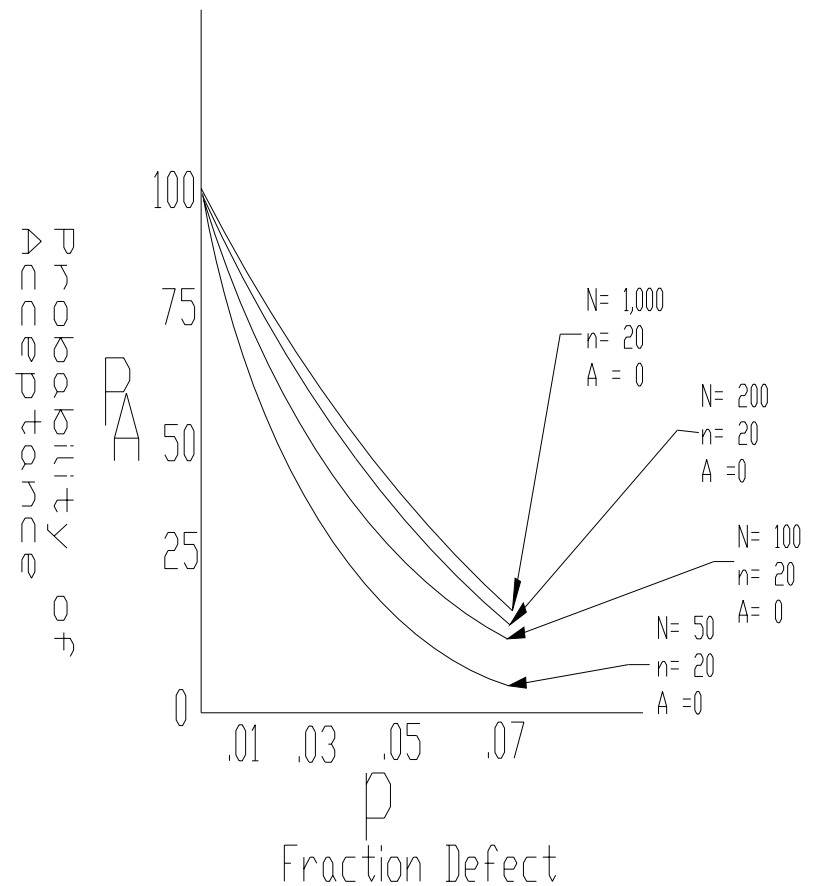
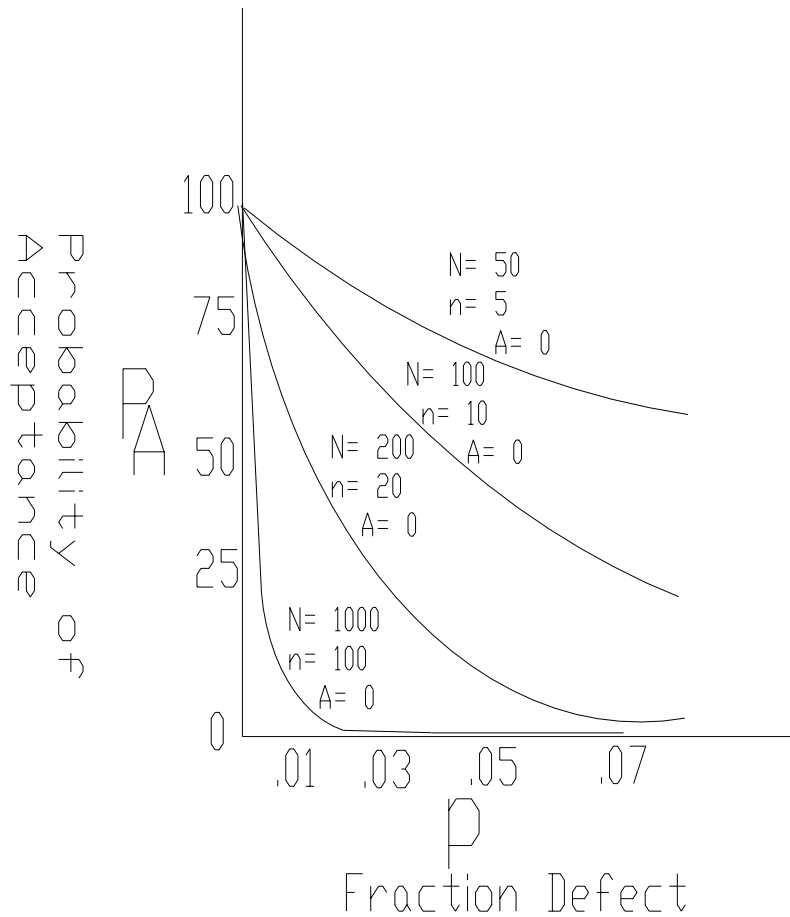
- Ideal curve would be perfectly perpendicular from 0 to 100% for a fraction defective = AQL
- It will accept every lot with $p \leq \text{AQL}$ and reject every lot with $p > \text{AQL}$



Properties of OC Curves

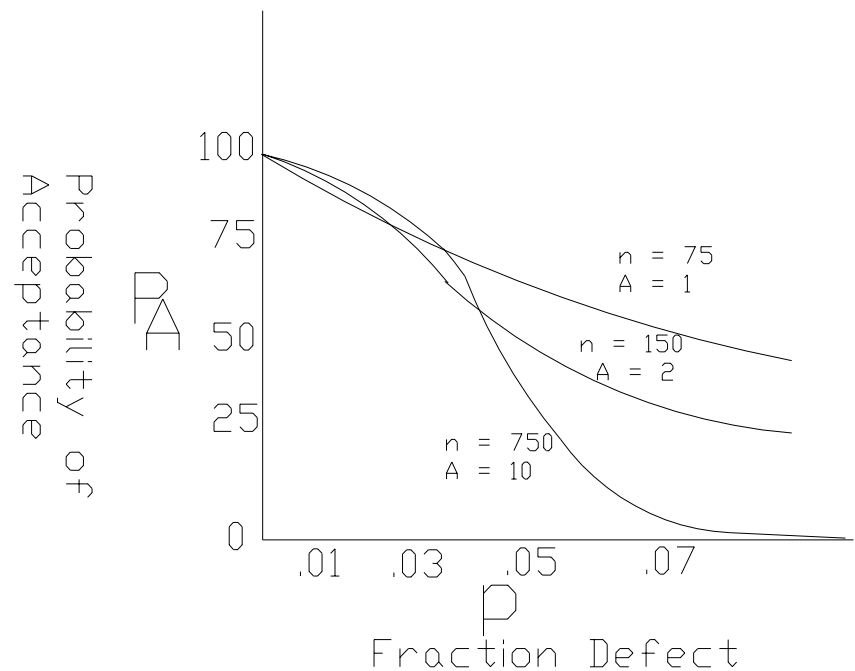
- The acceptance number c and sample size n are most important factors in defining the OC curve
- Decreasing the acceptance number is preferred over increasing sample size
- The larger the sample size the steeper is the OC curve (i.e., it becomes more discriminating between good and bad lots)

Properties of OC Curves



Properties of OC Curves

- If the acceptance level c is changed, the shape of the curve will change. All curves permit the same fraction of sample to be nonconforming.



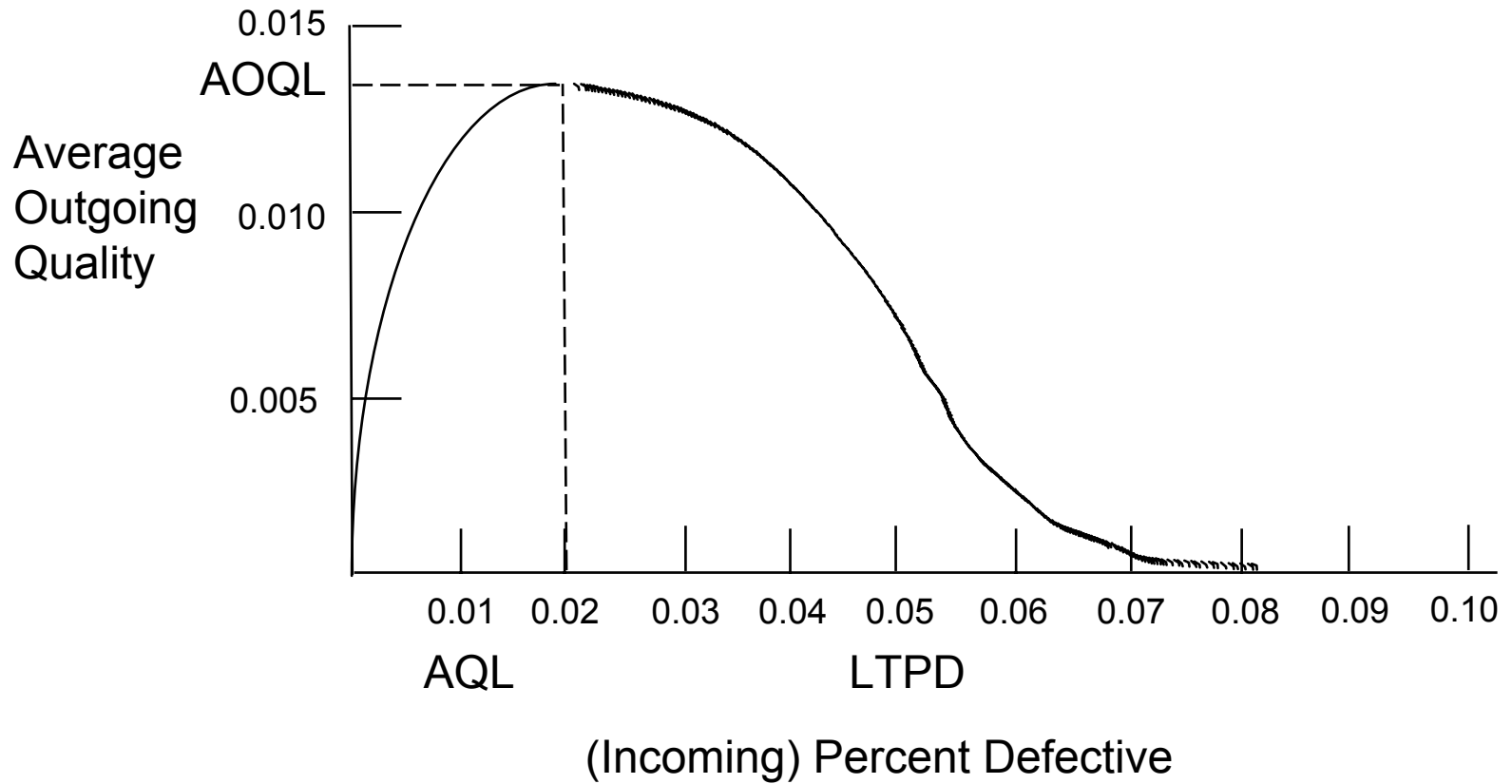
Average Outgoing Quality (AOQ)

- Expected proportion of defective items passed to customer

$$AOQ \text{ with rectifying inspection} = \frac{P_a p(N - n)}{N}$$

- Average outgoing quality limit (AOQL) is
 - The “maximum” point on AOQ curve

AOQ Curve



Double Sampling Plans

- Take small initial sample
 - If # defectives $<$ lower limit, accept
 - If # defectives $>$ upper limit, reject
 - If # defectives between limits, take second sample
- Accept or reject lot based on 2 samples
- Less inspection than in single-sampling

Multiple Sampling Plans

- Advantage: Uses smaller sample sizes
- Take initial sample
 - If # defectives $<$ lower limit, accept
 - If # defectives $>$ upper limit, reject
 - If # defectives between limits, re-sample
- Continue sampling until accept or reject lot based on all sample data

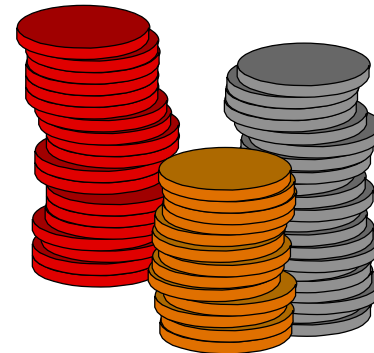
Sequential Sampling

- The ultimate extension of multiple sampling
- Items are selected from a lot one at a time
- After inspection of each sample a decision is made to accept the lot, reject the lot, or to select another item

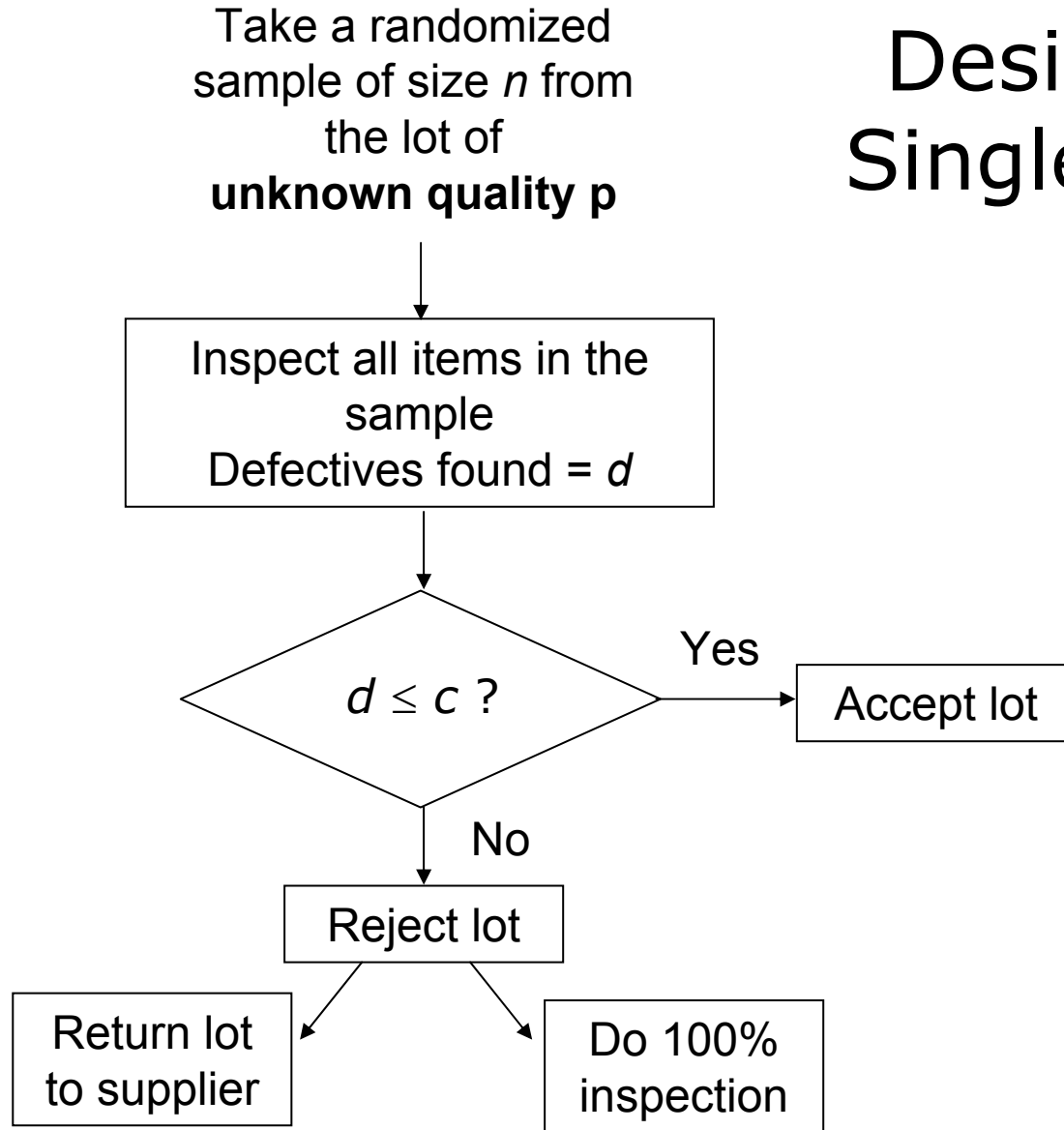
In Skip Lot Sampling only a fraction of the lots submitted are inspected

Choosing A Sampling Method

- An economic decision
- Single sampling plans
 - high sampling costs
- Double/Multiple sampling plans
 - low sampling costs



Designing The Single Sampling plan



Poisson distribution for Defects

- Poisson parameter: $\lambda = np$
- $P(r) = \frac{(np)^r e^{-np}}{r!} = \text{Prob}(\text{exactly } r \text{ defectives in } n)$
- This formula may be used to formulate equations involving AQL, RQL, α and β to given (n, c) .

We can use Poisson tables to approximately solve these equations. Poisson can approximate binomial probabilities if n is large and p small.

Q. If we sample 50 items from a large lot, what is the probability that 2 are defective if the defect rate (p) = .02? What is the probability that no more than 3 defects are found out of the 50?

Hypergeometric Distribution

- Hypergeometric formula:
$$P(r) = \frac{\binom{n-r}{N-M} \binom{r}{M}}{\binom{n}{N}}$$

r defectives in sample size n when M defectives are in N .

- This distribution is used when sampling from a small population. It is used when the lot size is not significantly greater than the sample size.
- (Can't assume here each new part picked is unaffected by the earlier samples drawn).

Q. A lot of 20 tires contains 5 defective ones (i.e., $p = 0.25$). If an inspector randomly samples 4 items, what is the probability of 3 defective ones?

Sampling Plan Design by Binomial Distribution

- Binomial distribution:

$$P(x \text{ defectives in } n) = [n!/(x!(n-x)!)]p^x(1-p)^{n-x}$$

Recall $n!/(x!(n-x)!) =$ ways to choose x in n

Q. If 4 samples (items) are chosen from a population with a defect rate = .1, what is the probability that

- a) exactly 1 out of 4 is defective?
- b) at most 1 out of 4 is defective?

Solving for (n, c)

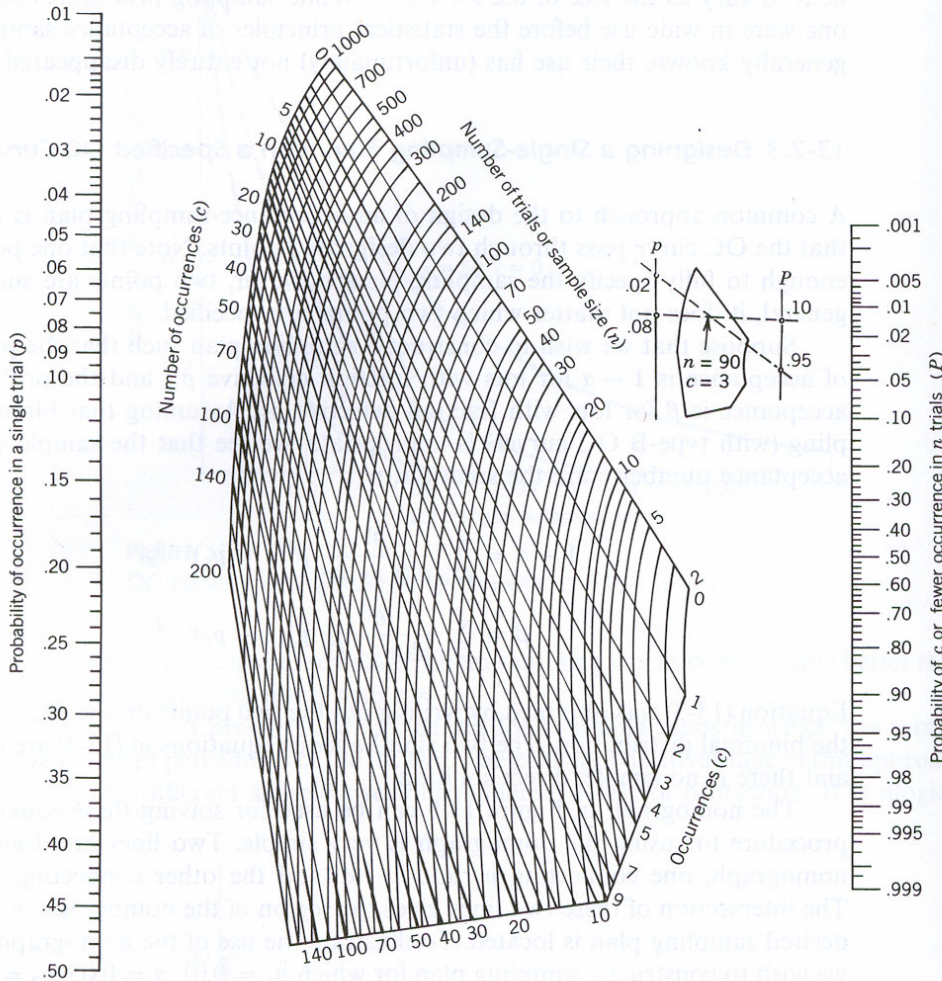
To design a single sampling plan we need two points. Typically these are $p_1 = \text{AQL}$, $p_2 = \text{LTPD}$ and α , β are the Producer's Risk (Type I error) and Consumer's Risk (Type II error), respectively. By binomial formulas, n and c are the solution to

$$1 - \alpha = \sum_{d=0}^c \frac{n!}{d!(n-d)!} p_1^d (1-p_1)^{n-d}$$

$$\beta = \sum_{d=0}^c \frac{n!}{d!(n-d)!} p_2^d (1-p_2)^{n-d}$$

These two simultaneous equations are nonlinear so there is no simple, direct solution. The Larson nomogram can help us here.

$$P(m \leq c) = \sum_{m=0}^c \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m} \quad \frac{n}{N} \leq 0.1$$



Note:

If p is less than 0.01, set $k \times p$ on the p -scale and multiply the values on the n -scale by k , where $k = 0.01/p$ (taking k to the next higher integer).

The Larson Nomogram

- Applies to single sampling plan
- Based on binomial distribution
- Uses
 - $1-\alpha = P_a$ at AQL
 - $\beta = P_a$ at RQL
- Can produce OC curve

Definitions and Terms

Reference: NIST Engineering Statistics Handbook

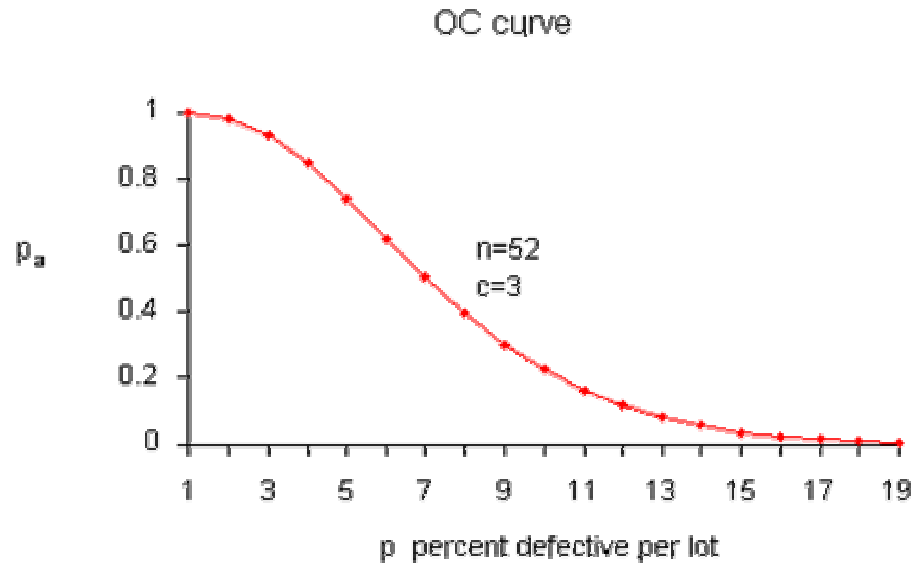
Acceptable Quality Level (AQL): The AQL is a percent defective that is the base line requirement for the quality of the producer's product. The producer would like to design a sampling plan such that there is a *high probability of accepting* a lot that has a defect level less than or equal to the AQL.

Lot Tolerance Percent Defective (LTPD) also called RQL (Rejection Quality Level): The LTPD is a designated high defect level that would be unacceptable to the consumer. The consumer would like the sampling plan to have a *low probability of accepting* a lot with a defect level as high as the LTPD.

Type I Error (Producer's Risk): This is the probability, for a given (n, c) sampling plan, of rejecting a lot that has a defect level equal to the AQL. The producer suffers when this occurs, because a lot with acceptable quality was rejected. The symbol α is commonly used for the Type I error and typical values for β range from 0.2 to 0.01.

Type II Error (Consumer's Risk): This is the probability, for a given (n, c) sampling plan, of accepting a lot with a defect level equal to the LTPD. The consumer suffers when this occurs, because a lot with unacceptable quality was accepted. The symbol β is commonly used for the Type II error and typical values range from 0.2 to 0.01.

Operating Characteristic (OC) Curve: This curve plots the probability of accepting the lot (Y-axis) versus the lot fraction or percent defectives (X-axis).



The OC curve is the primary tool for displaying and investigating the properties of a sampling plan.

Average Outgoing Quality (AOQ): A common procedure, when sampling and testing is non-destructive, is to 100% inspect rejected lots and replace all defectives with good units. In this case, all rejected lots are made perfect and the only defects left are those in lots that were accepted. AOQ's refer to the long term defect level for this combined LASP and 100% inspection of rejected lots process. If all lots come in with a defect level of exactly p , and the OC curve for the chosen (n,c) LASP indicates a probability p_a of accepting such a lot, over the long run the AOQ can easily be shown to be:

$$AOQ = \frac{p_a p (N - n)}{N}$$

where N is the lot size.

Average Outgoing Quality Level (AOQL): A plot of the AOQ (Y-axis) versus the incoming lot p (X-axis) will start at 0 for $p = 0$, and return to 0 for $p = 1$ (where every lot is 100% inspected and rectified). In between, it will rise to a maximum. This maximum, which is the worst possible long term AOQ, is called the AOQL.

Average Total Inspection (ATI): When rejected lots are 100% inspected, it is easy to calculate the *ATI* if lots come consistently with a defect level of p . For a sampling plan (n, c) with a probability p_a of accepting a lot with defect level p , we have

$$ATI = n + (1 - p_a) (N - n)$$

where N is the lot size.

Average Sample Number (ASN): For a single sampling plan (n, c) we know each and every lot has a sample of size n taken and inspected or tested. For double, multiple and sequential plans, the amount of sampling varies depending on the number of defects observed. For any given double, multiple or sequential plan, a long term *ASN* can be calculated assuming all lots come in with a defect level of p . A plot of the *ASN*, versus the incoming defect level p , describes the sampling efficiency of a given lot sampling scheme.

The MIL-STD-105E approach

A Query from a Practitioner: Selecting AQL (acceptable quality levels)

I'd like some guidance on selecting an acceptable quality level and inspection levels when using sampling procedures and tables. For example, when I use MIL-STD-105E, how do I to decide when I should use GI, GII or S2, S4?

-- Confused in Columbus, Ohio

W. Edwards Deming observed that the main purpose of MIL-STD-105 was to beat the vendor over the head.

"You cannot improve the quality in the process stream using this approach," cautions Don Wheeler, author of *Understanding Statistical Process Control* (SPC Press, 1992). "Neither can you successfully filter out the bad stuff. About the only place that this procedure will help is in trying to determine which batches have already been screened and which batches are raw, unscreened, run-of-the-mill bad stuff from your supplier. I taught these techniques for years but have repented of this error in judgment. The only appropriate levels of inspection are all or none. Anything else is just playing roulette with the product."

MIL-STD-105E

- Original version (MIL STD 105A) issued in 1950 as tables; Last version (MIL STD 105E) in 1989; ISO adopted it as ISO 2859
- Plan covers sampling by attributes for given lot size (N) and acceptable quality level (AQL).
- Prescribes sample size n , acceptance number c , and rejection number r
- *Standard* included three types of inspection—normal, tightened and reduced and gives switching rules
- Plans assure producer's risk (α) of 0.01 – 0.1. The only way to control the consumer's risk (β) is to change inspection level

AQL Acceptance Sampling by Attributes by MILSTD 105E

- Determine lot size N and AQL for the task at hand
- Decide the type of sampling—single, double, etc.
- Decide the state of inspection (e.g. normal)
- Decide the type of inspection level (usually II)
- Look at Table K for sample sizes
- Look at the sampling plans tables (e.g. Table IIA)
- Read n , A_c and R_e numbers

MIL-STD-105E

TABLE I—Sample size code letters

(see 4.9.1 and 4.9.2)

Lot or batch size			Special inspection levels				General inspection levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	B	
9	to	15	A	A	A	A	B	C	
16	to	25	A	A	B	B	C	D	
26	to	50	A	B	B	C	C	E	
51	to	90	B	B	C	C	D	F	
91	to	150	B	B	C	D	D	G	
151	to	280	B	C	D	E	E	H	
281	to	500	B	C	D	E	F	I	
501	to	1200	C	C	E	F	G	J	
1201	to	3200	C	D	E	G	H	K	
3201	to	10000	C	D	F	G	J	L	
10001	to	35000	C	D	F	H	K	M	
35001	to	150000	D	E	G	J	L	N	
150001	to	500000	D	E	G	J	M	P	
500001	and	over	D	E	H	K	N	Q	

MIL-STD-105E

How/When would you use Acceptance Sampling?

- Advantages of acceptance sampling
 - Less handling damages
 - Fewer inspectors to put on payroll
 - 100% inspection costs are too high
 - 100% testing would take too long
- Acceptance sampling has some disadvantages
 - Risk included in chance of bad lot “acceptance” and good lot “rejection”
 - Sample taken provides less information than 100% inspection

Summary

- There are many basic terms you need to know to be able to understand acceptance sampling
 - SPC, Accept a lot, Reject a lot, Complete Inspection, AQL, LTPD, Sampling Plans, Producer's Risk, Consumer's Risk, Alpha, Beta, Defect, Defectives, Attributes, Variables, ASN, ATI.

Useful links

<http://www.bioss.sari.ac.uk/smart/unix/mseqacc/slides/frames.htm>

Acceptance Sampling Overview Text and Audio

<http://iew3.technion.ac.il/sqconline/milstd105.html>

Online calculator for acceptance sampling plans

http://www.stats.uwo.ca/courses/ss316b/2002/accept_02red.pdf

Acceptance sampling mathematical background