Heat Transfer

Computational Laboratories

Heat Transfer From Extended Surfaces
(Laboratory I)

Exploring the role of geometrical and thermophysical design parameters on the fin performance

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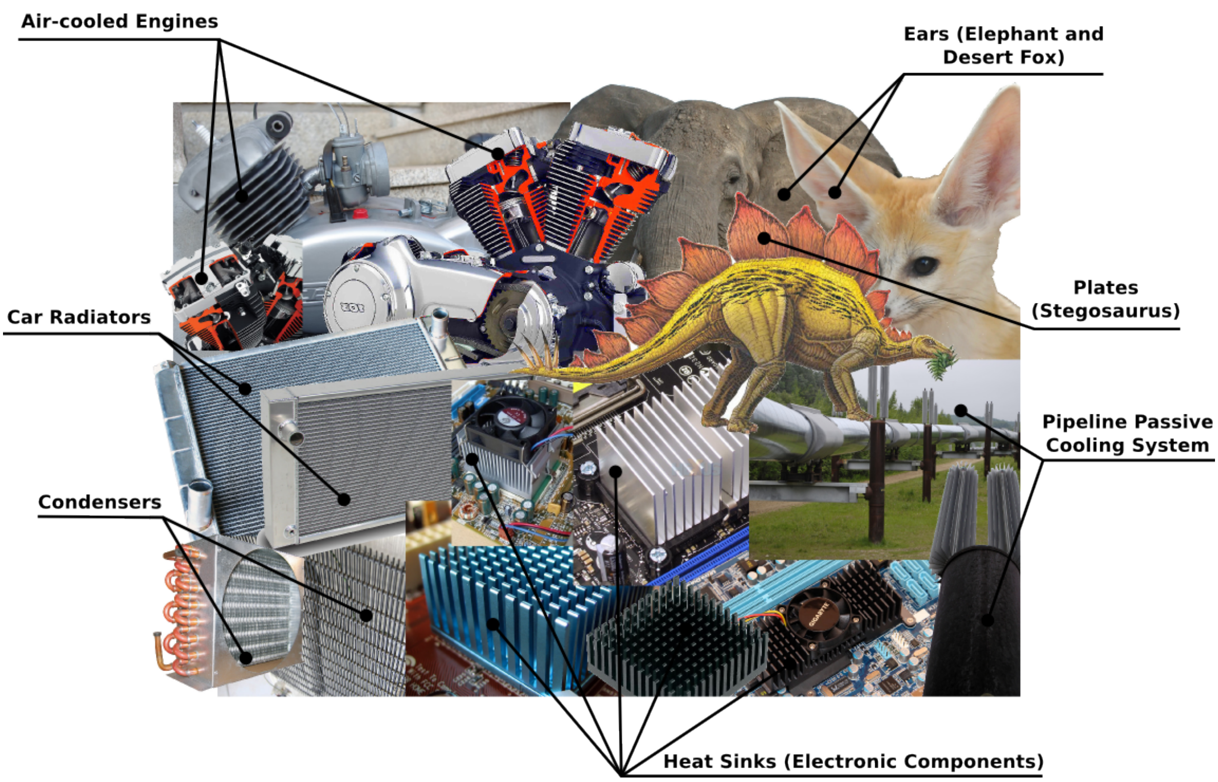
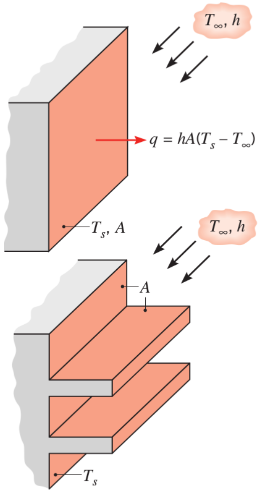
Extended Surfaces - Fins: Practical Applications



Extended Surfaces - Fins: Motivation for Application



To increase the rate of heat transfer (q) considering T_s constant:

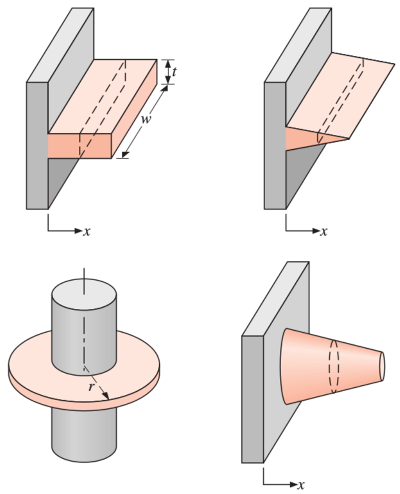
- Increase h and/or decrease T_∞;
- **Increase surface area (A).**

Motivation for Fin Application

Extended surfaces aim to **enhance heat transfer** by an increase in the available surface area for convection (and/or radiation). **Highly recommended when h is small** (gases under free-convection conditions).

Extended Surfaces - Fins: Configurations

Fin Configuration



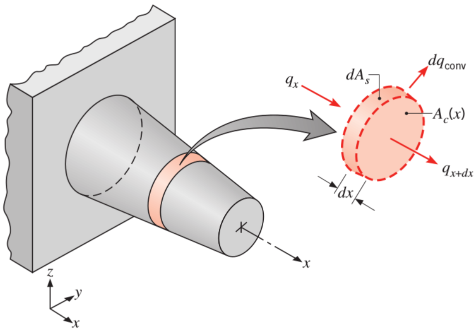
Typical Configurations:

- Straight fins:
  - Uniform Cross Section
  - Non-uniform Cross Section
- Annular fins (non-uniform cross section)
- Pin fins (non-uniform cross-section)

The **selection** of the fin design depends on: space, weight, cost, effect on fluid motion (and consequently on h and pressure drop).

Fin Equation: Governing Energy Balance Equation

- Equation derived from the application of an energy balance to a differential element.
- Assumptions:
  - one-dimensional conduction;
  - steady-state conditions;
  - constant thermal conductivity;
  - no thermal energy generation;
  - negligible radiative heat losses.



Fin Equation - General Form

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\left[T(x) - T_\infty\right] = 0$$

- The solution of the fin equation provides the temperature distribution along the axial position $T(x)$.

Fin Equation Applied to Uniform Fins

Fin Equation - General Form

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\left[T(x) - T_\infty\right] = 0$$

For fins of <u>uniform cross section</u>:

- $\frac{dA_c}{dx} = 0$

Considering:

- $m^2 = \frac{hP}{kA_c}$ $(P = dA_s/dx)$
- $\xi = x/L$
- $\Theta(\xi) = \frac{\theta(\xi)}{\theta_b} = \frac{T(\xi)-T_\infty}{T(0)-T_\infty}$

Fin Equation for Uniform Fins

$$\frac{d^2\Theta}{d\xi^2} - (mL)^2\,\Theta(\xi) = 0$$

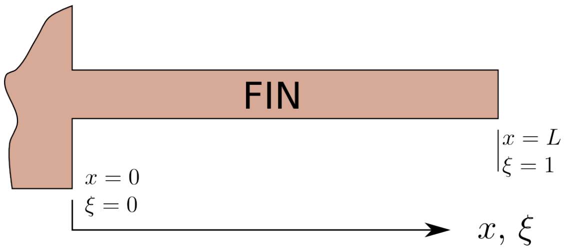
General Sol. for the Fin Eq. Applied to Uniform Fins

General Solution of the Fin Equation for Uniform Fins

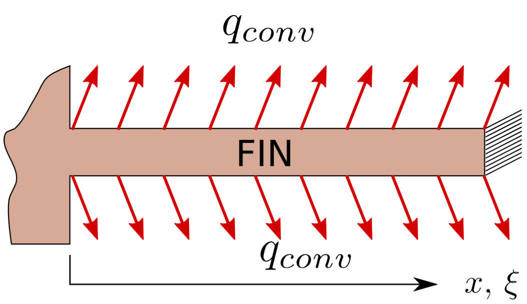
$$\Theta(\xi) = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$$

Boundary conditions (BC)

1. **Fin base ($\xi = 0$):**
   - $\Theta(0) = 1$ (Fixed temperature - Dirichlet BC)
2. **Fin tip ($\xi = 1$):**
   - $\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = 0$ (Adiabatic tip - Zero Neumann BC)
   - $-k\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = h_{tip} L \Theta(1)$ (Active tip - Convection BC)



Fin Eq. Solution for Uniform Fin with Adiabatic Tip



Normalized Excess Temperature

$$\Theta^a(\xi) = \frac{\cosh\left[mL(1-\xi)\right]}{\cosh(mL)}$$

Fin Heat Transfer Rate

$$q_f^a = M \tanh(mL)$$

Note that

- $mL = \sqrt{\frac{hP}{kA_c}}L$
- $M = \sqrt{hPkA_c}\theta_b$
  - M corresponds to the heat rate observed for an infinite fin (q_f^i)

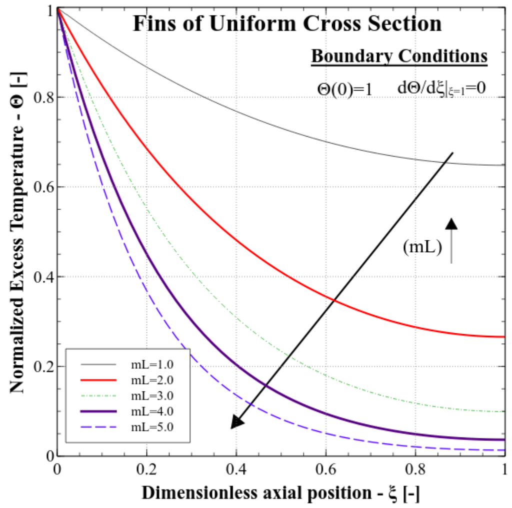
Fin Eq. Solution for Uniform Fin with Adiabatic Tip

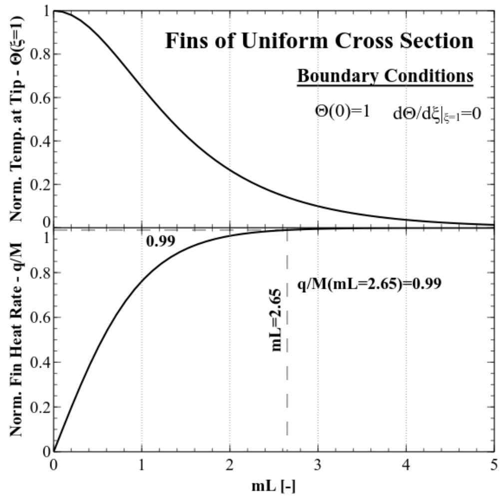
Normalized Excess Temperature

$$\Theta^a(\xi) = \frac{\cosh\left[mL(1-\xi)\right]}{\cosh(mL)}$$

Normalized Heat Transfer Rate

$$\frac{q_f^a}{M} = \tanh(mL)$$





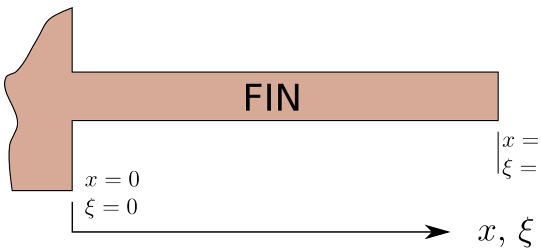
General Sol. for the Fin Eq. Applied to Uniform Fins

General solution of the Fin Equation for Uniform Fins

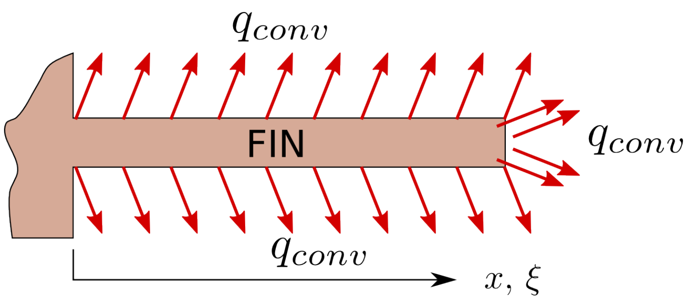
$$\Theta(\xi) = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$$

Boundary conditions (BC)

1. **Fin base ($\xi = 0$):**
   - $\Theta(0) = 1$ (Fixed temperature - Dirichlet BC)
2. **Fin tip ($\xi = 1$):**
   - $\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = 0$ (Adiabatic tip - Zero Neumann BC)
   - $-k\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = h_{tip}L\Theta(1)$ (Active tip - Convection BC)



Fin Eq. Solution for Uniform Fin with Convective Tip



Normalized Excess Temperature

$$\Theta^c(\xi) = \frac{\cosh[mL(1-\xi)] + (h_{tip}/km)\sinh[mL(1-\xi)]}{\cosh(mL) + (h_{tip}/km)\sinh(mL)}$$

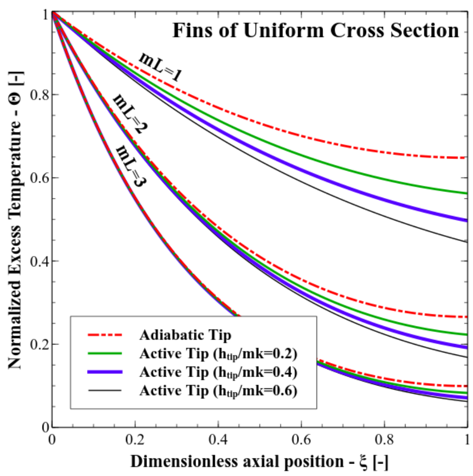
Fin Heat Transfer Rate

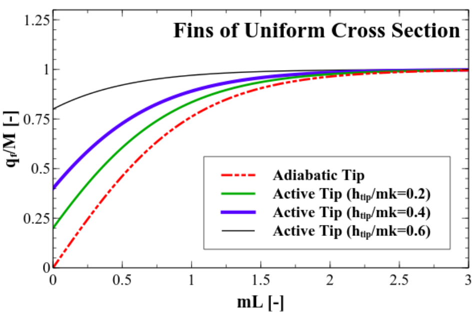
$$q_f^c = M\frac{(h_{tip}/mk) + \tanh(mL)}{1 + (h_{tip}/mk)\tanh(mL)}$$

Note that

- $h_{tip} = 0$: adiabatic tip solution
- Generally $h_{tip} = h$

Adiabatic *vs.* Convective Tip Solutions





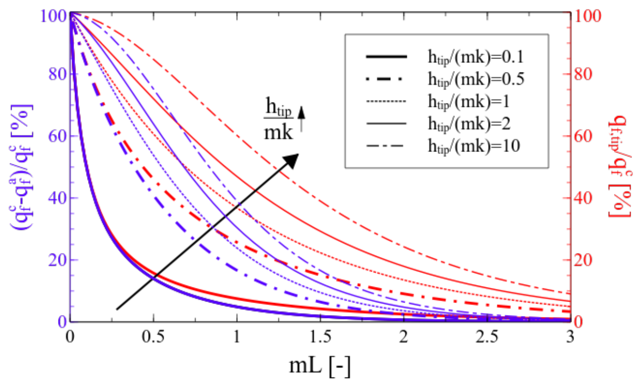
The difference $\Theta^a(\xi) - \Theta^c(\xi)$ increases by:

- increasing h_{tip} for a constant mL;
- decreasing L for a constant h_{tip}.

Increasing h_{tip}:

- $q_f^c - q_f^a$ increases;
- $\left.\frac{d\Theta}{d\xi}\right|_{\xi=1}$ increases, *i.e.* $q_{f,tip}$ increases, since $q_{f,tip} = -k\theta_b \frac{A_c}{L} \left.\frac{d\Theta}{d\xi}\right|_{\xi=1}$.

Relative importance of the tip convection on the overall heat transfer rate from the fin (1/2)



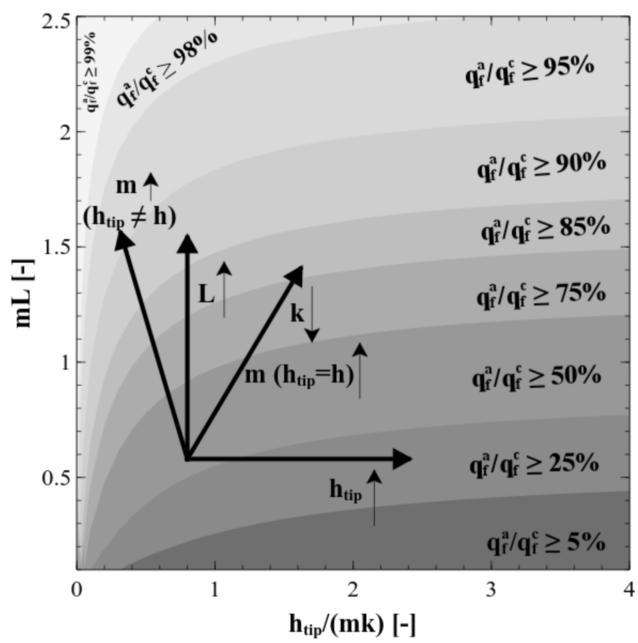
$q_{f,tip}/q_f^c$ decreases:

- increasing the fin length ($L \uparrow$);
- decreasing the convection coefficient at the fin tip ($h_{tip} \downarrow$).

The adiabatic tip solution becomes a suitable approximation for an actual active tip for **low values of $\mathbf{q_{f,tip}/q_f^c}$** or high values of q_f^a/q_f^c.

- $q_{f,tip} = \frac{q_f^c - q_f^a}{1-2\mathrm{sech}(mL)\sinh\left(\frac{mL}{2}\right)^2}$
  - $q_{f,tip} \approx q_f^c - q_f^a$ for low mL values.
- $q_{f,tip} = h_{tip} A_c \theta_b \Theta^c(\xi = 1)$
- $\frac{q_{f,tip}}{q_f^c} = \frac{(h_{tip}/km)}{(h_{tip}/km)\cosh(mL)+\sinh(mL)}$

Relative importance of the tip convection on the overall heat transfer rate from the fin (2/2)



The adiabatic tip solution becomes a suitable approximation for an actual active tip for low values of $q_{f,tip}/q_f^c$ or **high values of $\mathbf{q_f^a/q_f^c}$**.

- $\frac{q_f^a}{q_f^c} = \frac{\tanh(mL)\left[1+(h_{tip}/km)\tanh(mL)\right]}{(h_{tip}/km)+\tanh(mL)}$

q_f^a/q_f^c increases:

- increasing the fin length ($L \uparrow$);
- decreasing h_{tip};
- increasing m, that is:
  - increasing h/k;
  - increasing P/A_c.

Fin Performance Parameters

Fin Effectiveness (ε_f)

$$\varepsilon_f = \frac{q_f}{A_{c,b} h \theta_b}$$

$\boldsymbol{\varepsilon_f}$ - ratio between the actual fin heat rate and the heat rate observed without the fin application.

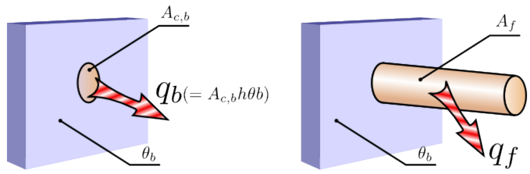
- The application of an extended surface may increase the heat transfer resistance (fin resistance) in relation to the convection resistance of the bare surface ($\varepsilon_f = R_{t,b}/R_{t,f}$).
- In general, $\varepsilon_f < 2$ do not justify the fin application.

Fin Efficiency (η_f)

$$\eta_f = \frac{q_f}{A_f h \theta_b}$$

$\boldsymbol{\eta_f}$ - ratio between the actual fin heat rate and the maximum (idealized) fin heat rate achieved for $\theta(x) = \theta_b$.

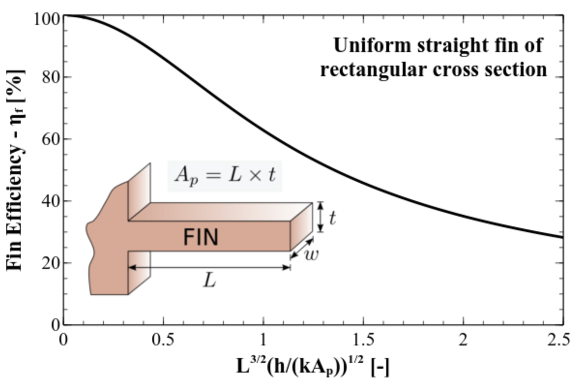
- An efficiency above 90% is attained for most of the fins used in practical applications.



Performance for the Uniform Fin with Adiabatic Tip

Fin Efficiency (η_f)

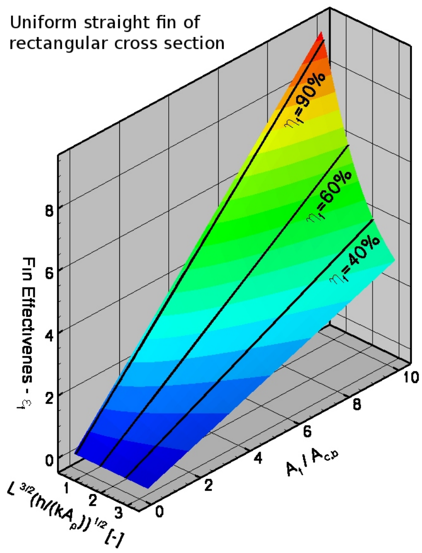
$$\eta_f^a = \frac{\tanh(mL)}{mL} = \int_0^1 \Theta d\xi$$



$$mL \cong L^{3/2} \left[2h/(kA_p)\right]^{1/2}, \; w >> t$$

Fin Effectiveness (ε_f)

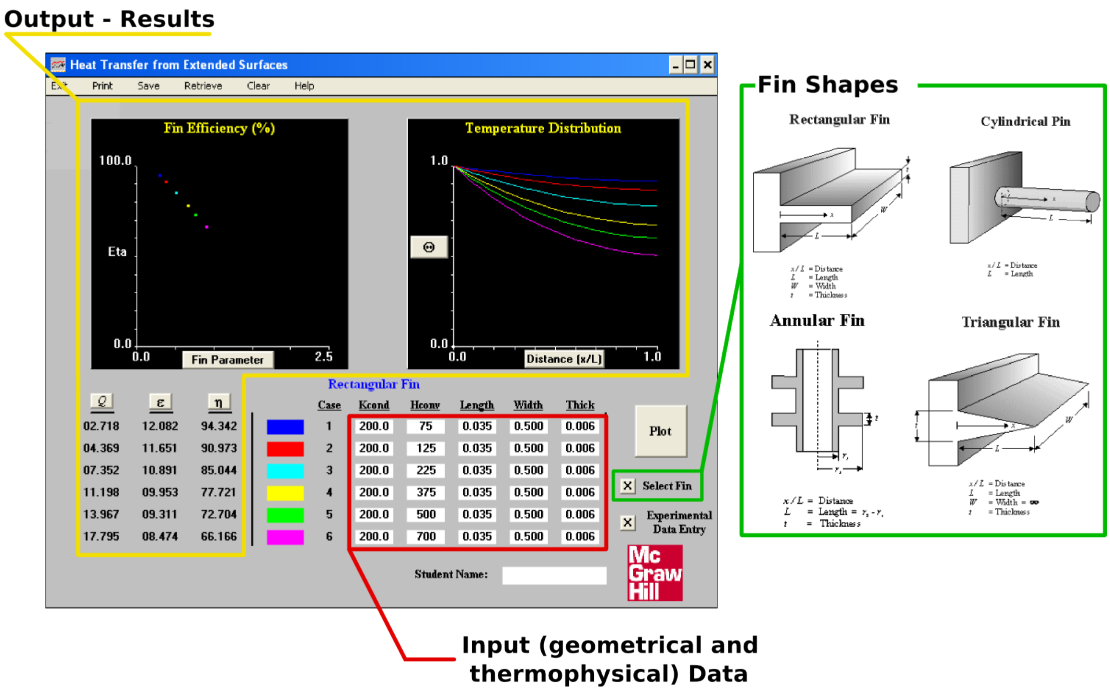
$$\varepsilon_f^a = \frac{A_f}{A_{c,b}} \eta_f^a$$



Final Remarks

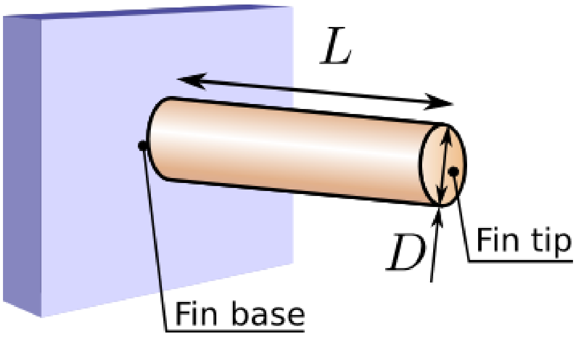
- **The temperature distribution** along the fin **is evaluated with the fin equation** through analytic or numerical means. From the evaluated temperature profile, the fin heat rate, efficiency and effectiveness can be computed.
- **The effect of fin design parameters and boundary conditions** on the fin temperature distribution, heat rate, and performance parameters **registered before for uniform fins is also observed** (eventually at a different extent) **for fins of non-uniform cross-sectional area.**
- **For non-uniform fins** the integration of the fin equation (governing energy equation) and the evaluation of integration constants result in **complex expressions for the temperature distribution**. For such reason, **only expressions for the fin efficiency are commonly provided** in the literature.

Exploring the Software



- **The software solves the fin equation through numerical methods** employing the finite volume method.

Exploring the Software - Uniform Pin Fin



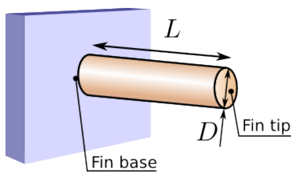
Exploring the Software - Uniform Pin Fin

Role of the thermal conductivity ("Kcond")

Consider a pin fin of uniform cross section ("Cylindrical Pin") with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$L = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$D = 0.015\,\mathrm{m}$



2. Thermal conductivity values

- $k_1 = 20\,\mathrm{W/(mK)}$
- $k_2 = 30\,\mathrm{W/(mK)}$
- $k_3 = 50\,\mathrm{W/(mK)}$
- $k_4 = 100\,\mathrm{W/(mK)}$
- $k_5 = 200\,\mathrm{W/(mK)}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

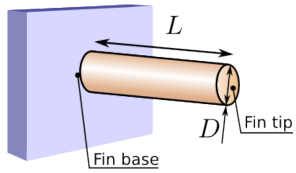
Exploring the Software - Uniform Pin Fin

Role of the convection heat transfer coefficient ("Hconv")

Consider a pin fin of uniform cross section ("Cylindrical Pin") with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$L = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$D = 0.015\,\mathrm{m}$



2. Convection coefficient values

- $h_1 = 100\,\mathrm{W/(m^2K)}$
- $h_2 = 150\,\mathrm{W/(m^2K)}$
- $h_3 = 200\,\mathrm{W/(m^2K)}$
- $h_4 = 300\,\mathrm{W/(m^2K)}$
- $h_5 = 400\,\mathrm{W/(m^2K)}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

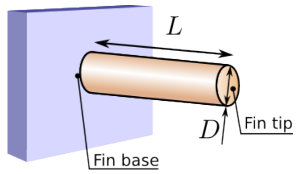
Exploring the Software - Uniform Pin Fin

Role of the fin length ("Length")

Consider a pin fin of uniform cross section ("Cylindrical Pin") with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$L = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$D = 0.015\,\mathrm{m}$



2. Fin length values

- $L_1 = 0.035\,\mathrm{m}$
- $L_2 = 0.04\,\mathrm{m}$
- $L_3 = 0.05\,\mathrm{m}$
- $L_4 = 0.06\,\mathrm{m}$
- $L_5 = 0.07\,\mathrm{m}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

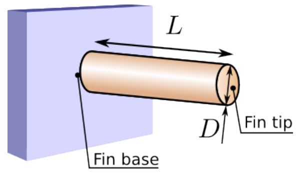
Exploring the Software - Uniform Pin Fin

Role of the fin diameter ("Diam")

Consider a pin fin of uniform cross section ("Cylindrical Pin") with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$L = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$D = 0.015\,\mathrm{m}$



2. Fin diameter values

- $D_1 = 0.015\,\mathrm{m}$
- $D_2 = 0.02\,\mathrm{m}$
- $D_3 = 0.03\,\mathrm{m}$
- $D_4 = 0.04\,\mathrm{m}$
- $D_5 = 0.05\,\mathrm{m}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

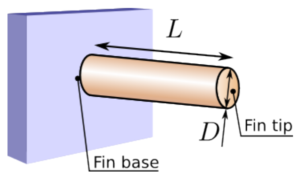
Exploring the Software - Uniform Pin Fin

Effect of the tip boundary condition

Consider a pin fin of uniform cross section ("Cylindrical Pin")

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$L = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$D = 0.015\,\mathrm{m}$



2. Prescribed boundary condition

- Adiabatic tip
- Convective tip

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

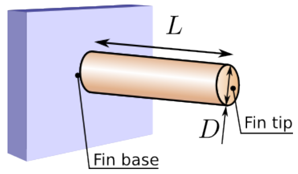
Exploring the Software - Uniform Pin Fin

Role of the fin diameter ("Diam") on q_f^a/q_f^c

Consider a pin fin of uniform cross section ("Cylindrical Pin")

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$L = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$D = 0.015\,\mathrm{m}$



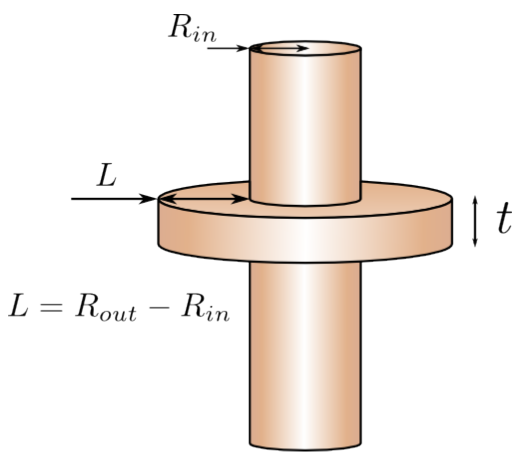
2. Diameter values

- $D_1 = 0.015\,\mathrm{m}$
- $D_2 = 0.02\,\mathrm{m}$
- $D_3 = 0.03\,\mathrm{m}$
- $D_4 = 0.04\,\mathrm{m}$
- $D_5 = 0.05\,\mathrm{m}$

3. Effect on q_f^a/q_f^c

D	$Q^a = q_f^a/\theta_b$	$Q^c = q_f^c/\theta_b$	q_f^a/q_f^c
D_1	0.110	0.115	$\approx$ **0.959**
D_2	0.160	0.169	$\approx$ 0.947
D_3	0.262	0.292	$\approx$ 0.897
D_4	0.368	0.430	$\approx$ 0.856
D_5	0.475	0.583	$\approx$ 0.815

Exploring the Software - Annular Fin



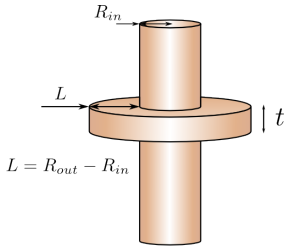
Exploring the Software - Annular Fin

Role of the thermal conductivity ("Kcond")

Consider an annular fin with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$R_{in} = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$R_{out} = 0.050\,\mathrm{m}$
	$t = 0.001\,\mathrm{m}$



2. Thermal conductivity values

- $k_1 = 20\,\mathrm{W/(mK)}$
- $k_2 = 30\,\mathrm{W/(mK)}$
- $k_3 = 50\,\mathrm{W/(mK)}$
- $k_4 = 100\,\mathrm{W/(mK)}$
- $k_5 = 200\,\mathrm{W/(mK)}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

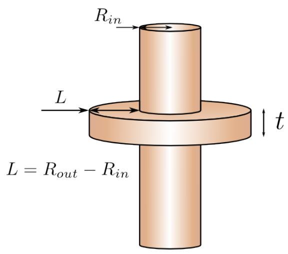
Exploring the Software - Annular Fin

Role of the convection heat transfer coefficient ("Hconv")

Consider an annular fin with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$R_{in} = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$R_{out} = 0.050\,\mathrm{m}$
	$t = 0.001\,\mathrm{m}$



2. Convection coefficient values

- $h_1 = 100\,\mathrm{W/(m^2K)}$
- $h_2 = 150\,\mathrm{W/(m^2K)}$
- $h_3 = 200\,\mathrm{W/(m^2K)}$
- $h_4 = 300\,\mathrm{W/(m^2K)}$
- $h_5 = 400\,\mathrm{W/(m^2K)}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

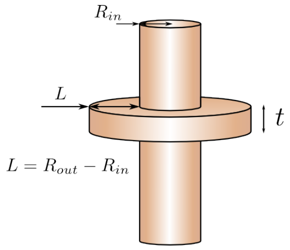
Exploring the Software - Annular Fin

Role of the fin outside radius ("Rout")

Consider an annular fin with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$R_{in} = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$R_{out} = 0.050\,\mathrm{m}$
	$t = 0.001\,\mathrm{m}$



2. Outside radius values

- $R_{out,1} = 0.050\,\mathrm{m}$
- $R_{out,2} = 0.055\,\mathrm{m}$
- $R_{out,3} = 0.060\,\mathrm{m}$
- $R_{out,4} = 0.070\,\mathrm{m}$
- $R_{out,5} = 0.080\,\mathrm{m}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

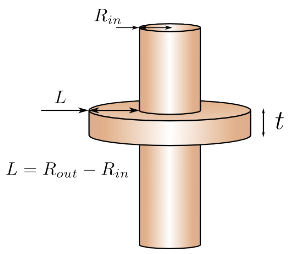
Exploring the Software - Annular Fin

Role of the fin thickness ("Thick")

Consider an annular fin with an adiabatic tip

1. Reference Data

Themo. Properties	Geom. Properties
$k = 20\,\mathrm{W/(mK)}$	$R_{in} = 0.035\,\mathrm{m}$
$h = 100\,\mathrm{W/(m^2K)}$	$R_{out} = 0.050\,\mathrm{m}$
	$t = 0.001\,\mathrm{m}$



2. Outside radius values

- $t_1 = 0.0010\,\mathrm{m}$
- $t_2 = 0.0015\,\mathrm{m}$
- $t_3 = 0.0020\,\mathrm{m}$
- $t_4 = 0.0030\,\mathrm{m}$
- $t_5 = 0.0040\,\mathrm{m}$

3. Effect on:

- Normalized Excess Temperature ($\Theta(\xi)$);
- Fin Heat Rate ($Q = q_f/\theta_b$);
- Fin Efficiency (η_f);
- Fin Effectiveness (ε_f).

Useful Relations

1. Fin Temperature Distribution - Θ, θ, T

$$\Theta(\xi) = \frac{\theta(\xi)}{\theta_b} = \frac{T(\xi) - T_\infty}{T(\xi = 0) - T_\infty}$$

$\Theta(\xi)$ - Normalized excess temperature
θ_b - Excess temperature at the fin base

2. Fin Heat Rate - q_f

$$q_f = \theta_b \, h \underbrace{\int_{A_f} \Theta(\xi) \, dA_s}_{Q = \frac{q_f}{\theta_b}}$$

q_f^a - fin heat rate considering an adiabatic tip
q_f^c - fin heat rate considering a convective tip
A_f - convective fin area

3. Fin Efficiency - η_f

$$\eta_f = \frac{q_f}{A_f h \theta_b} = \frac{Q}{A_f h}$$

4. Fin Effectiveness - ε_f

$$\varepsilon_f = \frac{q_f}{A_{c,b} h \theta_b} = \eta_f \frac{A_f}{A_{c,b}}$$