Heat Transfer

Computational Laboratories

Heat Transfer From Extended Surfaces
(Laboratory I)

Exploring the role of fin geometrical parameters and transport properties on the fin performance

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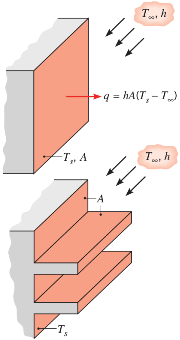
Extended Surfaces - Fins: Motivation for Application



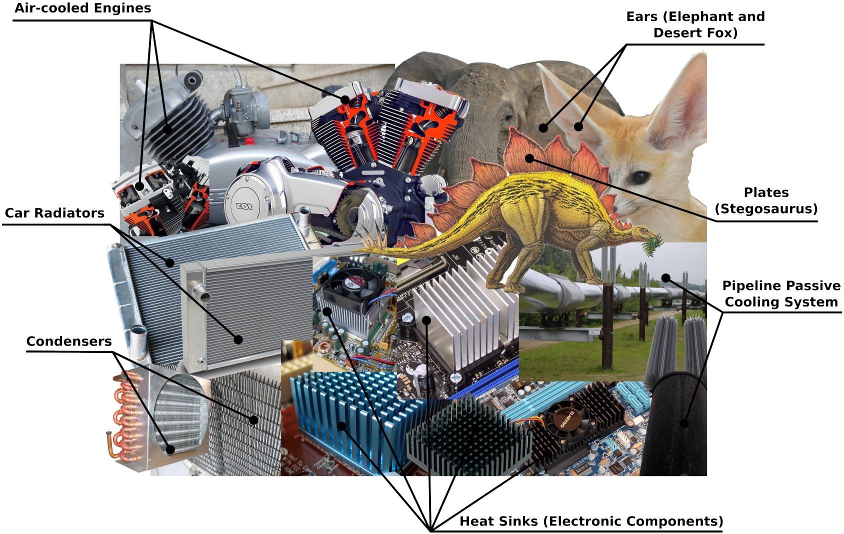
Procedures to increase the rate of heat transfer (q) from a wall at a constant temperature (T_s) to an adjoining fluid:

- Increase the convection heat transfer coefficient (h) and/or decrease the fluid temperature (T_∞) – costly and impractical solutions; and
- **Increase the effective surface area (A)**.

Motivation for Fin Application

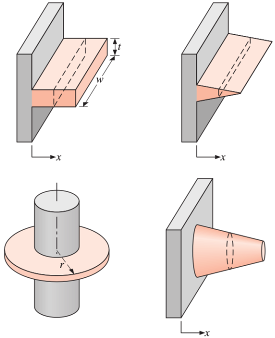
Extended surfaces aim to **enhance heat transfer** by an increase in the available surface area for convection (and/or radiation). **Highly recommended when h is small** (gases under free-convection conditions).

Extended Surfaces - Fins: Practical Applications



Extended Surfaces - Fins: Configurations

Fin Configuration



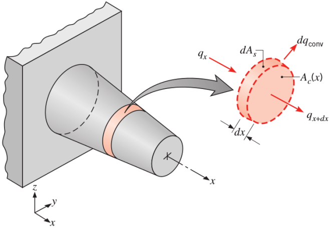
Typical Configurations:

- Straight fins:
  - Uniform Cross Section
  - Non-uniform Cross Section
- Annular fins (non-uniform cross section)
- Pin fins (non-uniform cross-section)

The **selection** of the fin design depends on: space, weight, cost, effect on fluid motion (and consequently on h and pressure drop).

Fin Equation: Governing Energy Balance Equation

- Equation derived from the application of an energy balance to a differential element.
- Assumptions:
  - one-dimensional conduction;
  - steady-state conditions;
  - constant thermal conductivity;
  - constant convection coefficient;
  - no thermal energy generation; and
  - negligible radiative heat losses.



Fin Equation - General Form

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

Fin Equation Applied to Uniform Fins

Fin Equation - General Form

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

For fins of uniform cross section:

- $\frac{dA_c}{dx} = 0$

Considering:

- $m^2 = \frac{hP}{kA_c}$ $(P = dA_s/dx)$
- $\xi = x/L$
- $\Theta(\xi) = \frac{\theta(\xi)}{\theta_b} = \frac{T(\xi)-T_\infty}{T(0)-T_\infty}$

Fin Equation for Uniform Fins

$$\frac{d^2\Theta}{d\xi^2} - (mL)^2\,\Theta = 0$$

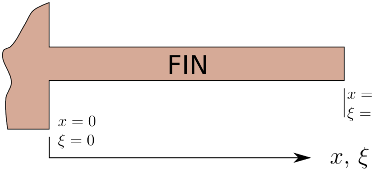
General Sol. for the Fin Eq. Applied to Uniform Fins

General Solution of the Fin Equation for Uniform Fins

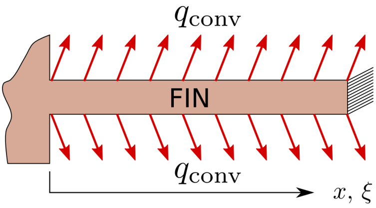
$$\Theta(\xi) = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$$

Boundary conditions (BCs)

1. **Fin base ($\xi = 0$):**
   - $\Theta(0) = 1$ (Fixed temperature - Dirichlet BC)
2. **Fin tip ($\xi = 1$):**
   - $\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = 0$ (Adiabatic tip - Zero Neumann BC)
   - $-k \left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = h_{\mathrm{tip}} L\Theta(1)$ (Active tip - Convection BC)



Fin Eq. Solution for Uniform Fin with **Adiabatic** Tip



Normalized Excess Temperature

$$\Theta^a(\xi) = \frac{\cosh\left[mL\left(1-\xi\right)\right]}{\cosh\left(mL\right)}$$

Fin Heat Transfer Rate

$$q_f^a = M\tanh(mL)$$

Note that

- $mL = \sqrt{\frac{hP}{kA_c}}L$
- $M = \sqrt{hPkA_c}\theta_b$
  - M corresponds to the heat rate observed for an infinite fin (q_f^i)

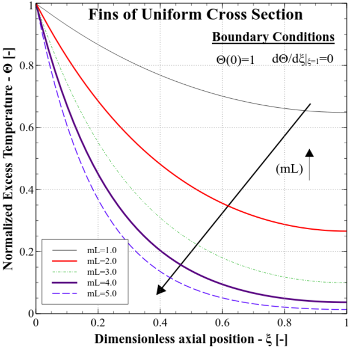
Fin Eq. Solution for Uniform Fin with **Adiabatic** Tip

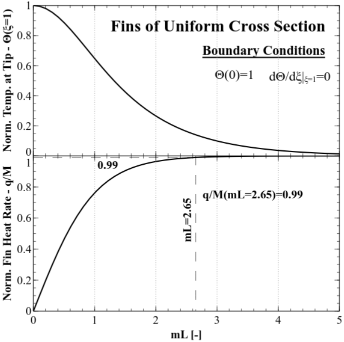
Normalized Excess Temperature

$$\Theta^a(\xi) = \frac{\cosh\left[mL(1-\xi)\right]}{\cosh(mL)}$$

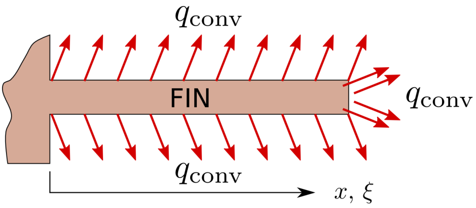
Normalized Heat Transfer Rate

$$\frac{q_f^a}{M} = \tanh(mL)$$





Fin Eq. Solution for Uniform Fin with **Convective** Tip



Normalized Excess Temperature

$$\Theta^c(\xi) = \frac{\cosh\left[mL(1-\xi)\right] + (h_{\text{tip}}/km)\sinh\left[mL(1-\xi)\right]}{\cosh(mL) + (h_{\text{tip}}/km)\sinh(mL)}$$

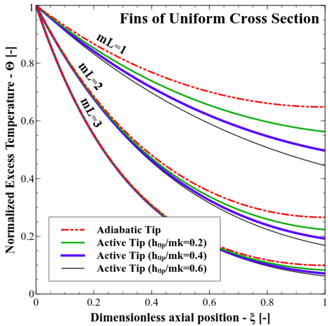
Fin Heat Transfer Rate

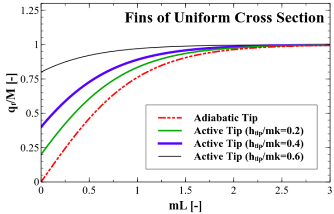
$$q_f^c = M\frac{(h_{\text{tip}}/mk) + \tanh(mL)}{1 + (h_{\text{tip}}/mk)\tanh(mL)}$$

Note that

- $h_{\text{tip}} = 0$: adiabatic tip solution
- Generally $h_{\text{tip}} = h$

Adiabatic *vs.* **Convective** Tip Solutions





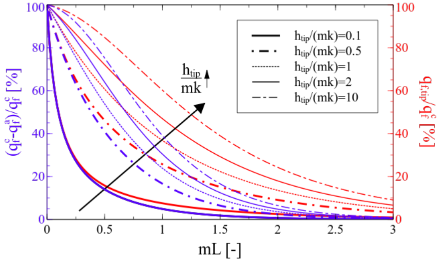
The difference $\Theta^a(\xi) - \Theta^c(\xi)$ increases by:

- increasing h_{tip} for a constant mL;
- decreasing L for a constant h_{tip}.

Increasing h_{tip}:

- $q_f^c - q_f^a$ increases;
- $\left.\frac{d\Theta}{d\xi}\right|_{\xi=1}$ increases, *i.e.* $q_{f,\mathrm{tip}}$ increases, since $q_{f,\mathrm{tip}} = -k\theta_b \frac{A_c}{L} \left.\frac{d\Theta}{d\xi}\right|_{\xi=1}$.

Relative Importance of Tip Convection on the Overall Fin Heat Transfer Rate (1/2)



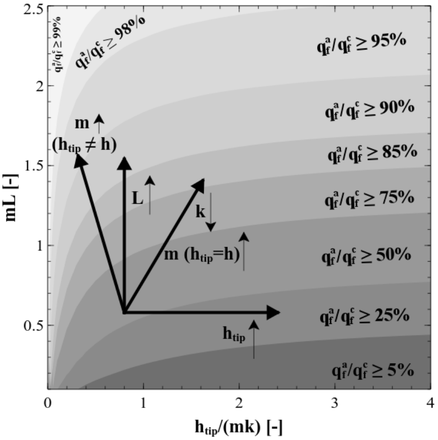
$q_{f,\text{tip}}/q_f^c$ decreases:

- increasing the fin length ($L \uparrow$);
- decreasing the convection coefficient at the fin tip ($h_{\text{tip}} \downarrow$).

The adiabatic tip solution becomes a suitable approximation for an actual active tip for **low values of $\mathbf{q}_{\mathbf{f},\text{tip}}/\mathbf{q}_\mathbf{f}^\mathbf{c}$** or high values of q_f^a/q_f^c.

- $q_{f,\text{tip}} = \frac{q_f^c - q_f^a}{1 - 2\text{sech}(mL)\sinh\left(\frac{mL}{2}\right)^2}$
  - $q_{f,\text{tip}} \approx q_f^c - q_f^a$ for low mL values.
- $q_{f,\text{tip}} = h_{\text{tip}} A_c \theta_b \Theta^c(\xi = 1)$
- $\frac{q_{f,\text{tip}}}{q_f^c} = \frac{(h_{\text{tip}}/km)}{(h_{\text{tip}}/km)\cosh(mL) + \sinh(mL)}$

Relative Importance of Tip Convection on the Overall Fin Heat Transfer Rate (2/2)



The adiabatic tip solution becomes a suitable approximation for an actual active tip for low values of $q_{f,\text{tip}}/q_f^c$ or **high values of $\mathbf{q_f^a/q_f^c}$**.

- $\dfrac{q_f^a}{q_f^c} = \dfrac{\tanh(mL)[1+(h_{\text{tip}}/km)\tanh(mL)]}{(h_{\text{tip}}/km)+\tanh(mL)}$

q_f^a/q_f^c increases:

- increasing the fin length ($L\uparrow$);
- decreasing h_{tip};
- increasing m, that is:
  - increasing h/k;
  - increasing P/A_c.

Fin Thermal Performance Parameters

Fin Effectiveness (ε_f)

$$\varepsilon_f = \frac{q_f}{A_{c,b} h \theta_b}$$

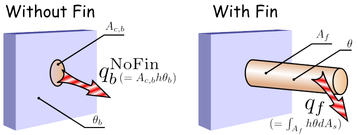
$\varepsilon_{\mathbf{f}}$ – ratio between the actual fin heat rate (q_f) and the heat rate observed without the fin application (q_b^{NoFin}).

- The application of an extended surface may increase the heat transfer resistance (fin resistance) in relation to the convection resistance of the bare surface ($\varepsilon_f = R_{t,b}/R_{t,f}$).
- In general, $\varepsilon_f < 2$ do not justify the fin application.

Fin Efficiency (η_f)

$$\eta_f = \frac{q_f}{A_f h \theta_b}$$

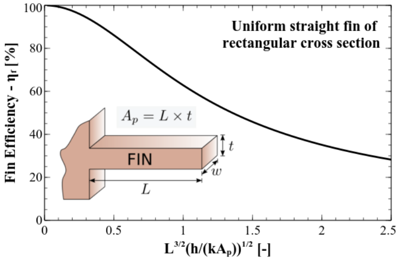
$\eta_{\mathbf{f}}$ – ratio between the actual fin heat rate (q_f) and the heat rate that would be observed if the entire fin were at the fin base temperature (maximum – idealized – fin heat rate, q_{max}).



Performance for the Uniform Fin with Adiabatic Tip

Fin Efficiency (η_f)

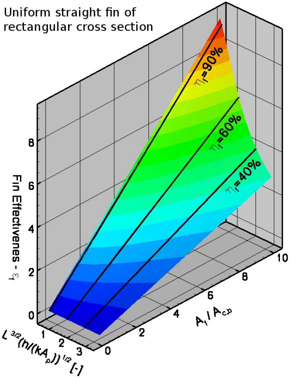
$$\eta_f^a = \frac{\tanh(mL)}{mL} = \int_0^1 \Theta d\xi$$



$$mL \cong L^{3/2}\left[2h/(kA_p)\right]^{1/2}, \; w >> t$$

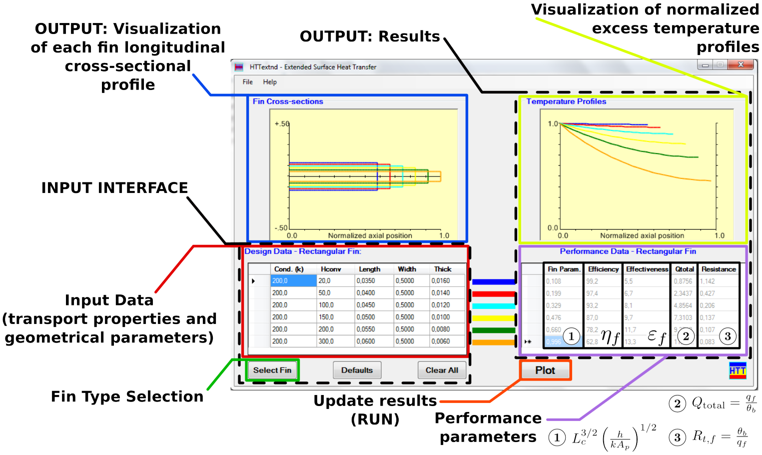
Fin Effectiveness (ε_f)

$$\varepsilon_f^a = \frac{A_f}{A_{c,b}} \eta_f^a$$



Exploring the Software Module (1/2)

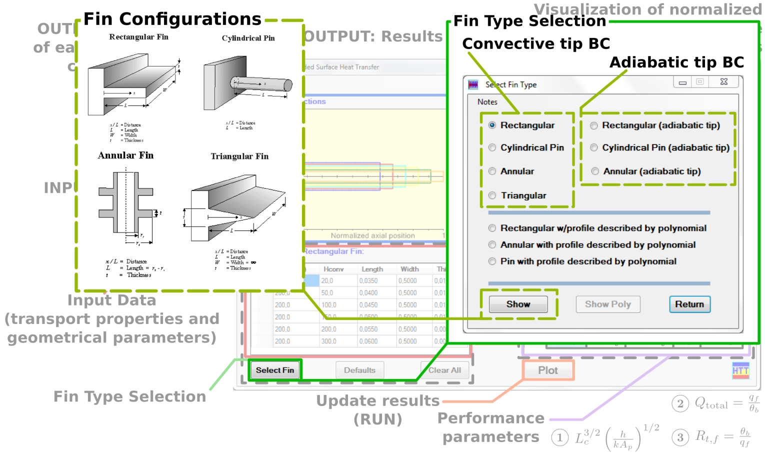
Software module – `HTTextnd.exe`



- The module solves the fin equation through a finite-volume approach.

Exploring the Software Module (2/2)

Software module – `HTTextnd.exe`

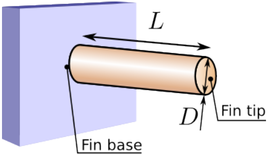


- **The module considers active and adiabatic tip BCs for different fin types.**

Exploring the Software Module - Uniform Pin Fin

Module Application Example I: Problem Statement

Consider a pin fin of uniform cross section with dimensions $L = 0.05\,\mathrm{m}$ and $D = 0.005\,\mathrm{m}$. The fin is made of 2024 aluminum alloy. All fin surfaces (lateral and tip surfaces) are surrounded by a fluid medium with a constant convection coefficient (h) and temperature (T_∞) equal to $250\,\mathrm{W.m^{-2}.K^{-1}}$ and $20\,^\circ\mathrm{C}$, respectively. The fin base temperature (T_b) is equal to $90\,^\circ\mathrm{C}$.



Evaluate the following using the software module:

1. temperature profile along the fin longitudinal direction, $T(x)$;
2. total fin heat transfer rate, q_f;
3. fin efficiency, η_f; and
4. fin effectiveness, ε_f.

Module Application Example I: Module Application and Results

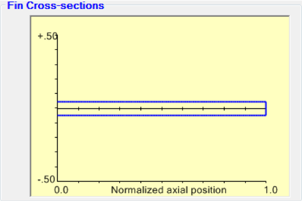
Module Input Data

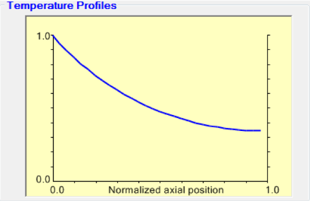
1 - Fin type
"Cylindrical Pin"

2 - Fin transport properties
k (Cond.): $178\,\mathrm{W.m^{-1}.K^{-1}}$
h (Hconv): $250\,\mathrm{W.m^{-2}.K^{-1}}$

3 - Fin geom. parameters
L (Length): $0.05\,\mathrm{m}$
D (Diam): $0.005\,\mathrm{m}$

Module Results





Design Data: Cylindrical Pin Fin

Cond. (k)	Hconv	Length	Diam
178,0	250,0	0,0500	0,0050

Performance Data: Cylindrical Pin Fin

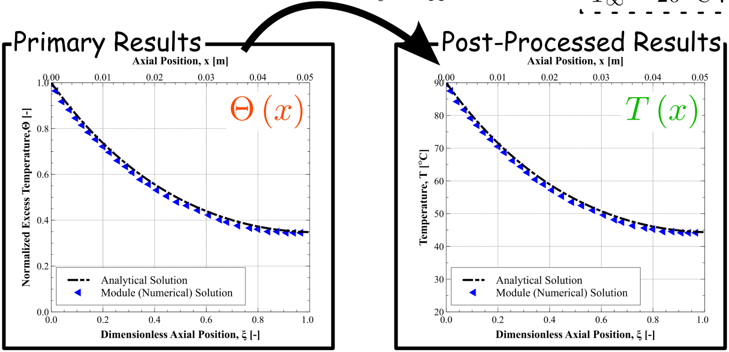
Fin Param.	Efficiency	Effectiveness	Qtotal	Resistance
0,859	54,6	22,4	0,1099	9,100

Module Application Example I: Results Analysis (1/4)

1. Temperature profile along the fin longitudinal direction, $T(x)$

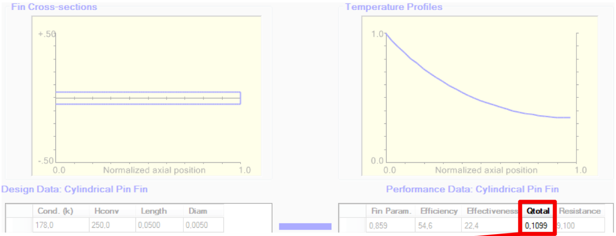
$$\Theta(x) = \frac{T(x) - T_\infty}{T_b - T_\infty}$$

$$T_b = 90\,^\circ\mathrm{C}$$
$$T_\infty = 20\,^\circ\mathrm{C}$$



Module Application Example I: Results Analysis (2/4)

2. Total fin heat transfer rate, q_f



$$\boxed{Q_{\text{total}} = 0.1099\,\text{W.K}^{-1}}$$

$$T_b = 90\,^{\circ}\text{C}$$
$$T_{\infty} = 20\,^{\circ}\text{C}$$

$$Q_{\text{total}} = \frac{q_f}{\theta_b} \Leftrightarrow$$

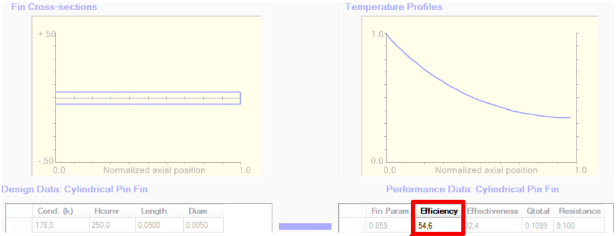
$$\Leftrightarrow q_f = Q_{\text{total}} \times \underbrace{(T_b - T_{\infty})}_{\theta_b} \Leftrightarrow \boxed{q_f \approx 7.69\,\text{W}}$$

$$(q_f)_{\text{Analytic Sol.}} \approx 7.70\,\text{W}$$

Exploring the Software Module - Uniform Pin Fin

Module Application Example I: Results Analysis (3/4)

3. Fin efficiency, η_f

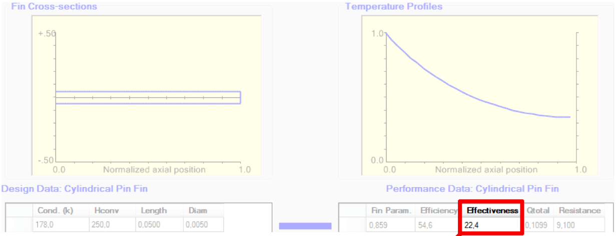


$$\boxed{\eta_f = 54.6\%}$$

Exploring the Software Module - Uniform Pin Fin

Module Application Example I: Results Analysis (4/4)

4. Fin effectiveness, ε_f

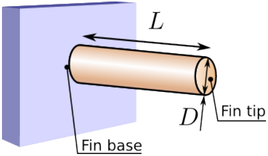


$$\boxed{\varepsilon_f = 22.4}$$

Exploring the Software Module - Uniform Pin Fin

Module Application Example II: Problem Statement

Consider for the same fin configuration, fin dimensions and temperatures of Example I the combination of three values of thermal conductivities – 5 (**k$_1$**), 178 (**k$_2$**), and 300 (**k$_3$**) $\mathrm{W.m^{-1}.K^{-1}}$ – with three values of convection heat transfer coefficients – 10 (**h$_1$**), 250 (**h$_2$**), and 1500 (**h$_3$**) $\mathrm{W.m^{-2}.K^{-1}}$.



For these nine combinations, evaluate the following using the module:

1. fin efficiency, η_f;
2. fin effectiveness, ε_f;
3. the suitability of the adiabatic tip assumption for the prediction of the fin heat rate for an actual active tip situation, q_f^a/q_f^c; and
4. the suitability of the 1D conduction assumption on the fin equation formulation in relation to the 3D conduction problem (advanced).

Exploring the Software Module - Uniform Pin Fin

Module Application Example II: Results Analysis (1/4)

1. Fin efficiency, η_f – Num.$_{(\text{Analyt.})}$ [%]

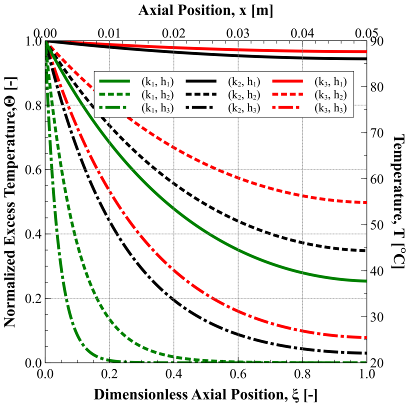
	h_1	h_2	h_3
k_1	$47.2_{(47.3)}$	$9.9_{(10.2)}$	$4.3_{(4.9)}$
k_2	$96.2_{(96.2)}$	$54.6_{(54.7)}$	$23.8_{(24.0)}$
k_3	$97.7_{(97.7)}$	$65.6_{(65.6)}$	$30.8_{(30.9)}$

2. Fin effectiveness, ε_f – Num.$_{(\text{Analyt.})}$ [–]

	h_1	h_2	h_3
k_1	$19.4_{(19.4)}$	$4.1_{(4.2)}$	**$1.8^{*}_{(2.0)}$**
k_2	$39.5_{(39.5)}$	$22.4_{(22.4)}$	$9.8_{(9.8)}$
k_3	$40.1_{(40.1)}$	$26.9_{(26.9)}$	$12.6_{(12.7)}$

Attention: The combination (k_1, h_3) is not recommended (and should be avoided) because the calculated fin effectiveness value (1.8) is less than 2.0.

Temp. Profiles, Θ and T – Analyt.



Exploring the Software Module - Uniform Pin Fin

Module Application Example II: Results Analysis (2/4)

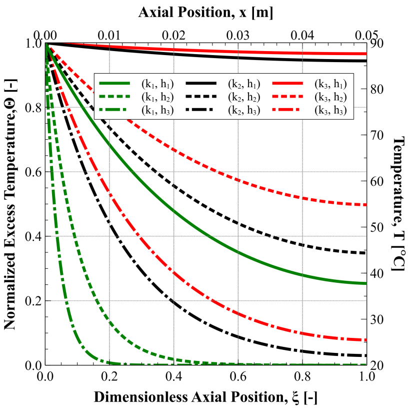
The ratio q_f^a/q_f^c is equal to $Q_{\mathrm{total}}^a/Q_{\mathrm{total}}^c$. Q_{total}^a (Q_{total}^c) is directly given by the module selecting the adiabatic (convective) fin tip boundary condition.

3. q_f^a/q_f^c – Num.$_{\text{(Analyt.)}}$ [–]

	h_1	h_2	h_3
k_1	$1.00_{(1.00)}$	$1.00_{(1.00)}$	$1.00_{(1.00)}$
k_2	$0.99_{(0.98)}$	$0.99_{(0.99)}$	$1.00_{(1.00)}$
k_3	$0.97_{(0.98)}$	$0.99_{(0.99)}$	$1.00_{(0.99)}$

The adiabatic tip assumption becomes well-suited for high values of q_f^a/q_f^c, *i.e.*, as the ratio h/k increases – in full accordance with Slide 13.

Temp. Profiles, Θ and T – Analyt.



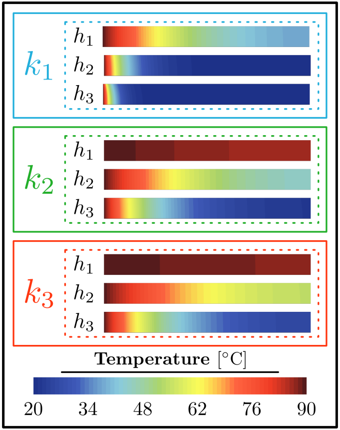
Exploring the Software Module - Uniform Pin Fin

Module Application Example II: Results Analysis (3/4)

4(1). 1D *vs.* multi-dim. heat conduction

- The 1D heat conduction assumption behind the fin equation formulation may be inadequate for accurate predictions under specific fin conditions.
- Since the fin configuration is axisymmetric the heat conduction may proceed along the longitudinal and radial directions (2D heat flow).
- 2D axisymmetric simulation results for the nine (k, h) combinations are presented at the right figure. For ease of visualization the vertical (radial) direction was stretched by a factor of 2.

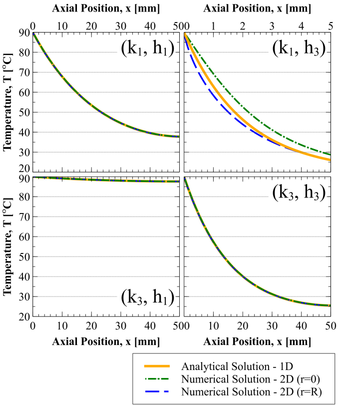
4(2). 2D temperature distribution



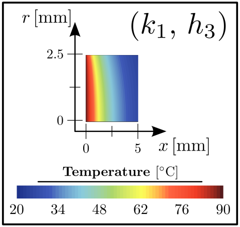
Exploring the Software Module - Uniform Pin Fin

Module Application Example II: Results Analysis (4/4)

4(3). Temp. profiles - extreme cases



4(4). Extreme case (k_1, h_3)



- The 1D conduction assumption on the formulation of the fin equation is less adequate for high h/k ratios.
- The multi-dimensional conduction effects are more pronounced near the fin base surface.

Useful Relations

1. Fin Temperature Profile - Θ, θ, T

$$\Theta(\xi) = \frac{\theta(\xi)}{\theta_b} = \frac{T(\xi) - T_\infty}{T(0) - T_\infty}$$

$\Theta(\xi)$ – Normalized excess temperature
θ_b – Excess temperature at the fin base
ξ $(= x/L)$ – Dimensionless axial position
L – Fin length

2. Fin Heat Rate - q_f

$$q_f = \theta_b \, \underbrace{h \int_{A_f} \Theta(\xi) \, dA_s}_{Q_{\text{total}} = \frac{q_f}{\theta_b} = R_{t,f}^{-1}}$$

q_f^a – Fin heat rate considering an adiabatic tip
q_f^c – Fin heat rate considering a convective tip
A_f – Convective fin area

3. Fin Effectiveness - ε_f

$$\varepsilon_f = \frac{q_f}{\underbrace{A_{c,b} h \theta_b}_{q_b^{\text{NoFin}}}} = \eta_f \frac{A_f}{A_{c,b}}$$

Fins with $\varepsilon_f < 2$ are not recommended for application

4. Fin Efficiency - η_f

$$\eta_f = \frac{q_f}{\underbrace{A_f h \theta_b}_{q_{\text{max}}}} = \frac{Q_{\text{total}}}{A_f h}$$

q_{max} – Heat transfer rate that would be observed if the entire fin were at the base temperature