Heat Transfer

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Practical Lecture 2 (Solved Problems)

1. A flat plate has an insulated surface and another exposed to the sun. The surface exposed to the sun absorbs radiation at a rate of $800\,\mathrm{W\,m^{-2}}$ and loses heat by convection and radiation to ambient air at $300\,\mathrm{K}$. If the surface emissivity is 0.9 and the convection coefficient is $12\,\mathrm{W\,m^{-2}\,{}^{\circ}C^{-1}}$, determine the plate temperature in steady-state.

Solution:

The flat plat steady-state temperature is computed by applying an energy balance. Equation (1) corresponds to the energy balance equation formulated on a time rate basis for a control volume (flat plate).

$$\dot{E}_{\mathrm{in}} - \dot{E}_{\mathrm{out}} + \dot{E}_{\mathrm{g}} = \dot{E}_{\mathrm{st}} \tag{1}$$

Equation (1) can be simplified to solve the problem by taking into account the following assumptions:

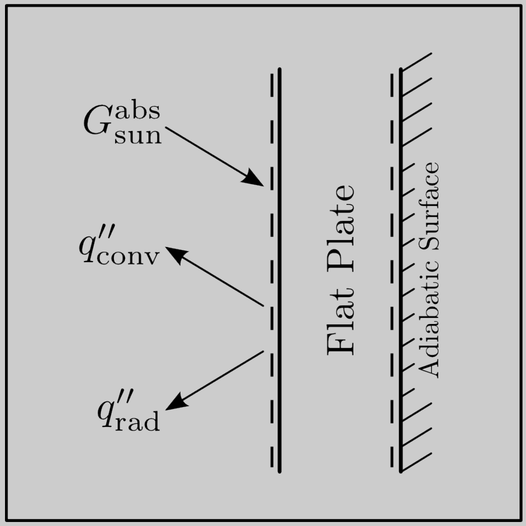
- $\dot{E}_{\mathrm{st}} = 0$ – the steady-state solution is in consideration, and consequently, the rate of energy storage vanishes.
- $\dot{E}_{\mathrm{g}} = 0$ – the generation rate of thermal energy in the control volume is not relevant in the steady-state flat plat temperature governing equation because there are no references in the problem statement to internal generation of thermal energy.

Equation (2) is obtained after applying the stated simplifying assumptions in Equation (1).

$$\dot{E}_{\mathrm{in}} = \dot{E}_{\mathrm{out}} \tag{2}$$

The terms $\dot{E}_{\mathrm{in}}$ e $\dot{E}_{\mathrm{out}}$ (Equation (2)) are obtained by considering the corresponding heat transfer rate contributions through the plate surface that is exposed to the sun radiation (see figure) according to Equations (3) and (4), respectively.

Heat Transf. Contri. to the Energy Bal.



$$\dot{E}_{\text{in}} = AG_{\text{sun}}^{\text{abs}} \tag{3}$$

$$\dot{E}_{\text{out}} = A\left[\underbrace{h\left(T_s - T_\infty\right)}_{q''_{\text{conv}}} + \underbrace{\epsilon\sigma\left(T_s^4 - T_{\text{sur}}^4\right)}_{q''_{\text{rad}}}\right] \tag{4}$$

In Equation (3), $G_{\text{sun}}^{\text{abs}}$ corresponds to the absorbed solar irradiation. In Equation (4), q''_{conv} e q''_{rad} are the convective and net radiative heat fluxes, respectively, from the exposed surface of the flat plate to the sun radiation. In the later equation, T_s, T_∞, and T_{sur} are the plate surface temperature, adjoining fluid temperature and surrounding surfaces temperature. (For the problem under consideration, $T_\infty = T_{\text{sur}} = 300\,\text{K}$.) Note that the calculation of the net radiative heat flux as stated in Equation (4) implies that the flat plate surface is exposed to very large surrounding surfaces – in order to approximate its irradiation as blackbody emission power ($G = \sigma T_{\text{sur}}^4$) – and that the plate surface has gray radiative properties ($\alpha = \epsilon$).

Finally, the flat plate steady-state temperature, T_p is obtained replacing Equations (3) and (4) in Equation (2) and solving for T_p – see Equation (5).

$$\begin{aligned} AG_{\text{sun}}^{\text{abs}} = Aq''_{\text{conv}} + Aq''_{\text{rad}} \Leftrightarrow G_{\text{sun}}^{\text{abs}} = h\left(T_p - T_\infty\right) + \epsilon\sigma\left(T_p^4 - T_{\text{sur}}^4\right) \Leftrightarrow \\ \Leftrightarrow 800 = 12\left(T_p - 300\right) + 0.9 \times 5.67 \times 10^{-8}\left(T_p^4 - 300^4\right) \Leftrightarrow \\ \Leftrightarrow -5.103 \times 10^{-8} T_p^4 - 12T_p + 4813.34 = 0 \Rightarrow \boxed{T_p \approx 342.556\,\text{K}} \end{aligned} \tag{5}$$

Note that in Equation (5), for the calculation of convective and radiative heat fluxes the surface temperature of the plate, T_s, was replaced by the plate temperature, T_p, because the entire plate is at the same temperature ($T_p = T_s$). The flat plate isothermal condition is due to the fact that only the steady-state condition is in consideration and because the opposite surface to the exposed one to the sun is an adiabatic surface (see figure above).

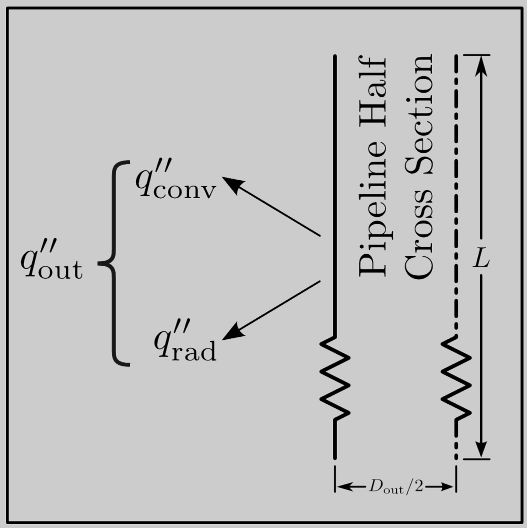
Please, bear in mind that for the calculation of radiative heat transport rates the temperature must be considered in the Kelvin scale – thermodynamic (absolute) temperature. (If in the problem statement T_{sur} were given as 26.85°C you should convert this temperature value to the Kelvin scale in order to correctly obtain T_p.) Also notice, that because temperature differences in the Celsius and Kelvin scales are equal – see Problem A(a) (Practical Lecture 1) – a convection heat transfer coefficient value given as $12\,\mathrm{W\,m^{-2}\,{}^{\circ}C^{-1}}$ is equal to $12\,\mathrm{W\,m^{-2}\,K^{-1}}$.

3. A 2 m long pipeline with an outer diameter of 5 cm transports hot water, dissipating heat by convection and radiation to the surroundings at 0°C. Let the temperature of the outer surface of the pipeline be 125°C, which can be assumed to behave as a blackbody. Determine the rate of heat transfer to the surrounding, knowing that the outer convection coefficient is $20\,\mathrm{W\,m^{-2}\,{}^{\circ}C^{-1}}$.

Solution:

The total heat transfer rate from the pipeline outer surface to the surroundings, q_{out}, have two contributions, namely convection and radiation heat transfer rates (see figure below), and is computed according to Equation (6).

Heat Fluxes from Pipeline Outer Surf.



$$\begin{aligned} q_{\mathrm{out}} = A\left(q''_{\mathrm{conv}} + q''_{\mathrm{rad}}\right) \Leftrightarrow q_{\mathrm{out}} = \pi D_{\mathrm{out}} L \left[h\left(T_s - T_\infty\right) + \epsilon\sigma\left(T_s^4 - T_{\mathrm{sur}}^4\right)\right] \Leftrightarrow \\ \Leftrightarrow q_{\mathrm{out}} = \pi \times 0.05 \times 2 \times \{20 \times (125 - 0) + \\ 1 \times 5.67 \times 10^{-8} \times \left[(125 + 273.15)^4 - (0 + 273.15)^4\right]\} \Leftrightarrow \\ \Leftrightarrow \boxed{q_{\mathrm{out}} \approx 1133.869\,\mathrm{W}} \end{aligned} \tag{6}$$

Note that for the net radiative heat rate expression (per unit surface area), q''_{rad}, the temperature values must be provided in the Kelvin scale.

5. (Homework) Under certain conditions, the temperature at the surface of a person's skin is 30°C, which is lower than the body temperature, *i.e.*, 36.5°C. The transition between these temperatures takes place in a skin layer of 1 cm thickness. The thermal conductivity is $0.42\,\mathrm{W\,m^{-1}\,K^{-1}}$.

   (a) Estimate the heat flux escaping through the skin, considering it as a conductive medium at rest.

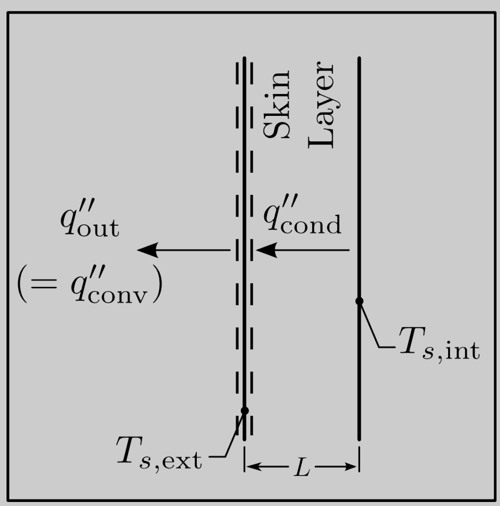
Solution:

The heat flux escaping through the skin can be evaluated with a surface energy balance formulated on a time rate basis and applied to the skin external surface as described by Equation (7). (The surface energy balance equation (Equation (7)) differs from the control volume energy balance equation (Equation (1)) by the absence of the terms related to the volume of the system. Because a control surface has no mass (volume) there are no terms related to the internal generation of energy ($\dot{E}_\mathrm{g}$) and to energy storage/accumulation ($\dot{E}_\mathrm{st}$)).

$$\dot{E}_\mathrm{in} - \dot{E}_\mathrm{out} = 0 \tag{7}$$

The terms $\dot{E}_\mathrm{in}$ e $\dot{E}_\mathrm{out}$ (Equation (7)) are obtained by considering the corresponding heat transfer rate contributions at the control surface (skin external surface) – see figure. Particularly, the inflow of thermal energy (heat) towards the control surface is due to conduction heat transfer through the skin. Therefore, the Fourier's law (heat conduction rate equation) is applied to evaluate $\dot{E}_\mathrm{in}$ – see Equation (8). In Equation (8), $T_{s,\mathrm{int}}$ and $T_{s,\mathrm{ext}}$ are the internal and external skin layer temperatures, respectively (see figure) and k and L are the skin layer thermal conductivity and thickness, respectively. $\dot{E}_\mathrm{out}$ is given by Equation (9) in which q''_out corresponds to the heat flux escaping through the skin.

Heat Transf. Contri. to the Energy Bal.



$$\dot{E}_\mathrm{in} = A\underbrace{\left(k\frac{T_{s,\mathrm{int}} - T_{s,\mathrm{ext}}}{L}\right)}_{q''_\mathrm{cond}} \tag{8}$$

$$\dot{E}_\mathrm{out} = Aq''_\mathrm{out} \tag{9}$$

Finally, the heat flux escaping through the skin is obtained by replacing Equations (8) and

(9) in Equation (7) and solving for q''_{out} as shown in Equation (10).

$$Aq''_{\text{cond}} = Aq''_{\text{out}} \Leftrightarrow k\frac{(T_{s,\text{int}} - T_{s,\text{ext}})}{L} = q''_{\text{out}} \Leftrightarrow q''_{\text{out}} = 0.42 \times \frac{(36.5 - 30)}{0.01} \Leftrightarrow \Leftrightarrow \boxed{q''_{\text{out}} = 273\,\text{W}\,\text{m}^{-2}} \quad (10)$$

(b) Assuming that ambient air is at 20°C, determine the convection coefficient.

Solution:

Considering convection heat transfer as the only mode of heat removal from the skin external surface then Equation (11) applies, where q''_{out} corresponds to the heat flux calculated previously – see Equation (10).

$$q''_{\text{out}} = q''_{\text{conv}} \quad (11)$$

Replacing q''_{conv} by the corresponding rate expression (Newton's law of cooling), and consequently, T_{ext} and T_∞ by the corresponding values according to the problem statement (30°C and 20°C, respectively) as well as q''_{out} obtained in Equation (10), the convection heat transfer coefficient is obtained – see Equation (12).

$$q''_{\text{out}} = q''_{\text{conv}} \Leftrightarrow q''_{\text{out}} = h\left(T_{\text{ext}} - T_\infty\right) \Leftrightarrow \Leftrightarrow h = \frac{q''_{\text{out}}}{(T_{\text{ext}} - T_\infty)} \Leftrightarrow h = \frac{273}{(30 - 20)} \Leftrightarrow \boxed{h = 27.3\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}} \quad (12)$$