Heat Transfer

Computational Laboratories

One-Dimensional, Transient Conduction

(Laboratory II)

Space- and time-dependent conduction heat transfer in large plane walls, long rods, and spheres initiated by convection heat transfer accross its boundaries

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Transient Conduction - Introduction

- A transient (unsteady or time-dependent) heat conduction process is initiated whenever a thermal equilibrium state of a system is perturbed.
- A perturbation on a system thermal equilibrium state can be induced by a change in:
  - surface convection conditions (T_∞ or h);
  - surface radiation conditions (T_{sur} or h_r);
  - surface heat flux ($q_s^{''}$) or surface temperature (T_s); and
  - internal energy generation ($\dot{q}$).
- Transient heat conduction processes can be modelled through analytical or numerical means:
  - Lumped system analysis (overall energy balance);
  - Exact solutions to the heat diffusion equation; and
  - Finite difference, finite element or finite volume methods.

Transient Conduction - Temperature Gradients

Importance of Solid Temperature Spatial Resolution

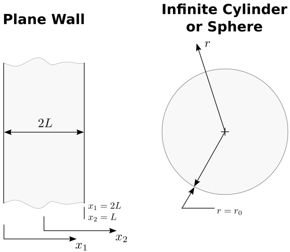
- For a transient conduction process in a solid driven by convection heat transfer across its boundaries, Biot number (Bi) determines if the spatial thermal gradients within the solid are negligible or not.

$$Bi = \frac{hL_c}{k} = \frac{\text{Conduction resistance within the solid}}{\text{Convection resistance between the solid and the fluid}}$$

- For $\mathbf{Bi} < \mathbf{0.1}$ the solid temperature distribution can be considered spatially uniform (depends only on the time): $T(\mathbf{x}, t) \approx T(t)$.
  - **The lumped capacitance method provides a solution for $\mathbf{T}(\mathbf{t})$.**
- For $\mathbf{Bi} \geq \mathbf{0.1}$ the local solid temperatures depend on the position and time.
  - **$\mathbf{T}(\mathbf{x}, \mathbf{t})$ solutions to the heat diffusion equation can be evaluated by analytical (exact and approximate) or numerical means.**

One-Dimensional, Transient Conduction – Gov. Eqs.

Transient conduction can be described in 1D for the case of a plane wall, infinite cylinder and a sphere through the heat equation.



Simplifying assumptions:

- no thermal energy generation; and
- constant thermal conductivity.

Heat Diffusion Equation

$$\boldsymbol{\nabla \cdot (k \nabla T) + \dot{q} = \rho c \frac{\partial T}{\partial t}}$$

Plane Wall

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Infinite Cylinder

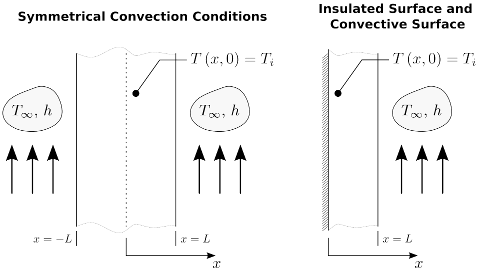
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Sphere

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\alpha = k/(\rho c)$ – Thermal diffusivity

One-Dimensional, Transient Conduction in a Plane Wall



Governing Equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial Condition

$$T(x, t = 0) = T_i$$

Boundary Conditions

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$$

8 Independent Variables

$$T = f(x, \alpha, t, T_i, k, L, h, T_\infty)$$

One-Dimensional, Transient Conduction in a Plane Wall

Non-dimensionalization:

- $\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$
  - $0 \le \theta^* \le 1$
- $x^* = \frac{x}{L}$
  - $0 \le x^* \le 1$
- $Fo = t^* = \frac{\alpha t}{L^2}$
- $Bi = \frac{hL}{k}$

Governing Equation

$$\frac{\partial^2 \theta^*}{\partial {x^*}^2} = \frac{\partial \theta^*}{\partial Fo}$$

Initial Condition

$$\theta^*\left(x^*, 0\right) = 1$$

3 Independent Variables

$$\theta^* = f\left(x^*, Fo, Bi\right)$$

θ^* – Dimensionless local temperature difference
Fo – Fourier number
x^* – Dimensionless position

Boundary Conditions

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\theta^*\left(1, Fo\right)$$

One-Dimensional, Transient Conduction in a Plane Wall

Dimensionless Local Temperature Difference

- **Exact Solution**

  The exact solution for the problem is given in the form of an infinite series.

$$\theta^*\left(x^*, Fo\right) = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right)\cos\left(\zeta_n x^*\right)$$

  For the geometry under consideration (plane wall), C_n and ζ_n are functions of Bi. C_n and ζ_n are commonly given in tables.

- **Approximate Solution: One-term Approx. (Valid for Fo > 0.2)**

$$\theta^*\left(x^*, Fo\right) = \frac{T\left(x^*, Fo\right) - T_\infty}{T_i - T_\infty} = \underbrace{C_1 \exp\left(-\zeta_1^2 Fo\right)}_{\theta_0^*(Fo)=\theta^*(0,Fo)} \cos\left(\zeta_1 x^*\right)$$

θ_0^* – Midplane ($x^* = 0$) dimensionless temperature difference

One-Dimensional, Transient Conduction in a Plane Wall

Dimensionless Mean Temperature Difference

- **Exact Solution**

  The exact solution for the problem is given in the form of an infinite series.

$$\overline{\theta^*}(Fo) = \sum_{n=1}^{\infty} \frac{\sin(\zeta_n)}{\zeta_n} C_n \exp\left(-\zeta_n^2 Fo\right)$$

  - **Bi** $\rightarrow$ **0** – The exact solution becomes equal to the lumped capacitance method (LCM) solution (considering Bi and Fo defined with $L_c = V/A_s$):

$$\overline{\theta^*}(Fo) = \theta^*_{\mathrm{LCM}}(Fo) = \exp\left(-Bi.Fo\right)$$

- **Approximate Solution: One-term Approx. (Valid for Fo $>$ 0.2)**

$$\overline{\theta^*}(Fo) = \frac{\overline{T}(Fo) - T_\infty}{T_i - T_\infty} = \frac{\sin\zeta_1}{\zeta_1}\theta_0^*(Fo)$$

One-Dimensional, Transient Conduction in a Plane Wall

Fractional Energy Loss/Gain to/from the Surrounding Fluid

$$\frac{Q\left(Fo\right)}{Q_0} = 1 - \overline{\theta^*}\left(Fo\right)$$

- $Q\left(Fo\right)\left[= \rho Vc\left(T_i - \overline{T}\left(Fo\right)\right)\right]$ – Total thermal energy transfer from/to the wall over the time interval $t\left[= FoL^2/\alpha\right]$.
- $Q_0\left[= \rho Vc\left(T_i - T_\infty\right)\right]$ – Initial thermal energy of the wall relative to the fluid temperature, *i.e.*, maximum possible energy transfer from/to the wall if the process continues to time $t = \infty$.

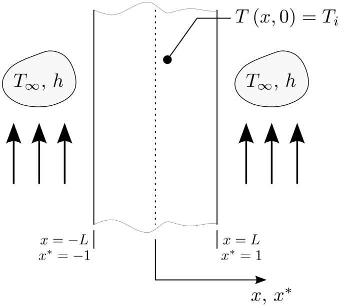
Boundary Condition at $x^* = 1$: **Constant Surface Temperature**

The foregoing solutions for θ^*, $\overline{\theta^*}$, and Q/Q_0 are also applicable for a prescribed temperature boundary condition at $x = L$ $(T(L,t) = T_s)$ since this is equivalent to consider $h = \infty$ $(Bi = \infty)$ and $T_\infty = T_s$.

One-Dimensional, Transient Conduction in a Plane Wall

Heat Removal ($T_i > T_\infty$): Convection Cooling

Numerical and One-Term Approximation Solutions



3 Case Studies:

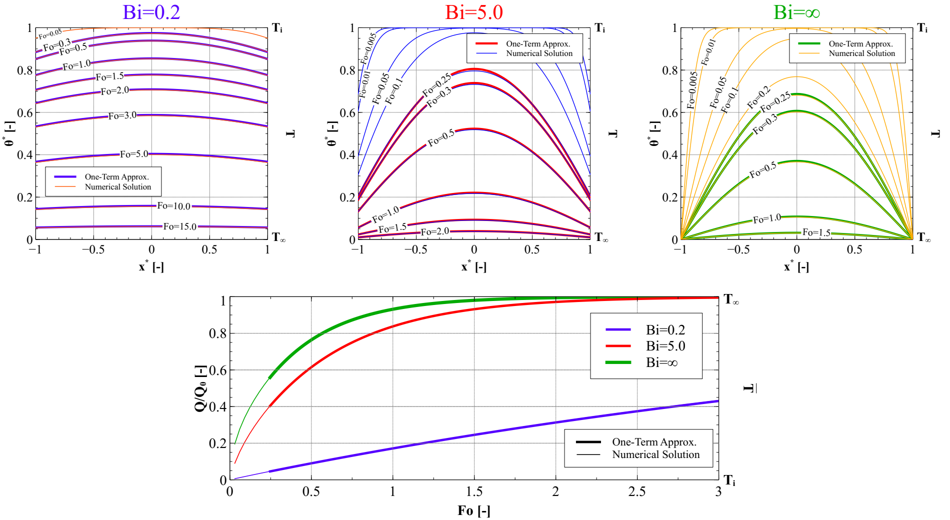
- $Bi = 0.2$;
- $Bi = 5.0$; and
- $Bi = \infty$.
  - Negligible convection resistance: equivalent to prescribe a constant surface temperature (T_s) equal to T_∞.

$$\Delta E_{st} = -Q, \quad Q > 0$$

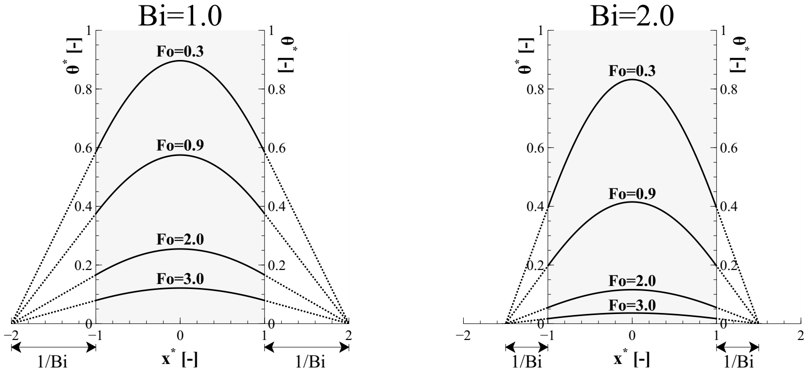
ΔE_{st} – Change in the thermal energy stored

One-Dimensional, Transient Conduction in a Plane Wall

Heat Removal – Numerical and One-Term Approximation Solutions



One-Dimensional, Transient Conduction in a Plane Wall

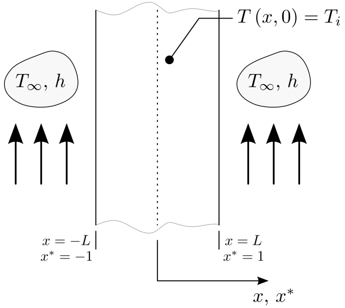


- At any time instant during an unsteady conduction process, the extensions of the tangents to the curves at the points $x^* = \pm 1$ intersect the axis perpendicular to $\theta^* = 0$ at the points $\pm\left(1 + \frac{1}{Bi}\right)$.
- This evidence is also observed for long rods and spheres and is due to the mathematical formulation of the convective surface boundary condition.

One-Dimensional, Transient Conduction in a Plane Wall

Heat Removal ($T_i > T_\infty$): Convection Cooling

Numerical and One-Term Approximation Solutions



3 Case Studies:

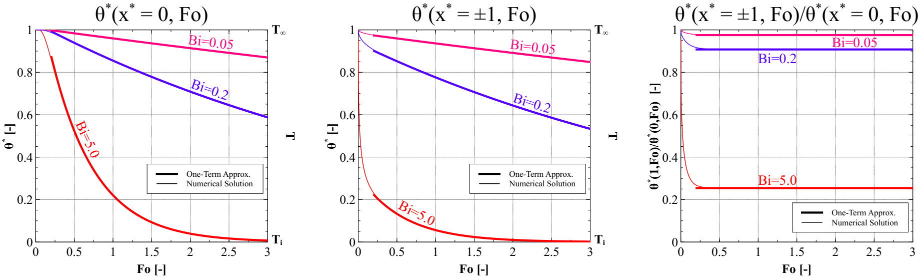
- $Bi = 0.05$;
- $Bi = 0.2$; and
- $Bi = 5.0$.

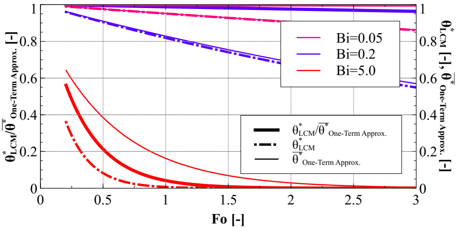
$$\Delta E_{st} = -Q, \quad Q > 0$$

ΔE_{st} – Change in the thermal energy stored

One-Dimensional, Transient Conduction in a Plane Wall

Heat Removal – Numerical and One-Term Approximation Solutions





$$\frac{\theta^*(1, t^*)}{\theta^*(0, t^*)} = f(Bi, Fo).$$

However, as $Fo \uparrow$ $\dfrac{\theta^*(1, t^*)}{\theta^*(0, t^*)} \longrightarrow \cos(\zeta_1)$

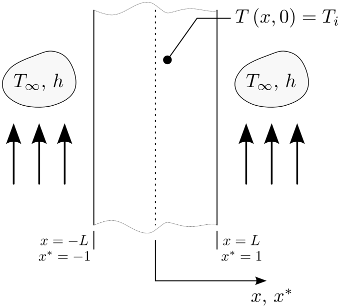
One-Term Approximation:

$$\frac{\theta^*(1, t^*)}{\theta^*(0, t^*)} = \begin{cases} 0.976 & Bi = 0.05 \\ 0.908 & Bi = 0.2 \\ 0.254 & Bi = 5.0 \end{cases}$$

One-Dimensional, Transient Conduction in a Plane Wall

Heat Addition ($T_\infty > T_i$): Convection Heating

Numerical and One-Term Approximation Solutions



3 Case Studies:

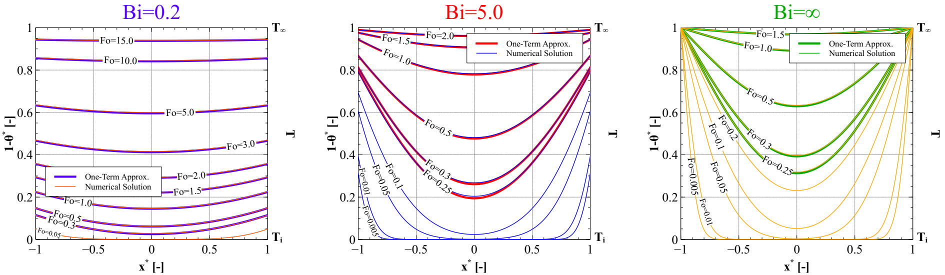
- $Bi = 0.2$;
- $Bi = 5.0$; and
- $Bi = \infty$.
  - Negligible convection resistance: equivalent to prescribe a constant surface temperature (T_s) equal to T_∞.

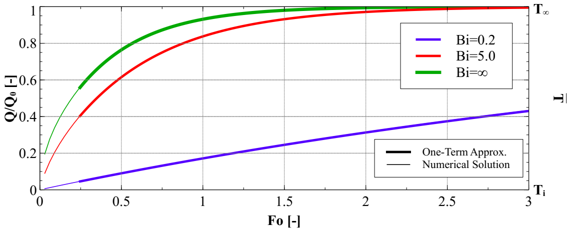
$$\Delta E_{st} = -Q, \quad Q < 0$$

ΔE_{st} – Change in the thermal energy stored

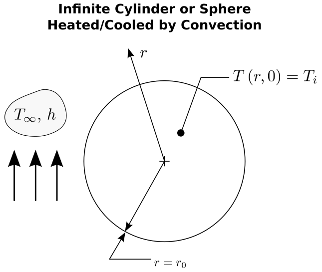
One-Dimensional, Transient Conduction in a Plane Wall

Heat Addition – Numerical and One-Term Approximation Solutions





One-Dimen., Transient Conduction in Radial Systems



Infinite Cylinder - Gov. Equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Sphere - Governing Equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial Condition

$$T(r, t = 0) = T_i$$

Boundary Conditions

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = h\left[T(r_0, t) - T_\infty\right]$$

One-Dimen., Transient Conduction in Radial Systems

Non-dimensionalization:

- $\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$
  - $0 \leq \theta^* \leq 1$
- $r^* = \frac{r}{r_0}$
  - $0 \leq r^* \leq 1$
- $Fo = t^* = \frac{\alpha t}{r_0^2}$
- $Bi = \frac{h r_0}{k}$

Initial Condition

$$\theta^* \left(r^*, 0\right) = 1$$

Infinite Cylinder - Gov. Equation

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial \theta^*}{\partial r^*} \right) = \frac{1}{\alpha} \frac{\partial \theta^*}{\partial Fo}$$

Sphere - Governing Equation

$$\frac{1}{{r^*}^2} \frac{\partial}{\partial r^*} \left({r^*}^2 \frac{\partial \theta^*}{\partial r^*} \right) = \frac{1}{\alpha} \frac{\partial \theta^*}{\partial Fo}$$

Boundary Conditions

$$\left. \frac{\partial \theta^*}{\partial r^*} \right|_{r^*=0} = 0$$

$$\left. \frac{\partial \theta^*}{\partial r^*} \right|_{r^*=1} = -Bi \theta^* \left(1, Fo\right)$$

One-Dimen., Transient Conduction in Radial Systems

Dimensionless Local Temperature Difference – Exact Solutions

The exact solutions for the infinite cylinder and sphere are given in the form of infinite series.

Infinite Cylinder

$$\theta^* \left(r^*, Fo\right) = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right) J_0\left(\zeta_n r^*\right)$$

Sphere

$$\theta^* \left(r^*, Fo\right) = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right) \frac{1}{\zeta_n r^*} \sin\left(\zeta_n r^*\right)$$

C_n and ζ_n are functions of Bi and the geometry under consideration (long rod or sphere). C_n and ζ_n are commonly given in tables.

One-Dimen., Transient Conduction in Radial Systems

Approximate Solutions: One-term Approximation (**Valid for Fo > 0.2**)

	Infinite Cylinder	**Sphere**
$\theta^*(r^*, Fo)$	$\theta_0^*(Fo)\, J_0(\zeta_1 r^*)$	$\theta_0^*(Fo)\, \frac{1}{\zeta_1 r^*} \sin(\zeta_1 r^*)$
$\theta_0^*(Fo)$	$C_1 \exp\left(-\zeta_1^2 Fo\right)$	
$\overline{\theta^*}(Fo)$	$\frac{2 J_1(\zeta_1)}{\zeta_1} \theta_0^*(Fo)$	$\frac{3\theta_0^*(Fo)}{\zeta_1^3} \left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right]$
$\frac{Q(Fo)}{Q_0}$	$1 - \overline{\theta^*}(Fo)$	

- θ_0^* - centerline [centerpoint] dimensionless temperature difference for an infinite cylinder [sphere].

One-Dimen., Transient Conduction in Radial Systems

Dimensionless Temperature Difference for $Bi \to 0$

As $Bi \to 0$ the exact solution for $\theta^* (r^*, Fo)$ becomes equal to the lumped capacitance method solution (considering Bi and Fo defined with $L_c = V/A_s$ – L_c is equal to $r_0/2$ and $r_0/3$ for a long cylinder and sphere, respectively):

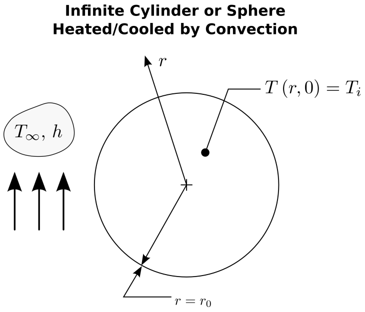
$$\theta^* (r^*, Fo) \to \overline{\theta^*} (Fo) = \exp(-Bi.Fo)$$

Boundary Condition at $r^* = 1$: **Constant Surface Temperature**

The foregoing solutions for θ^*, $\overline{\theta^*}$, and Q/Q_0 are also applicable for a prescribed temperature boundary condition at $r = r_0$ ($T(r_0, t) = T_s$) since this is equivalent to consider $h = \infty$ ($Bi = \infty$) and $T_\infty = T_s$.

Heat Removal ($T_i > T_\infty$): Convection Cooling

Numerical and One-Term Approximation Solutions



3 Case Studies:

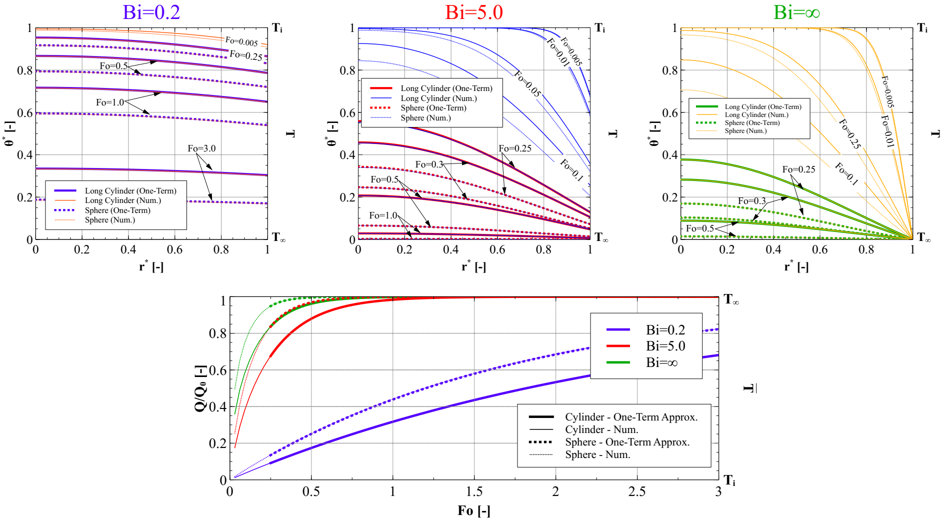
- $Bi = 0.2$;
- $Bi = 5.0$; and
- $Bi = \infty$.
  - Negligible convection resistance: equivalent to prescribe a constant surface temperature (T_s) equal to T_∞

$$\Delta E_{st} = -Q, \quad Q > 0$$

ΔE_{st} – Change in the thermal energy stored

One-Dimen., Transient Conduction in Radial Systems

Heat Removal – Numerical and One-Term Approximation Solutions



Final Remarks (1/2)

- Biot number provides an estimation for the relevance of temperature spatial gradients in a heat conduction process within a solid concurrent with convection across its boundaries.

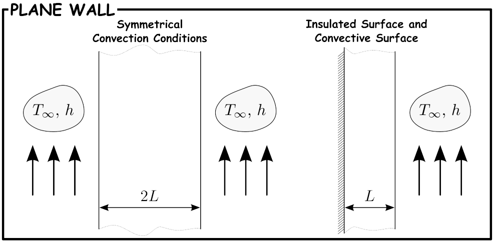
  For a one-dimensional, transient heat conduction process if:
  - $Bi < 0.1$: the spatial gradients are not relevant; consequently, the lumped capacitance method can be applied;
  - $Bi \geq 0.1$: the spatial gradients are relevant; consequently, the one-term approximation to the exact solution – particularly recommended for $Fo > 0.2$ – or a numerical procedure should be applied to evaluate the temporal and spatial solid temperature distribution profiles.

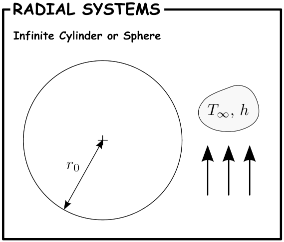
- The one-term approx. for $Fo > 0.2$ results in an error below 2%.

- Heisler/Gröber charts (transient temperature and heat transfer charts) provide a graphical representation for θ_0^*, θ^*/θ_0^*, and Q/Q_0 obtained with the single-term approximation of the exact solution.

Final Remarks (2/2) – L_c for Biot and Fourier Numbers

	L_c – Characteristic length[1]		
	Plane Wall	Inf. Cylinder	Sphere
Conservative Bi Criterion (relevance of temp. spatial gradients)	L	r_0	r_0
Lumped capacitance method – $L_c = V/A_s$	L	$r_0/2$	$r_0/3$
Analytical and numerical solutions for $\theta^*(x^*, Fo)$	L	r_0	r_0

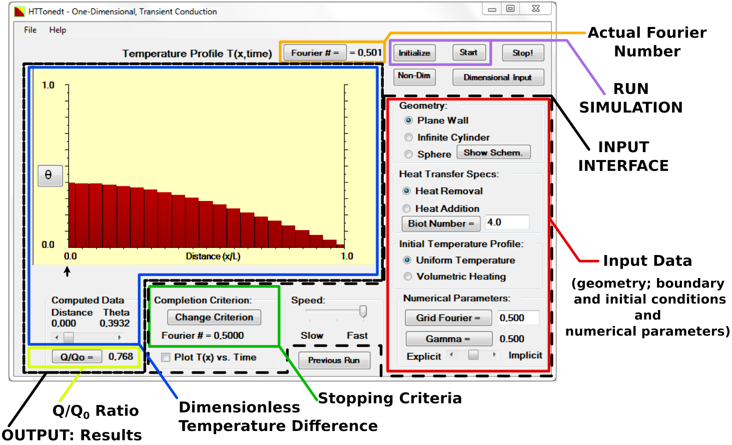




[1]Find the L_c value (L and r_0) in accordance with the accompanying figure.

Exploring the Software Module (1/4)

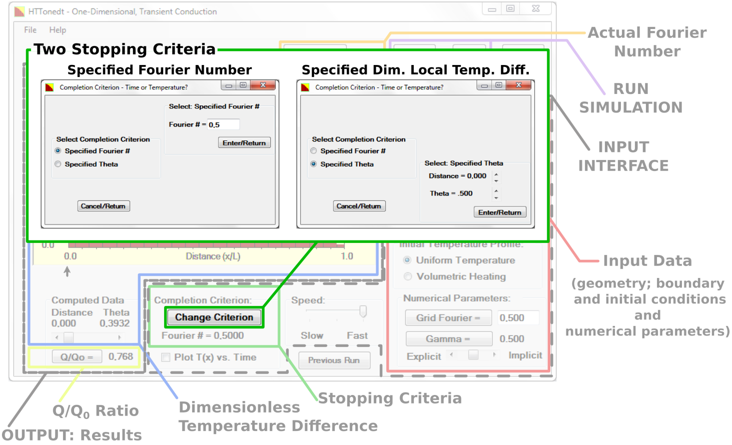
Software module – `HTTonedt.exe (Version 5.0.0.2)`



- The module solves the one-dimensional, transient heat equation through a finite-volume approach for a plane wall, infinite cylinder, and sphere.

Exploring the Software Module (2/4)

Software module – `HTTonedt.exe (Version 5.0.0.2)`



- The module ends the simulation for two possible criteria: (a) specified Fourier number; and (b) specified dimensional local temperature difference.

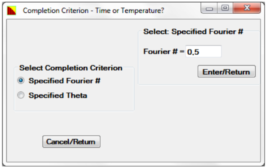
Exploring the Software Module (3/4)

Completion Criteria

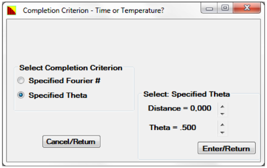
The module terminates the simulation for two possible criteria:

1. **Specified Fourier number – Fo; and**
   - For evaluation of the temperature distribution profiles and the ratio Q/Q_0 at a specific time instant.
2. **Specified dimensionless local temperature difference – $\theta^* \left(\mathbf{x}^*, \mathbf{Fo}\right)$.**
   - For the evaluation of the elapsed time, temperature distribution profiles, and the ratio Q/Q_0.

1. Specified Fourier Number



2. Specified Dimensionless Local Temperature Difference



Exploring the Software Module (4/4)

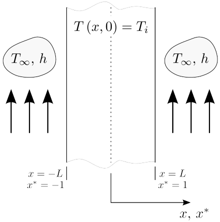
Spatial Discretization

- Two spatial discretization schemes (meshes) are available in the software module.
- The finest mesh has two times the cell count of the default mesh and, consequently, should provide more accurate results; however, at the expense of longer computation times.
- The finest grid is taken into account (activated) for the calculations once the default window size is changed.
- To revert to the default mesh, the user should restart the software module.
- The module application examples that follow (next slides) consider the default mesh.

Exploring the Module - Cooling a Plane Wall

Module Application Example I: Problem Statement

Consider a $0.1\,\mathrm{m}$ ($2L$) thick plane wall initially at $T_i = 180\,^{\circ}\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 20\,^{\circ}\mathrm{C}$ and with $h = 2200\,\mathrm{W.m^{-2}.K^{-1}}$. The wall material has a thermal conductivity (k), density (ρ), and specific heat (c) equal to $110\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$, $8530\,\mathrm{kg.m^{-3}}$, and $380\,\mathrm{J.kg^{-1}.K^{-1}}$, respectively.



After 40 s of cooling, evaluate the following using the module:

1. temperature distribution profile, $T(-L \leq x \leq L)$;
2. fractional energy loss, Q/Q_0;
3. average wall temperature, $\overline{T}$; and
4. compare the average wall temperature computed with the module with the temperature predicted by the lumped capacitance method.

Exploring the Module - Cooling a Plane Wall

Module Application Example I: Module Application

Preliminary Calculations

Biot Number	Thermal Diffusivity	Fourier Number
$Bi = 1.00$	$\alpha = 3.39 \times 10^{-5}\,\mathrm{m}^2.\mathrm{s}^{-1}$	$Fo\,(t = 40\,\mathrm{s}) \approx 0.54$

Module Input Data

1 - Geometry:
"Plane Wall"

2 - Heat Transfer Specs:
"Heat Removal"
"Biot Number = 1"

3 - Initial Temperature Profile:
"Uniform Temperature"

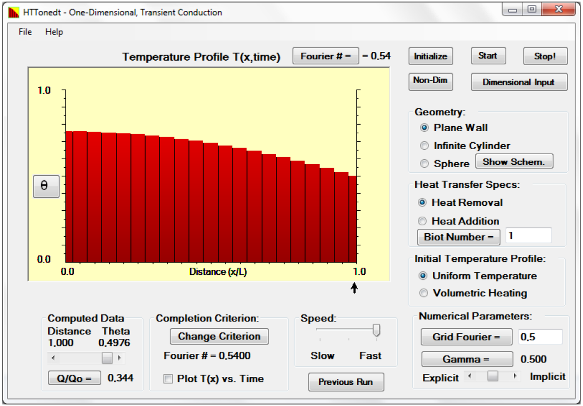
4 - Numerical Parameters:
"Grid Fourier = 0,5"
"Gamma = 0.500"
"Default Mesh"

5 - Completion Criteria:
"Fourier # = 0,5400"

Exploring the Module - Cooling a Plane Wall

Module Application Example I: Module Application and Results

Module Results

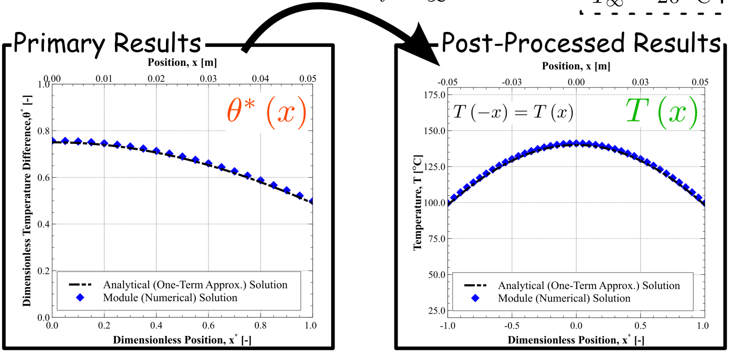


Module Application Example I: Results Analysis (1/3)

1. Temperature distribution profile, $T\,(-L \leq x \leq L)$

$$\theta^*\,(x) = \frac{T\,(x) - T_\infty}{T_i - T_\infty}$$

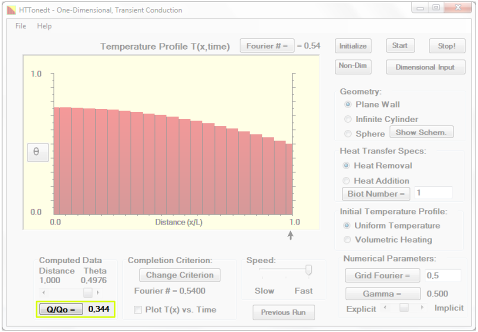
$T_i = 180\,°\mathrm{C}$
$T_\infty = 20\,°\mathrm{C}$



Exploring the Module - Cooling a Plane Wall

Module Application Example I: Results Analysis (2/3)

2. Fractional energy loss, Q/Q_0



$$\boxed{Q/Q_0 = 0.344} \qquad \left((Q/Q_0)_{\text{Analytic Sol.}} \approx 0.339\right)$$

Exploring the Module - Cooling a Plane Wall

Module Application Example I: Results Analysis (3/3)

3. Average wall temperature, $\overline{T}$

$$\overline{\theta^*} = \int_0^1 \theta^*(x^*)\, dx^* = $$
$$= 1 - Q/Q_0 = 0.656$$

$$\overline{\theta^*} = \frac{\overline{T} - T_\infty}{T_i - T_\infty} \Leftrightarrow$$
$$\Leftrightarrow \boxed{\overline{T} = 124.96\,^\circ\mathrm{C}}$$

$(\overline{T}_{\mathrm{Analytic\,Sol.}} \approx 125.79\,^\circ\mathrm{C})$

4. Average wall temperature – module *vs.* lumped capacitance method results

$$\theta^*_{\mathrm{LCM}} = \exp(-Bi \cdot Fo) = \exp\left(-\frac{h\alpha t}{kL}\right) =$$
$$= \exp\left(-\frac{2200 \times 3.39 \times 10^{-5} \times 40}{110 \times 0.05}\right) \Leftrightarrow$$
$$\Leftrightarrow \theta^*_{\mathrm{LCM}} \approx 0.581$$

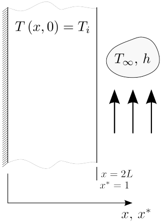
$$\theta^*_{\mathrm{LCM}} = \frac{T_{\mathrm{LCM}} - T_\infty}{T_i - T_\infty} \Leftrightarrow \boxed{T_{\mathrm{LCM}} = 112.960\,^\circ\mathrm{C}}$$

A relative **error of about 10%** is observed for T_{LCM} in relation to the module solution ($\overline{T}_{\mathrm{Mod.}} = 124.96\,^\circ\mathrm{C}$).

Exploring the Module - Heating a Plane Wall

Module Appl. Example II: Problem Statement

Consider the same plane wall of Example I (same thermophysical properties and geometrical parameters) initially at $T_i = 20\,^\circ\mathrm{C}$. One surface is perfectly insulated while the other is suddenly exposed to a fluid at $T_\infty = 180\,^\circ\mathrm{C}$ and with $h = 2200\,\mathrm{W.m^{-2}.K^{-1}}$.



Evaluate the following using the module:

1. elapsed time, t, to observe a temperature equal to $100\,^\circ\mathrm{C}$ at the insulated surface (*i.e.*, $T(x = 0, t) = 100\,^\circ\mathrm{C}$);
2. temperature at $x = 0.08\,\mathrm{m}$ and at the time instant of 1.; and
3. thermal energy absorbed per unit active surface area, Q/A_s, at the time instant of 1.

Exploring the Module - Heating a Plane Wall

Module Appl. Example II: Module Application

Preliminary Calculations

Biot Number	Thermal Diffusivity	Dim. Local Temp. Diff.
$Bi = 2.00$	$\alpha = 3.39 \times 10^{-5}\, \mathrm{m}^2.\mathrm{s}^{-1}$	$\theta^* (0, Fo) = 0.5$

Module Input Data

1 - Geometry:

"Plane Wall"

2 - Heat Transfer Specs:

"Heat Addition"
"Biot Number = 2"

3 - Initial Temperature Profile:

"Uniform Temperature"

4 - Numerical Parameters:

"Grid Fourier = 0,05"
"Gamma = 0.500"
"Default Mesh"

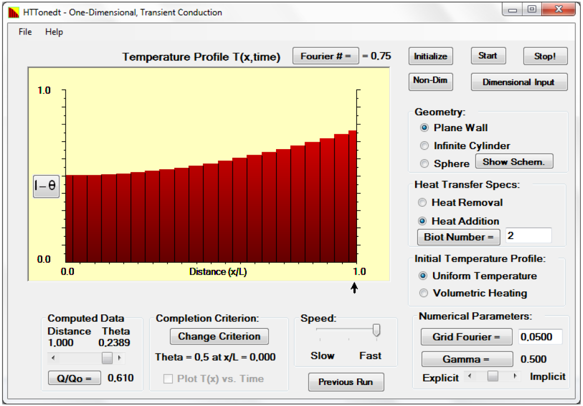
5 - Completion Criteria:

"Theta=0.5 at x/L=0.000"

Exploring the Module - Heating a Plane Wall

Module Appl. Example II: Module Application and Results

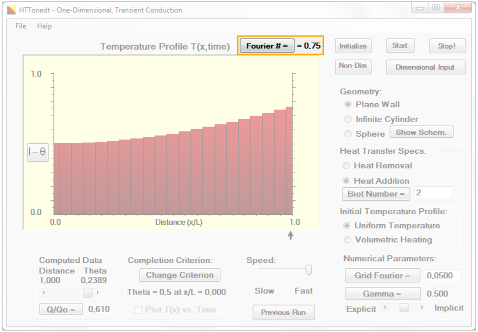
Module Results



Exploring the Module - Heating a Plane Wall

Module Application Example II: Results Analysis (1/3)

1. Elapsed time to observe $T(x = 0, t) = 100\,^\circ\mathrm{C}$



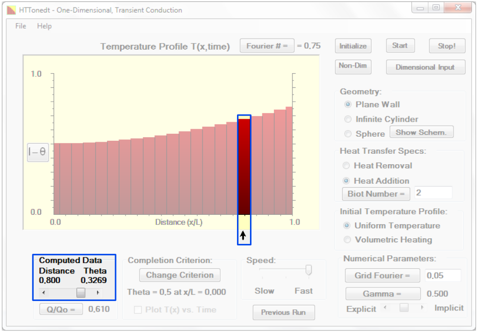
$$Fo = 0.75 \Leftrightarrow t = \frac{0.1^2}{3.39 \times 10^{-5}} \times 0.75 \Leftrightarrow \boxed{t \approx 3\,\text{min. and } 41\,\text{sec.}}$$

$$\left(\left(\theta^*(0, Fo)\right)_{\text{Analytic Sol.}} = 0.5 \Rightarrow t_{\text{Analytic Sol.}} \approx 3\,\text{min. and } 38\,\text{sec.} \right)$$

Exploring the Module - Heating a Plane Wall

Module Application Example II: Results Analysis (2/3)

2. Temperature at $x = 0.08\,\mathrm{m}$ when $T(x = 0, t) = 100\,^{\circ}\mathrm{C}$



$$\theta^{*}\left(x^{*} = 0.8, Fo\right) = 0.3269 \Rightarrow \boxed{T\left(x = 0.08\,\mathrm{m}, t\right) \approx 127.7\,^{\circ}\mathrm{C}}$$

$$\left(\left(T\left(x = 0.08\,\mathrm{m}, t\right)\right)_{\mathrm{Analytic\ Sol.}} \approx 127.9\,^{\circ}\mathrm{C}\right)$$

Exploring the Module - Heating a Plane Wall

Module Application Example II: Results Analysis (3/3)

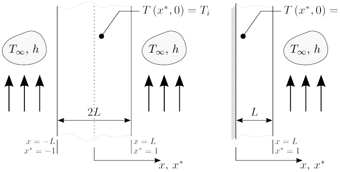
3. Absorbed energy per unit active surface area, Q/A_s, when $T(x=0,t) = 100\,^\circ\mathrm{C}$

$$Q/A_s = \rho L_c c\,(Q/Q_0)\,\theta_i = 8530 \times 0.1 \times 380 \times 0.610 \times (20 - 180) \Leftrightarrow$$

$$\Leftrightarrow \boxed{Q/A_s \approx -3.16 \times 10^7\ \mathrm{J\cdot m^{-2}}} \qquad \left((Q/A_s)_{\mathrm{Analytic\ Sol.}} \approx -3.07 \times 10^7\ \mathrm{J\cdot m^{-2}}\right)$$

Useful Relations

Plane Wall



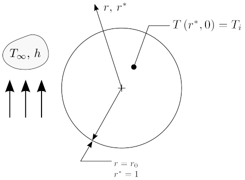
$$\theta^* = \frac{T(x^*, Fo) - T_\infty}{T(x^*, 0) - T_\infty}$$

$$x^* = \frac{x}{L}$$

$$Fo = \frac{\alpha t}{L^2}$$

$$Bi = \frac{hL}{k}$$

Radial Systems



$$\theta^* = \frac{T(r^*, Fo) - T_\infty}{T(r^*, 0) - T_\infty}$$

$$r^* = \frac{r}{r_0}$$

$$Fo = \frac{\alpha t}{r_0^2}$$

$$Bi = \frac{h r_0}{k}$$

$$\alpha = k/(\rho c) \qquad Q_0 = \rho V c (T_i - T_\infty)$$