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Heat Transfer

Exam 2 – Problem 1

February 9, 2021 (15h00)

A straight, circular tube with a diameter equal to 6.0 cm is subjected at the external side to a cross flow of atmospheric air with upstream velocity and temperature equal to v_{wind} and 20°C, respectively. An oil (Eastman Therminol 62) circulates inside the tube, entering the tube with uniform velocity and temperature profiles with values equal to $0.1\,\text{m}\,\text{s}^{-1}$ and 60°C, respectively. Consider the following (constant) thermophysical properties for the oil: $\rho = 926\,\text{kg}\,\text{m}^{-3}$, $c_p = 2050\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $\mu = 5.38 \times 10^{-3}\,\text{N}\,\text{s}\,\text{m}^{-2}$, and $k = 1.195 \times 10^{-1}\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$. Consider steady-state conditions and neglect the tube thickness and radiative heat transfer.

(a) (3.0 v.) Estimate the external (air-side) convection heat transfer coefficient knowing that the oil releases 1.5 kW to the surrounding air along a distance equal to 10 m from the inlet section.

Solution:

The external (air-side) convection heat transfer coefficient (h_e) can be calculated applying Equation (1). In this equation, q is the thermal power loss from the oil to the surrounding air, $T_{m,i}$ and $T_{m,L}$ correspond to the oil mean temperatures at the inlet section ($x = 0$) and at $x = L = 10\,\text{m}$, respectively, and h_i is the internal (oil-side) convection heat transfer coefficient.

$$q = \frac{\Delta T_{\text{lm}}}{R_{t,\text{tot}}} \Leftrightarrow q = \frac{\frac{\Delta T_L - \Delta T_i}{\ln(\Delta T_L/\Delta T_i)}}{\frac{1}{h_i A} + \frac{1}{h_e A}} \Leftrightarrow q = \frac{\frac{(T_{m,L}-T_\infty)-(T_{m,i}-T_\infty)}{\ln\left[(T_{m,L}-T_\infty)/(T_{m,i}-T_\infty)\right]}}{\frac{1}{\pi D L}\left(\frac{1}{h_i}+\frac{1}{h_e}\right)} \Leftrightarrow$$
$$\Leftrightarrow h_e = \left(\pi D L \frac{\frac{T_{m,L}-T_{m,i}}{\ln\left[(T_{m,L}-T_\infty)/(T_{m,i}-T_\infty)\right]}}{q} - \frac{1}{h_i}\right)^{-1} \tag{1}$$

In order to compute h_e by applying Equation (1), $T_{m,L}$ and h_i must be previously calculated. $T_{m,L}$ is evaluated in Equation (2) and h_i is evaluated with Equations (3) – (6).

Since the total rate of heat transfer lost to the surrounding environment (air) is known, as well as the oil inlet temperature and the heat capacity rate ($\dot{m}c_p$), the oil mean temperature at the distance of 10 m from the inlet section can be determined through Equation (2).

$$q = \dot{m}c_p\left(T_{m,i} - T_{m,L}\right) \Leftrightarrow T_{m,L} = T_{m,i} - \frac{4q}{\rho u_m \pi D^2 c_p} \Leftrightarrow$$
$$\Leftrightarrow T_{m,L} = 60 - \frac{4 \times 1500}{926 \times 0.1 \times \pi \times 0.06^2 \times 2050} \Leftrightarrow \tag{2}$$
$$\Leftrightarrow T_{m,L} \approx 57.205°\text{C}$$

The Reynolds number is calculated in Equation (3). The Reynolds number is lower than the critical value (≈ 2300), and consequently, the flow regime is laminar.

$$Re_D = \frac{\rho u_m D}{\mu} \Leftrightarrow Re_D = \frac{926 \times 0.1 \times 0.06}{5.38 \times 10^{-3}} \Leftrightarrow Re_D \approx 1032.714 < 2300 \tag{3}$$

The oil Prandtl number is evaluated with the provided thermophysical properties in Equation (4)

$$Pr = \frac{\nu}{\alpha} \Leftrightarrow Pr = \frac{c_p \mu}{k} \Leftrightarrow Pr = \frac{2050 \times 5.38 \times 10^{-3}}{1.195 \times 10^{-1}} \Leftrightarrow Pr \approx 92.293 \tag{4}$$

The length of the entrance region is estimated in Equation (5). Since the entry length is much higher than the tube length under consideration ($L = 10\,\mathrm{m}$), laminar developing flow conditions are expected over the corresponding tube length.

$$x_{\mathrm{fd,t}} \approx 0.05 Re_D Pr D \Leftrightarrow x_{\mathrm{fd,t}} \approx 0.05 \times 1032.714 \times 92.293 \times 0.06 \Leftrightarrow \Leftrightarrow x_{\mathrm{fd,t}} \approx 285.937\,\mathrm{m} >> L = 10\,\mathrm{m} \tag{5}$$

Since along the distance under consideration, thermally – or combined, since $Pr >> 5$ – developing flow conditions are anticipated, then the following empirical Nusselt number correlation is applied to obtain the average convection heat transfer coefficient – see Equation (6).

$$\begin{aligned} \overline{Nu}_D \equiv \frac{h_i D}{k} = 3.66 + \frac{0.0668\,[(D/L)\,Re_D Pr]}{1 + 0.04\,[(D/L)\,Re_D Pr]^{2/3}} \Leftrightarrow \\ \Leftrightarrow h_i = \frac{k}{D}\left(3.66 + \frac{0.0668\,[(D/L)\,Re_D Pr]}{1 + 0.04\,[(D/L)\,Re_D Pr]^{2/3}}\right) \Leftrightarrow \\ \Leftrightarrow h_i = \frac{1.195 \times 10^{-1}}{0.06}\left(3.66 + \frac{0.0668\,[(0.06/10) \times 1032.714 \times 92.293]}{1 + 0.04\,[(0.06/10) \times 1032.714 \times 92.293]^{2/3}}\right) \Leftrightarrow \\ \Leftrightarrow h_i \approx 27.547\,\mathrm{W\,m^{-2}\,K^{-1}} \end{aligned} \tag{6}$$

Finally, the external convection heat transfer coefficient is calculated replacing the parameters, properties, and variables by the corresponding values in Equation (1) – see Equation (7).

$$\begin{aligned} h_e = \left(\pi D L \frac{\frac{T_{m,L} - T_{m,i}}{\ln\left[\left(T_{m,L} - T_\infty\right)/\left(T_{m,i} - T_\infty\right)\right]}}{q} - \frac{1}{h_i}\right)^{-1} \Leftrightarrow \\ \Leftrightarrow h_e = \left(\pi \times 0.06 \times 10 \times \frac{\frac{57.205 - 60}{\ln[(57.205 - 20)/(60 - 20)]}}{1500} - \frac{1}{27.547}\right)^{-1} \Leftrightarrow \\ \Leftrightarrow \boxed{h_e \approx 82.058\,\mathrm{W\,m^{-2}\,K^{-1}}} \end{aligned} \tag{7}$$

(b) (2.0 v.) Determine the heat transfer rate to the surrounding air per unit tube length considering $v_{\mathrm{wind}} = 10\,\mathrm{m\,s^{-1}}$ and neglecting the internal (oil-side) convection resistance in relation to the external (air-side) convection resistance – condition achieved, for instance, considering a very high oil mass flow rate.

Solution:

Since the internal convection resistance is negligible in comparison to the external convection resistance, the tube surface is at a constant temperature (T_s) and equal to the oil inlet temperature (60°C). Consequently, the heat transfer rate to the surrounding air per unit tube length is computed through Equation (8).

$$q' = \pi D \overline{h}_e \left(T_s - T_\infty\right) \tag{8}$$

Three average Nusselt number correlations for a circular cylinder in cross flow are commonly considered in heat transfer textbooks: (*i*) Zukauskas correlation; (*ii*) Hilpert correlation; and (*iii*) Churchill correlation. These correlations are herein considered to estimate the average (external) convection heat transfer coefficient required in Equation (8). The thermophysical properties required by the correlations are gathered from Table A-4.

- Zukauskas correlation

For air at atmospheric pressure and at the air upstream temperature, *i.e.*, at $20°\mathrm{C} = 293.15\,\mathrm{K}$ (T_∞): $\nu \approx 1.528 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$, $k \approx 2.575 \times 10^{-2}\,\mathrm{W\,m^{-1}\,K^{-1}}$, and $Pr \approx 0.709$. For air at atmospheric pressure and at the tube surface temperature, *i.e.*, at $60°\mathrm{C} = 333.15\,\mathrm{K}$ (T_s): $Pr_s \equiv Pr \approx 0.702$.

The Reynolds number is calculated in Equation (9), the average convection heat transfer coefficient is computed in Equation (10), and finally, the heat rate per unit tube length is determined in Equation (11).

$$Re_D = \frac{v_{\mathrm{wind}} D}{\nu} \Leftrightarrow Re_D = \frac{10 \times 0.06}{1.528 \times 10^{-5}} \Leftrightarrow Re_D \approx 39267.016 \tag{9}$$

$$\begin{aligned} \overline{Nu}_D \equiv \frac{\overline{h}_e D}{k} = 0.26 Re_D^{0.6} Pr^{0.37} \left(\frac{Pr}{Pr_s}\right)^{1/4} \Leftrightarrow \\ \Leftrightarrow \overline{h}_e = \frac{2.575 \times 10^{-2}}{0.06} \times 0.26 \times 39267.016^{0.6} \times 0.709^{0.37} \times \left(\frac{0.709}{0.702}\right)^{1/4} \Leftrightarrow \\ \Leftrightarrow \overline{h}_e \approx 56.212\,\mathrm{W\,m^{-2}\,K^{-1}} \end{aligned} \tag{10}$$

$$q' = \pi D \overline{h}_e (T_s - T_\infty) \Leftrightarrow q' = \pi \times 0.06 \times 56.212 \times (60 - 20) \Leftrightarrow \boxed{q' \approx 423.828\,\mathrm{W\,m^{-1}}} \tag{11}$$

- <u>Hilpert correlation</u>

For air at atmospheric pressure and at the film temperature, *i.e.*, at $[(60 + 20)/2]°\,\mathrm{C} = 40°\mathrm{C} = 313.15\,\mathrm{K}$ (T_f): $\nu \approx 1.721 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$, $k \approx 2.727 \times 10^{-2}\,\mathrm{W\,m^{-1}\,K^{-1}}$, and $Pr \approx 0.705$.

The Reynolds number is calculated in Equation (12), the average convection heat transfer coefficient is computed in Equation (13), and finally, the heat rate per unit tube length is determined in Equation (14).

$$Re_D = \frac{v_{\mathrm{wind}} D}{\nu} \Leftrightarrow Re_D = \frac{10 \times 0.06}{1.721 \times 10^{-5}} \Leftrightarrow Re_D \approx 34863.451 \tag{12}$$

$$\begin{aligned} \overline{Nu}_D \equiv \frac{\overline{h}_e D}{k} = C Re_D^m Pr^{1/3} \Leftrightarrow \\ \Leftrightarrow \overline{h}_e = \frac{2.727 \times 10^{-2}}{0.06} \times 0.193 \times 34863.451^{0.618} \times 0.705^{1/3} \Leftrightarrow \overline{h}_e \approx 50.081\,\mathrm{W\,m^{-2}\,K^{-1}} \end{aligned} \tag{13}$$

$$q' = \pi D \overline{h}_e (T_s - T_\infty) \Leftrightarrow q' = \pi \times 0.06 \times 50.081 \times (60 - 20) \Leftrightarrow \boxed{q' \approx 377.602\,\mathrm{W\,m^{-1}}} \tag{14}$$

- <u>Churchill correlation</u>

For air at atmospheric pressure and at the film temperature, *i.e.*, at $[(60 + 20)/2]°\,\mathrm{C} = 40°\mathrm{C} = 313.15\,\mathrm{K}$ (T_f): $\nu \approx 1.721 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$, $k \approx 2.727 \times 10^{-2}\,\mathrm{W\,m^{-1}\,K^{-1}}$, and $Pr \approx 0.705$.

The Reynolds number is calculated in Equation (15), the average convection heat transfer coefficient is computed in Equation (16), and finally, the heat rate per unit tube length is determined in Equation (17).

$$Re_D = \frac{v_{\text{wind}} D}{\nu} \Leftrightarrow Re_D = \frac{10 \times 0.06}{1.721 \times 10^{-5}} \Leftrightarrow Re_D \approx 34863.451 \tag{15}$$

$$\begin{gathered} \overline{Nu}_D \equiv \frac{\overline{h}_e D}{k} = \\ 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5} \Leftrightarrow \overline{h}_e = \frac{2.727 \times 10^{-2}}{0.06} \times \\ \left\{0.3 + \frac{0.62 \times 34863.451^{1/2} \times 0.705^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{34863.451}{282000}\right)^{5/8}\right]^{4/5}\right\} \Leftrightarrow \\ \Leftrightarrow \overline{h}_e \approx 49.920\,\mathrm{W\,m^{-2}\,K^{-1}} \end{gathered} \tag{16}$$

$$q' = \pi D \overline{h}_e \left(T_s - T_\infty\right) \Leftrightarrow q' = \pi \times 0.06 \times 49.920 \times (60 - 20) \Leftrightarrow \boxed{q' \approx 376.388\,\mathrm{W\,m^{-1}}} \tag{17}$$