Heat Transfer

Computational Laboratories

One-Dimensional, Transient Conduction

(Laboratory II)

Space- and time-dependent conduction heat transfer in large plane walls, long rods and spheres

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Transient Conduction - Introduction

- A transient, unsteady, heat conduction process is initiated whenever an initial thermal equilibrium state is perturbed.
- A perturbation on a thermal equilibrium state can be induced by a change in:
  - surface convection conditions (T_∞ or h);
  - surface radiation conditions (T_{sur} or h_r);
  - surface heat flux ($q_s^{''}$) or surface temperature (T_s);
  - internal energy generation ($\dot{q}$).
- Transient heat conduction processes can be modelled through analytic or numerical means:
  - Lumped system analysis (overall energy balance);
  - Exact solutions for the heat diffusion equation;
  - Finite difference, finite element or finite volume methods.

Transient Conduction - Temperature Gradients

Importance of the Spatially Resolution for Temperature Distribution

- During a transient heat conduction process, Bi number determine if the temperature gradients within the solid are negligible or not.

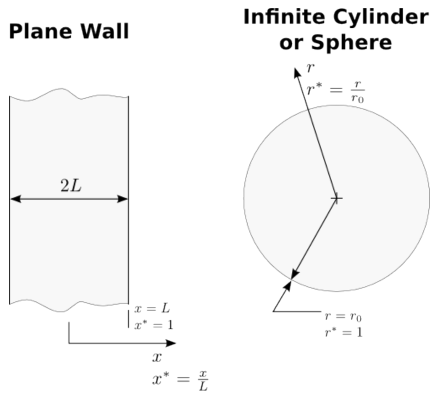
$$Bi = \frac{hL_c}{k} = \frac{\text{Conduction resistance within the solid}}{\text{Convection resistance between the solid and the fluid}}$$

- For **Bi** < 0.1 the temperature of the solid can be considered spatially uniform (depends only on the time): $T(\vec{x}, t) \approx T(t)$.
  - **The lumped capacitance method provides a solution for T(t).**

- For **Bi** ≥ 0.1 the temperature distribution within the solid depend on the position and time.
  - **Approximate solutions for appropriate forms of the heat equation can be evaluated through exact or numerical means.**

One-Dimensional, Transient Conduction without Thermal Energy Generation

Transient conduction can be described in 1D for the case of a plane wall, infinite cylinder and a sphere through the heat equation.



$\alpha = \frac{k}{\rho c}$ - Thermal diffusivity

Heat Diffusion Equation

$$\nabla \cdot (k \nabla T) + \dot{q} = \rho c \frac{\partial T}{\partial t}$$

Plane Wall

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

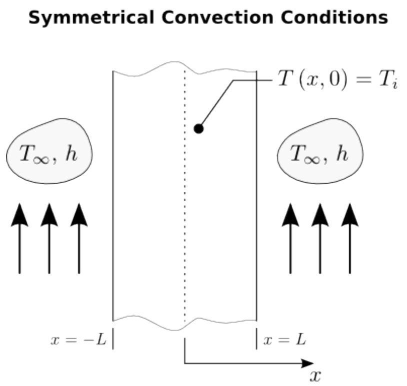
Infinite Cylinder

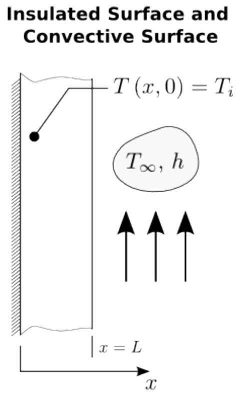
$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Sphere

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

One-Dimensional, Transient Conduction in a Plane Wall





Governing Equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Initial Condition

$$T(x, t = 0) = T_i$$

Boundary Conditions

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$$

8 Independent Variables

$$T = f(x, \alpha, t, T_i, k, L, h, T_\infty)$$

One-Dimensional, Transient Conduction in a Plane Wall

Non-dimensionalization:

- $\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}, \qquad 0 \leq \theta^* \leq 1$
- $x^* = \frac{x}{L}, \quad 0 \leq x^* \leq 1$
- $t^* = Fo = \frac{\alpha t}{L^2}$
- $Bi = \frac{hL}{k}$

Governing Equation

$$\frac{\partial^2 \theta^*}{\partial {x^*}^2} = \frac{\partial \theta^*}{\partial t^*}$$

Initial Condition

$$\theta^*(x^*, 0) = 1$$

Boundary Conditions

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\theta^*(1, t^*)$$

3 Independent Variables

$$\theta^* = f(x^*, Fo, Bi)$$

One-Dimensional, Transient Conduction in a Plane Wall

Exact Solution - Dimensionless Temperature Difference

The exact solution for the problem is given in the form of an infinite series.

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right) \cos\left(\zeta_n x^*\right)$$

C_n and ζ_n are functions of Bi number and the geometry under consideration (large plane wall). C_n and ζ_n are commonly given in tables.

Approximate Solution: One-term Approximation (**Valid for Fo > 0.2**)

$$\theta^*\left(x^*, t^*\right) = \frac{\theta\left(x^*, t^*\right)}{\theta_i} = \frac{T\left(x^*, t^*\right) - T_\infty}{T_i - T_\infty} = \underbrace{C_1 \exp\left(-\zeta_1^2 Fo\right)}_{\theta_0^*} \cos\left(\zeta_1 x^*\right)$$

$$\theta_0^* = \frac{T\left(0, t^*\right) - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

θ_0^* - midplane dimensionless temperature difference

One-Dimensional, Transient Conduction in a Plane Wall

Exact Solution - Dimensionless Mean Temperature Difference

The exact solution for the problem is given in the form of an infinite series.

$$\overline{\theta^*}\left(t^*\right) = \frac{1}{x^*} \int_0^1 \theta^*\left(x^*, t^*\right) dx^* = \sum_{n=1}^{\infty} \frac{\sin\left(\zeta_n\right)}{\zeta_n} C_n \exp\left(-\zeta_n^2 Fo\right)$$

- **Bi** $\rightarrow$ **0**: $\overline{\theta^*}\left(t^*\right) = \exp\left(-Bi.Fo\right)$
  - Lumped capacitance method solution for the dimensionless temperature difference: $\theta^*\left(t\right) = \exp\left(-\frac{t}{\tau_t}\right) = \exp\left(-Bi.Fo\right)$.

Approximate Solution: One-term Approximation (**Valid for Fo > 0.2**)

$$\overline{\theta^*}\left(t^*\right) = \frac{\sin\zeta_1}{\zeta_1} \theta_0^*\left(t^*\right)$$

One-Dimensional, Transient Conduction in a Plane Wall

Approximate Solution: One-term Approximation (**Valid for Fo > 0.2**)

$$\frac{Q(t)}{Q_0} = 1 - \overline{\theta^*} \quad \text{with} \quad \overline{\theta^*}(t^*) = \frac{\sin\zeta_1}{\zeta_1}\theta_0^*(t^*)$$

- $Q(t)\left[= \rho V c\left(T_i - \overline{T}_t(t)\right)\right]$ - Total energy transfer from/to the wall over the time interval t.
- $Q_0\left[= \rho V c\left(T_i - T_\infty\right)\right]$ - Initial thermal energy of the wall relative to the fluid temperature, *i.e.*, maximum possible energy transfer from/to the wall if the process continues to time $t = \infty$.

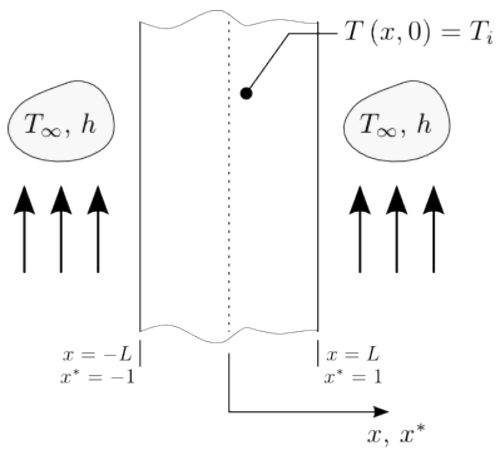
Boundary Condition at $x^* = 1$: **Constant Surface Temperature**

The foregoing solutions for θ^*, $\overline{\theta^*}$ and Q/Q_0 are also applicable for a fixed surface temperature boundary condition at $x^* = 1$ since it is equivalent to consider $h = \infty$ ($Bi = \infty$) and $T_\infty = T_s$.

One-Dimensional, Transient Conduction in a Plane Wall

Heat Removal ($T_i > T_\infty$)

Numerical and One-Term Approximation Solutions



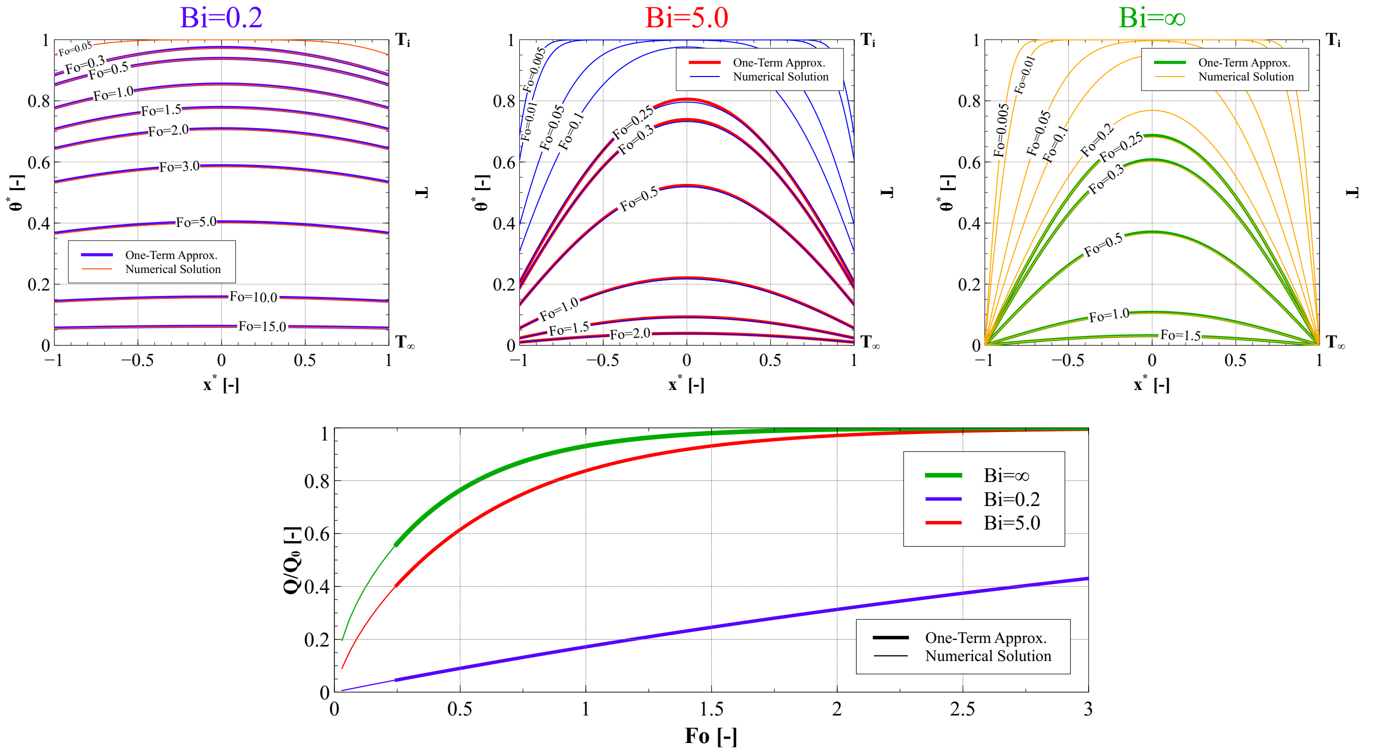
3 Case Studies:

- $Bi = 0.2$;
- $Bi = 5.0$;
- $Bi = \infty$.
  - Negligible convection resistance: equivalent to prescribe a constant surface temperature (T_s) equal to T_∞.

$$\Delta E_{st} = -Q, \quad Q > 0$$

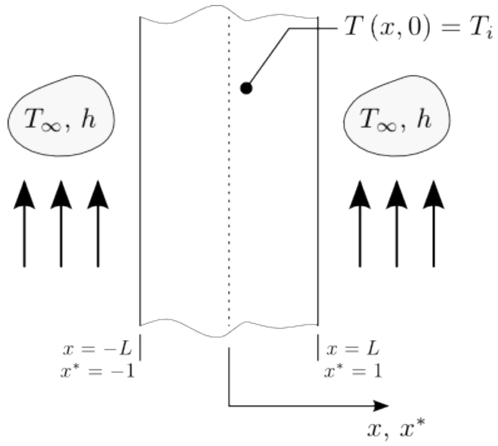
ΔE_{st} - change in thermal energy storage

Heat Removal - Numerical and One-Term Approximation Solutions



Heat Removal ($T_i > T_\infty$)

One-Term Approximation Solutions



3 Case Studies:

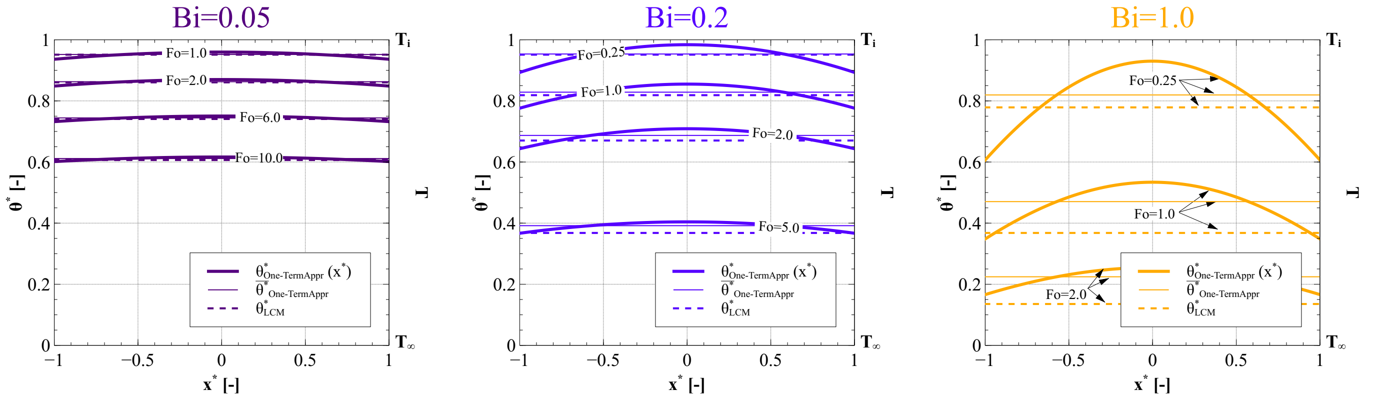
- $Bi = 0.05$;
- $Bi = 0.2$;
- $Bi = 1.0$.

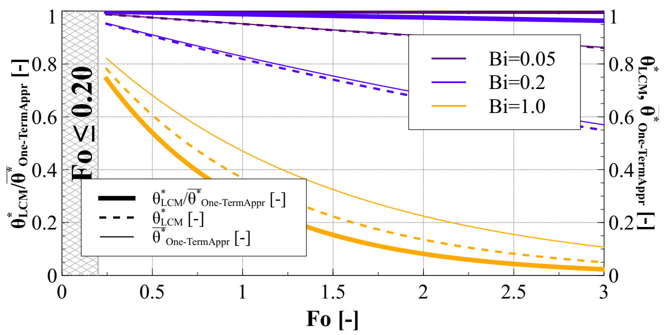
$$\Delta E_{st} = -Q, \quad Q > 0$$

ΔE_{st} - change in thermal energy storage

One-Dimensional, Transient Conduction in a Plane Wall

Heat Removal - One-Term Approximation Solutions





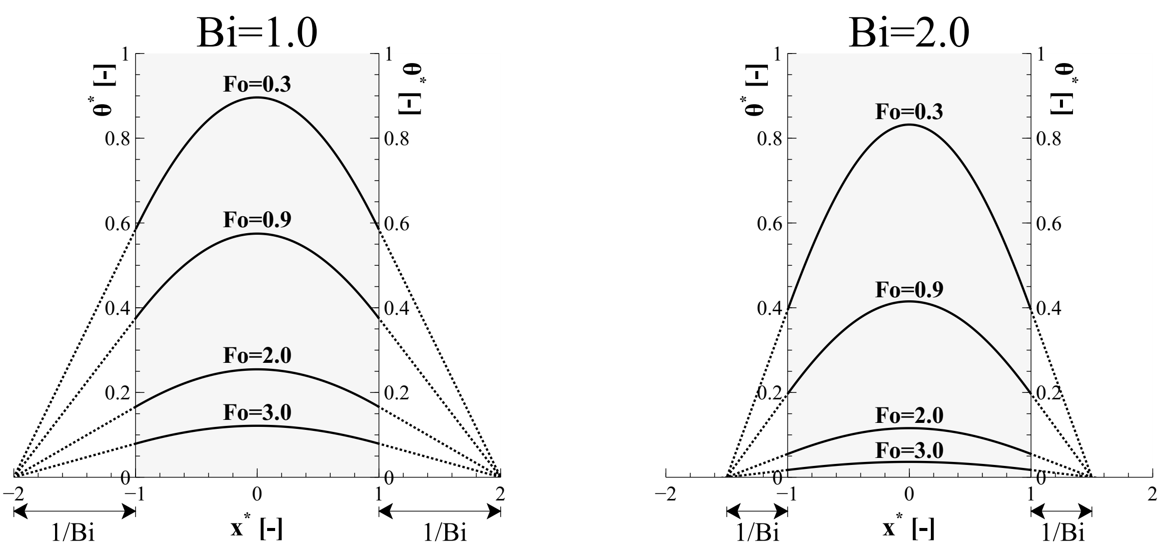
$$\frac{\theta^*(1,t^*)}{\theta^*(0,t^*)} = f(Bi, Fo).$$

However, as Fo $\uparrow$ $\frac{\theta^*(1,t^*)}{\theta^*(0,t^*)} \longrightarrow \cos(\zeta_1)$

One-Term Approximation:

$$\frac{\theta^*(1,t^*)}{\theta^*(0,t^*)} = \begin{cases} 0.976 & Bi = 0.05 \\ 0.908 & Bi = 0.2 \\ 0.652 & Bi = 1.0 \end{cases}$$

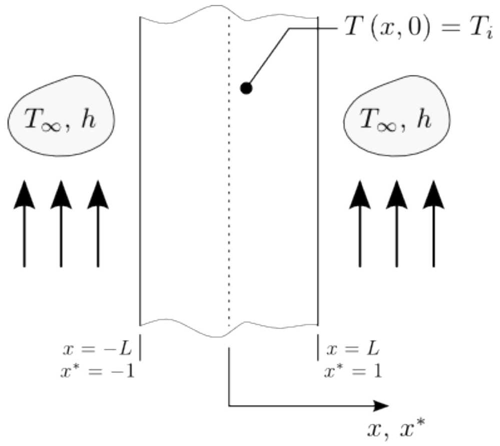
One-Dimensional, Transient Conduction in a Plane Wall



- At any time during an unsteady conduction process, the extensions of the tangents to the curves at the points $x^* = \pm 1$ intersect the axis perpendicular to $\theta^* = 0$ at the points $\pm\left(1 + \frac{1}{Bi}\right)$.
- This evidence is also observed for long rods and spheres.

Heat Addition ($T_\infty > T_i$)

One-Term Approximation Solutions



3 Case Studies:

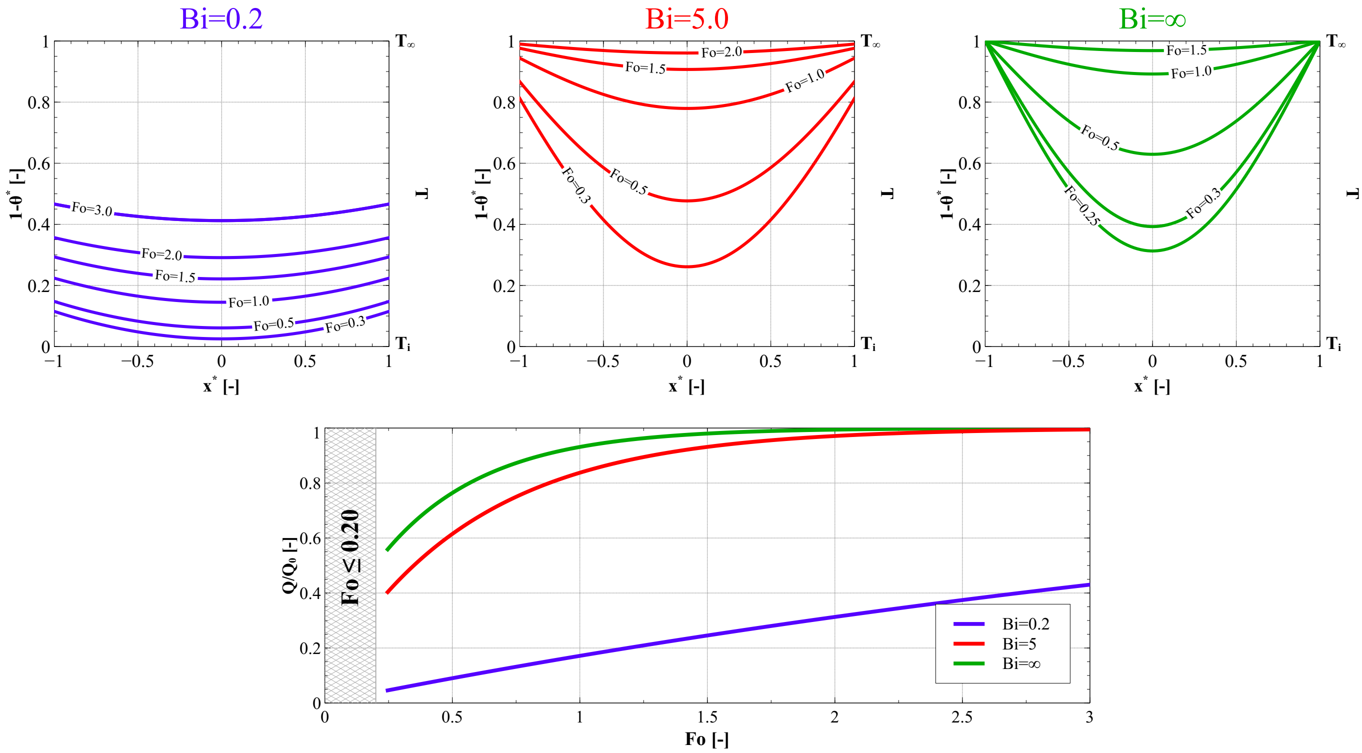
- $Bi = 0.2$;
- $Bi = 5.0$;
- $Bi = \infty$.
  - Negligible convection resistance: equivalent to prescribe a constant surface temperature (T_s) equal to T_∞

$$\Delta E_{st} = -Q, \quad Q < 0$$

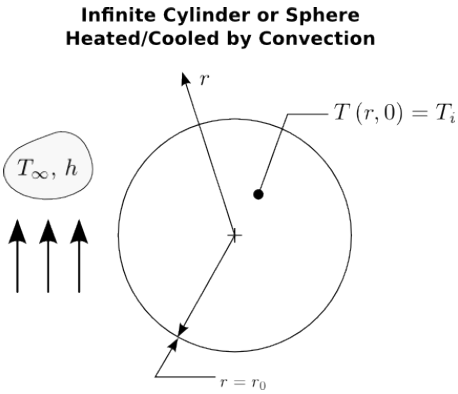
ΔE_{st} - change in thermal energy storage

$$1 - \theta^* = \frac{T(x^*,t^*) - T(x^*,0)}{T_\infty - T(x^*,0)}$$

Heat Addition - One-Term Approximation Solutions



One-Dimensional, Transient Conduction in Radial Systems



Initial Condition

$$T(r, t=0) = T_i$$

Infinite Cylinder - Gov. Equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Sphere - Governing Equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = h\left[T(r_0, t) - T_\infty\right]$$

One-Dimensional, Transient Conduction in Radial Systems

Non-dimensionalization:

- $\theta^* = \frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty}, \quad 0 \leq \theta^* \leq 1$
- $r^* = \frac{r}{r_0}, \quad 0 \leq r^* \leq 1$
- $t^* = Fo = \frac{\alpha t}{r_0^2}$
- $Bi = \frac{hr_0}{k}$

Initial Condition

$$\theta^*(r^*, 0) = 1$$

Infinite Cylinder - Gov. Equation

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta^*}{\partial r^*}\right) = \frac{1}{\alpha}\frac{\partial \theta^*}{\partial t^*}$$

Sphere - Governing Equation

$$\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial \theta^*}{\partial r^*}\right) = \frac{1}{\alpha}\frac{\partial \theta^*}{\partial t^*}$$

Boundary Conditions

$$\left.\frac{\partial \theta^*}{\partial r^*}\right|_{r^*=0} = 0$$

$$\left.\frac{\partial \theta^*}{\partial r^*}\right|_{r^*=1} = -Bi\theta^*(1, t^*)$$

One-Dimensional, Transient Conduction in Radial Systems

Exact Solutions - Dimensionless Temperature Difference

The exact solutions for the infinite cylinder and sphere are given in the form of infinite series.

Infinite Cylinder

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right) J_0\left(\zeta_n r^*\right)$$

Sphere

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right) \frac{1}{\zeta_n r^*} \sin\left(\zeta_n r^*\right)$$

C_n and ζ_n are functions of Bi number and the geometry under consideration (long rod or sphere). C_n and ζ_n are commonly given in tables.

One-Dimensional, Transient Conduction in Radial Systems

Approximate Solutions: One-term Approximation (**Valid for Fo > 0.2**)

Infinite Cylinder	**Sphere**
$\theta^* = \theta_0^* J_0\left(\zeta_1 r^*\right)$	$\theta^* = \theta_0^* \frac{1}{\zeta_1 r^*} \sin\left(\zeta_1 r^*\right)$
$\overline{\theta^*}\left(t^*\right) = \frac{2J_1(\zeta_1)}{\zeta_1}\theta_0^*$	$\overline{\theta^*}\left(t^*\right) = \frac{3\theta_0^*}{\zeta_1^3}\left[\sin\left(\zeta_1\right) - \zeta_1 \cos\left(\zeta_1\right)\right]$
$\frac{Q}{Q_0} = 1 - \frac{2J_1(\zeta_1)}{\zeta_1}\theta_0^*$	$\frac{Q}{Q_0} = 1 - \frac{3\theta_0^*}{\zeta_1^3}\left[\sin\left(\zeta_1\right) - \zeta_1 \cos\left(\zeta_1\right)\right]$

$$\theta_0^* = \frac{T\left(0, t^*\right) - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

- θ_0^* - centerline [centerpoint] dimensionless temperature difference for an infinite cylinder [sphere].

One-Dimensional, Transient Conduction in Radial Systems

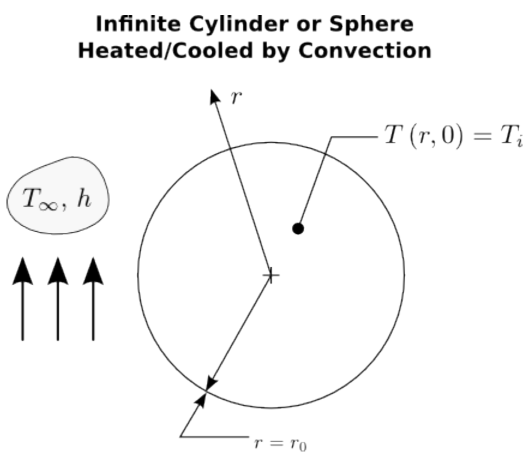
Boundary Condition at $r^* = 1$: Constant Surface Temperature

The foregoing solutions for θ^*, $\overline{\theta^*}$ and Q/Q_0 are also applicable for a fixed surface temperature boundary condition at $r^* = 1$ since it is equivalent to consider $h = \infty$ ($Bi = \infty$) and consequently $T_\infty = T_s$.

One-Dimen., Transient Conduction in Radial Systems

Heat Removal ($T_i > T_\infty$)

Numerical and One-Term Approximation Solutions



3 Case Studies:

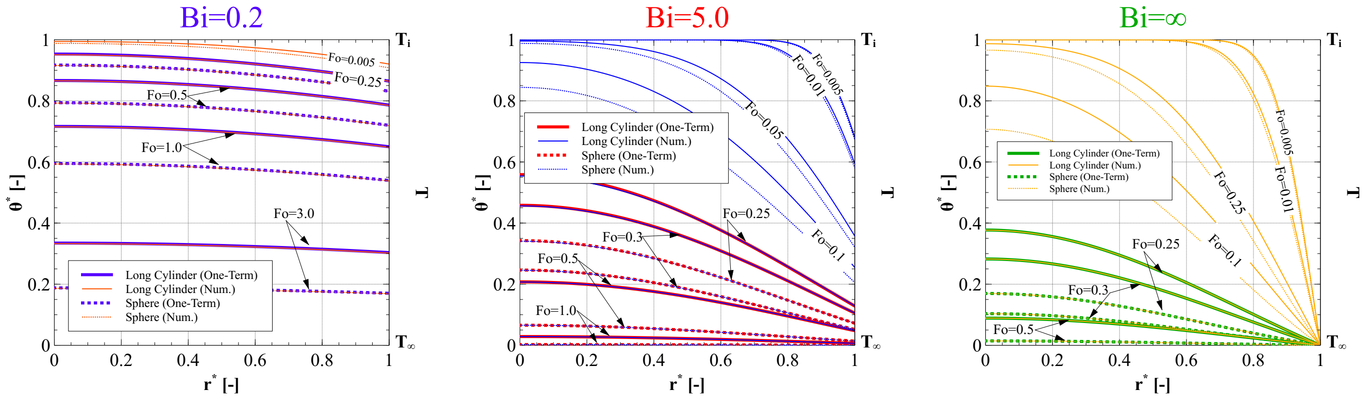
- $Bi = 0.2$;
- $Bi = 5.0$;
- $Bi = \infty$.
  - Negligible convection resistance: equivalent to prescribe a constant surface temperature (T_s) equal to T_∞

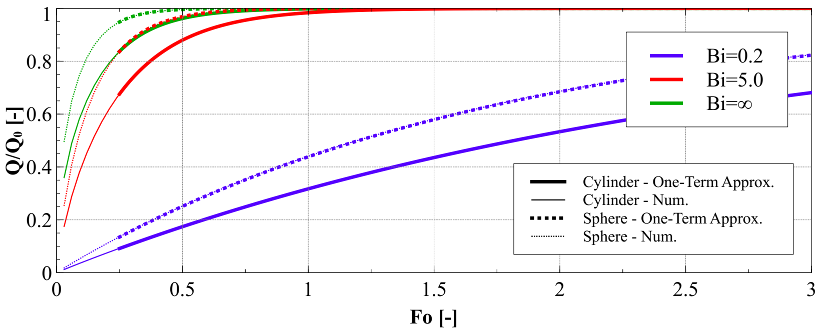
$$\Delta E_{st} = -Q, \quad Q > 0$$

ΔE_{st} - change in thermal energy storage

One-Dimen., Transient Conduction in Radial Systems

Heat Removal - Numerical and One-Term Approximation Solutions





Final Remarks

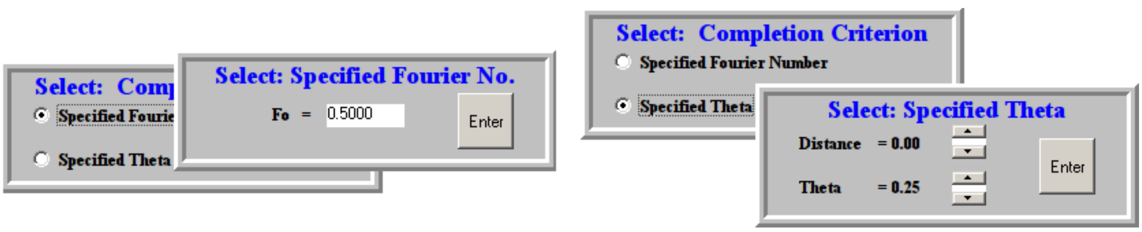
- The evaluation of temperature distribution profiles ($T(\vec{x}, t)$) during a transient heat conduction process with an appropriate form of the heat equation (and initial and boundary conditions) through numerical or approximated analytical solutions require a Biot number computed with L_c equal to L for large plane walls and r_0 for long cylinders and spheres.
  - Bi number with a characteristic length (L_c) equal to V/A_s is only considered for lumped system analysis.
- The one-term approximation for Fo>0.2 results in an error below 2%.
- Heisler/Gröber charts (transient temperature and heat transfer charts) provide a graphical representation for θ_0^*, θ^*/θ_0^* and Q/Q_0 obtained with the single-term approximation of the exact solution.

- **The software solves the one-dimensional, transient heat equation through numerical methods** employing the finite volume method.

Completion Criteria

The program ends its computations for two possible stopping criteria:

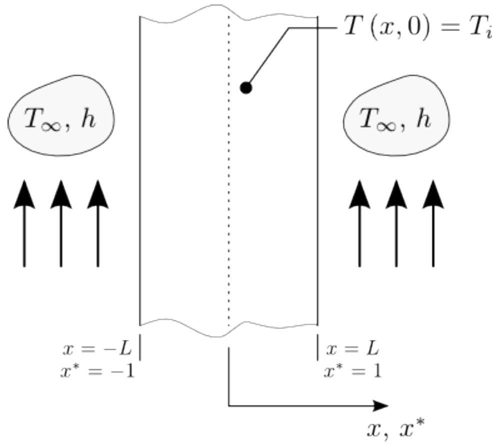
1. **Specified Fourier number (Fo);**
   - For evaluation of the temperature distribution profiles and the ratio Q/Q_0 at a specific time instant.
2. **Specified θ^* (x*, t*).**
   - For the evaluation of the elapsed time, temperature distribution profiles and the ratio Q/Q_0.



Exploring the Software - Case Study I

Cooling of a Plane Wall

Using Fo Number as the Stopping Criterion



Objectives:

1. Calculation of the temperature distribution at any time instant, $T(x,t)$;
2. Calculation of the fractional energy loss, Q/Q_0, at any time instant, $Q/Q_0(t)$.

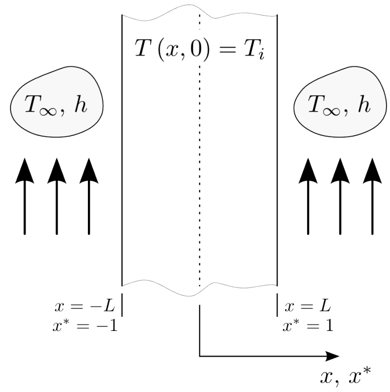
Exploring the Software - Cooling of a Plane Wall (1/3)

Temperature distribution and heat lost to the fluid after a specific time interval (Fo number as the stopping criterion)

Consider a plane wall initially at $T_i = 180\,^\circ\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 20\,^\circ\mathrm{C}$ and with $h = 2500\,\mathrm{W.m^{-2}.K^{-1}}$.

Reference Data

Thermoph. Properties	Geom. Properties
$k = 110\,\mathrm{W/(m.K)}$	
$\rho = 8530\,\mathrm{W/(m^2.K)}$	$L = 0.05\,\mathrm{m}$
$c = 380\,\mathrm{W/(m^2.K)}$	



1. Determine the Temperature (T) at

- $x = 0\,\mathrm{m}$ and after $t = 20\,\mathrm{s}$
- $x = L/2\,\mathrm{m}$ and after $t = 40\,\mathrm{s}$

2. Determine Q/Q_0 after

- $t = 20\,\mathrm{s}$
- $t = 40\,\mathrm{s}$

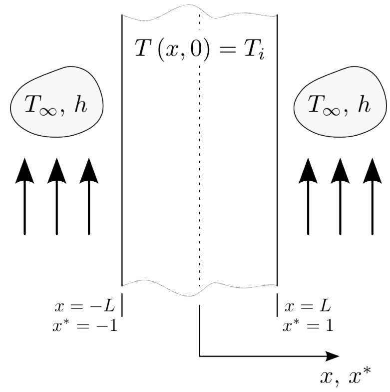
Exploring the Software - Cooling of a Plane Wall (2/3)

Temperature distribution and heat lost to the fluid after a specific time interval (Fo number as the stopping criterion)

Consider a plane wall initially at $T_i = 180\,^{\circ}\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 20\,^{\circ}\mathrm{C}$ and with $h = 2500\,\mathrm{W.m^{-2}.K^{-1}}$.

Reference Data

Thermal Diffusivity	Biot Number
$\alpha = 3.39 \times 10^{-5}\,\mathrm{m^2.s^{-1}}$	$Bi = 1.14$



1. Determine the Temperature (T) for

- $x^* = 0$ and $t^* = Fo = 0.27$
- $x^* = 0.5$ and $t^* = Fo = 0.54$

2. Determine Q/Q_0 after

- $t^* = Fo = 0.27$
- $t^* = Fo = 0.54$

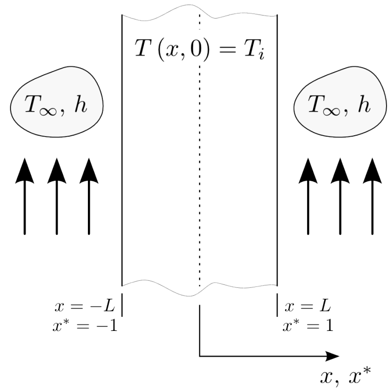
Exploring the Software - Cooling of a Plane Wall (3/3)

Temperature distribution and heat lost to the fluid after a specific time interval (Fo number as the stopping criterion)

Consider a plane wall initially at $T_i = 180\,^{\circ}\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 20\,^{\circ}\mathrm{C}$ and with $h = 2500\,\mathrm{W.m^{-2}.K^{-1}}$.

Reference Data

Thermal Diffusivity	Biot Number
$\alpha = 3.39 \times 10^{-5}\,\mathrm{m^2.s^{-1}}$	$Bi = 1.14$



1. Determine the Temperature (T)

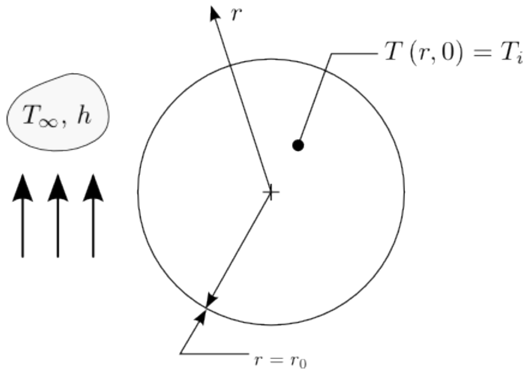
- $\theta^*(0, 0.27) = 0.9011 \Rightarrow \mathbf{T = 164.2}\,^{\circ}\mathrm{C}$
- $\theta^*(0.5, 0.54) = 0.6565 \Rightarrow \mathbf{T = 125.0}\,^{\circ}\mathrm{C}$

2. Determine Q/Q_0

- $\mathbf{Q/Q_0\,(0.27) = 0.210}$
- $\mathbf{Q/Q_0\,(0.54) = 0.366}$

Cooling of a Sphere

Using a Specified $\theta^*(x^* = x_1^*, t^*)$ Value as the Stopping Criterion



Objectives:

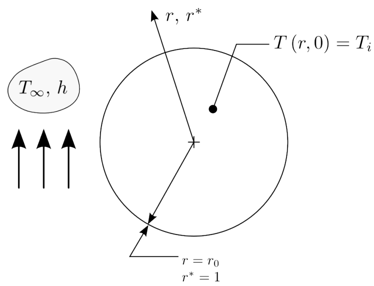
1. Calculation of the elapsed time, t, for achieving $\theta^*(x^* = x_1^*, t^*)$ equal to a specified value;
2. Calculation of the temperature distribution, $T(x,t)$, when $\theta^*(x^* = x_1^*, t^*)$ is equal to a specified value;
3. Calculation of the ratio Q/Q_0 when $\theta^*(x^* = x_1^*, t^*)$ is equal to a specified value.

Temperature distribution, heat lost to the fluid and elapsed time for a specific value of $\theta^*(\mathbf{x}^*, \mathbf{t}^*)$ ($\theta^*(\mathbf{x}^*, \mathbf{t}^*)$ as the stopping criteria)

Consider a sphere initially at $T_i = 260\,^\circ\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 20\,^\circ\mathrm{C}$ and with $h = 3000\,\mathrm{W.m^{-2}.K^{-1}}$.

Reference Data

Thermoph. Properties	Geom. Properties
$k = 110\,\mathrm{W/(m.K)}$	
$\rho = 8530\,\mathrm{W/(m^2.K)}$	$r_0 = 0.02\,\mathrm{m}$
$c = 380\,\mathrm{W/(m^2.K)}$	



1. Determine the time instant, t, for which

- $T(r = 0, t) = 200\,^\circ\mathrm{C}$
- $T(r = r_0, t) = 80\,^\circ\mathrm{C}$

2. Determine Q/Q_0 when

- $T(r = 0, t) = 200\,^\circ\mathrm{C}$
- $T(r = r_0, t) = 80\,^\circ\mathrm{C}$

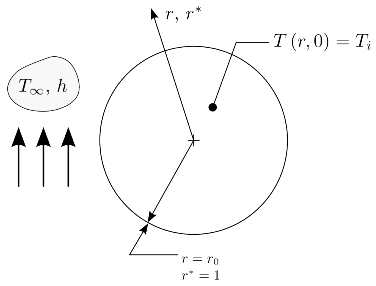
Exploring the Software - Cooling of a Sphere (2/3)

Temperature distribution, heat lost to the fluid and elapsed time for a specific value of $\theta^*(\mathbf{x}^*, \mathbf{t}^*)$ ($\theta^*(\mathbf{x}^*, \mathbf{t}^*)$ as the stopping criteria)

Consider a sphere initially at $T_i = 260\,^\circ\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 25\,^\circ\mathrm{C}$ and with $h = 3000\,\mathrm{W.m^{-2}.K^{-1}}$.

Reference Data

Thermal Diffusivity	Biot Number
$\alpha = 3.39 \times 10^{-5}\,\mathrm{m^2.s^{-1}}$	$Bi = 0.55$



1. Determine the time instant, t, for which

- $\theta^*(0, t^*) = 0.75$
- $\theta^*(1, t^*) = 0.25$

2. Determine Q/Q_0 when

- $\theta^*(0, t^*) = 0.75$
- $\theta^*(1, t^*) = 0.25$

Exploring the Software - Cooling of a Sphere (3/3)

Temperature distribution, heat lost to the fluid and elapsed time for a specific value of $\theta^*(\mathbf{x}^*, \mathbf{t}^*)$ ($\theta^*(\mathbf{x}^*, \mathbf{t}^*)$ as the stopping criteria)

Consider a sphere initially at $T_i = 260\,^\circ\mathrm{C}$ that is suddenly cooled with a fluid at $T_\infty = 25\,^\circ\mathrm{C}$ and with $h = 3000\,\mathrm{W.m^{-2}.K^{-1}}$.

Reference Data

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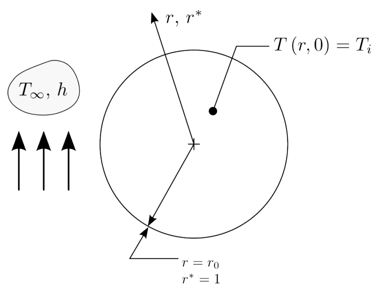


1. Determine the time instant, t, for which

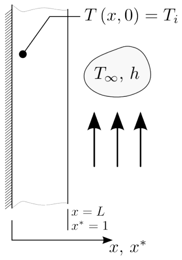
- $\theta^*(0, t^*) = 0.75 \Rightarrow Fo = 0.2930 \Rightarrow$ $\mathbf{t} = \mathbf{3.46}\,\mathrm{s}$
- $\theta^*(0, t^*) = 0.25 \Rightarrow Fo = 0.8595 \Rightarrow$ $\mathbf{t} = \mathbf{10.14}\,\mathrm{s}$

2. Determine Q/Q_0

- $\mathbf{Q/Q_0\,(0.2930) = 0.355}$
- $\mathbf{Q/Q_0\,(0.8595) = 0.721}$

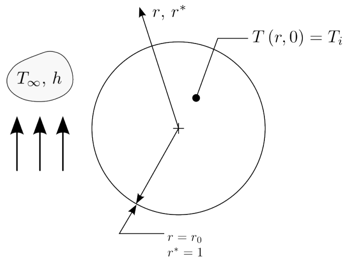
Useful Relations

Plane Wall



$$\theta^* = \frac{T(x^*, t^*) - T_\infty}{T(x^*, 0) - T_\infty}$$
$$x^* = \frac{x}{L}$$
$$t^* = Fo = \frac{\alpha t}{L^2}$$
$$Bi = \frac{hL}{k}$$

Radial Systems



$$\theta^* = \frac{T(r^*, t^*) - T_\infty}{T(r^*, 0) - T_\infty}$$
$$r^* = \frac{r}{r_0}$$
$$t^* = Fo = \frac{\alpha t}{r_0^2}$$
$$Bi = \frac{hr_0}{k}$$

$$\alpha = k/\rho c \qquad Q_0 = \rho V c \left(T_i - T_\infty\right)$$