Heat Transfer

Computational Laboratories

Forced Convection over a Flat Plate

(Laboratory III)

Convective heat transfer over a flat plate (external flow) with a parallel flow under steady, incompressible, and constant-property fluid conditions

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Convection Heat Transfer – Introduction

- Convection heat transfer between a fluid and a surface with different temperatures requires simultaneously the following mechanisms:
  - energy transfer by bulk (macroscopic) fluid motion over the surface (**advection**); and
  - energy transfer by microscopic random motion of fluid molecules (**conduction**).
- Convection heat transfer occurs along a fluid region over a surface (thermal boundary layer) where temperature gradients exist.

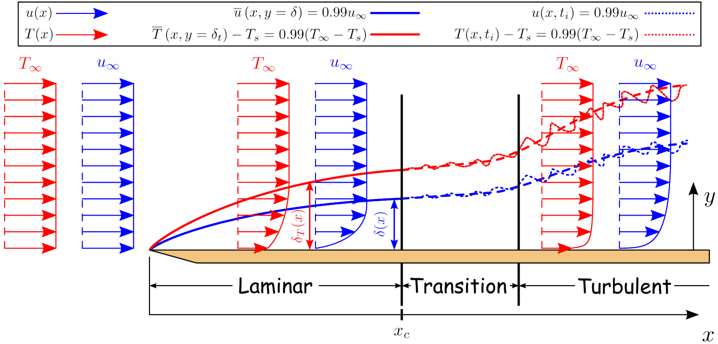
Problem of convection heat transfer

Determination of local or average convection heat transfer coefficients h_x or $\overline{h}$, respectively. This is performed by determining (experimentally or theoretically) local or average Nusselt (Nu_x, $\overline{Nu}$) correlations suitable for a broad range of conditions. **The theoretical fashion requires the solution of the appropriate form of the boundary layer equations.**

Velocity and Thermal Boundary Layers – Convective Momentum and Heat Transfer (1/2)

External flow configuration

Parallel flow over an isothermal flat plate ($T_\infty > T_s$)



Velocity and Thermal Boundary Layers – Convective Momentum and Heat Transfer (2/2)

- The velocity boundary layer starts growing from the beginning of the plate ($x = 0$) – no slip condition;
- The transition to the turbulent boundary layer occurs when small disturbances (perturbations) are no longer damped by viscous forces and are amplified;
  - A critical (transition) Reynolds number ($Re_{x,c}$) is defined based on the flow configuration, surface geometry, free stream turbulence level and on surface roughness.
- The thermal boundary layer develops from the position above the plate where the surface temperature (T_s) differs from the free stream temperature (T_∞) – not necessarily from the plate's leading edge ($x = 0$).
- The temperature at the surface of the plate may differ of the free stream value as a consequence of an imposed surface temperature or an imposed surface heat flux.

Boundary Layer (BL) Governing Equations

Governing equations in Cartesian coordinates for the fluid and heat flow in the BLs

1. Continuity Equation – Overall Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2. x-Momentum Equation – Conservation of Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + (\nu + \varepsilon_M)\frac{\partial^2 u}{\partial y^2}$$

3. Thermal Energy Equation – Conservation of Energy

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (\alpha + \varepsilon_H)\frac{\partial^2 T}{\partial y^2} + \frac{\nu + \varepsilon_M}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

- $\vec{V} = (u, v)$ where $u = f(x, y)$ and $v = f(x, y)$.
- $T = f(x, y)$
- ε_M and ε_H – Eddy diffusivities for momentum and heat, respectively.

Assumptions: (1) steady-state conds.; (2) 2D flow; (3) incomp. flow; (4) constant-property fluid; (5) negligible body forces; (6) no thermal energy generation; (7) $\partial^2 u/\partial y^2 >> \partial^2 u/\partial x^2$; (8) $\partial^2 T/\partial y^2 >> \partial^2 T/\partial x^2$; and (9) $dp/dx \approx dp_\infty/dx$

BL Equations and Boundary Conditions – Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

Simplifying assumptions:

- $\frac{dp_\infty}{dx} = 0 \Rightarrow$ since $u_\infty = \mathrm{Cnt}$
- $\left(\frac{\partial u}{\partial y}\right)^2 \simeq 0 \Rightarrow$ for low-speed flows

1. Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2. Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (\nu + \varepsilon_M)\frac{\partial^2 u}{\partial y^2}$$

3. Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (\alpha + \varepsilon_H)\frac{\partial^2 T}{\partial y^2}$$

Boundary Conditions (BCs)

- Continuity and Momentum Equations
  1. $u(0, y) = u_\infty$
  2. $u(x, 0) = 0$
  3. $v(x, 0) = 0$
  4. $u(x, \infty) = u_\infty$
- Energy Equation
  1. $T(0, y) = T_\infty$
  2. $T(x, 0) = T_s$
  3. $T(x, \infty) = T_\infty$

  x – any x position along the plate ($x > 0$)

Boundary Layers Parameters – Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

- For a fluid with constant properties (ρ, μ, c_p, k_f) the velocity field, $\vec{v}$ is evaluated exclusively with the overall continuity and momentum equations (uncoupled from the energy equation);
- The temperature along the thermal boundary layer is evaluated with the solution for $\vec{v}$ through the energy equation.

Velocity Boundary Layer

$$u = f\,(x, y, u_\infty, \rho, \mu)^\dagger$$

$$\tau_s = \mu\, \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$\tau_s = f\,(x, u_\infty, \rho, \mu)^\dagger$$

τ_s - surface shear stress

† Laminar flow

Thermal Boundary Layer

$$T = f\,(x, y, u_\infty, T_\infty, T_s, \rho, \mu, c_p, k_f)^\dagger$$

$$q_s'' = -k_f\, \frac{\partial T}{\partial y}\bigg|_{y=0} \qquad q_s'' = h\,(T_s - T_\infty)$$

$$h = \frac{k_f}{(T_\infty - T_s)}\, \frac{\partial T}{\partial y}\bigg|_{y=0}$$

$$h = f\,(x, u_\infty, T_\infty, T_s, \rho, \mu, c_p, k_f)^\dagger$$

Normalized BL Equations and BCs – Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L}$

$u^* = \frac{u}{u_\infty} \qquad v^* = \frac{v}{u_\infty} \qquad T^* = \frac{T - T_s}{T_\infty - T_s}$

1. Continuity Equation

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

2. Momentum Equation

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L}\left(1 + \frac{\varepsilon_M}{\nu}\right)\frac{\partial^2 u^*}{\partial y^{*2}}$$

3. Energy Equation

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\left(1 + \frac{\varepsilon_H}{\alpha}\right)\frac{\partial^2 T^*}{\partial y^{*2}}$$

Boundary Conditions (BCs)

- Continuity and Momentum Equations
  1. $u^*\left(0, y^*\right) = 1$
  2. $u^*\left(x^*, 0\right) = 0$
  3. $v^*\left(x^*, 0\right) = 0$
  4. $u^*\left(x^*, \infty\right) = 1$
- Energy Equation
  1. $T^*\left(0, y^*\right) = 1$
  2. $T^*\left(x^*, 0\right) = 0$
  3. $T^*\left(x^*, \infty\right) = 1$

  x^* – any x^* position along the plate ($x^* > 0$)

Normalized BL Parameters - Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

$$x^* = \frac{x}{L} \quad y^* = \frac{y}{L} \quad u^* = \frac{u}{u_\infty} \quad v^* = \frac{v}{u_\infty} \quad T^*\left(x^*, y^*\right) = \frac{T(x^*,y^*) - T_s}{T_\infty - T_s}$$

Velocity Boundary Layer

$$u^* = f\left(x^*, y^*, Re_L\right)^\dagger$$

$$\tau_{s,x} = \left(\frac{\mu u_\infty}{L}\right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$C_{f,x} = \frac{\tau_s}{\frac{1}{2}\rho u_\infty^2} = \frac{2}{Re_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$C_{f,x} = f\left(x^*, Re_L\right)^\dagger$$

$C_{f,x}$ – local friction coefficient

Thermal Boundary Layer

$$T^* = f\left(x^*, y^*, Re_L, Pr\right)^\dagger$$

$$h_x = \left(\frac{k_f}{L}\right) \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$Nu_x = \frac{h_x x}{k_f} = x^* \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$Nu_x = f\left(x^*, Re_L, Pr\right)^\dagger$$

Nu_x – local Nusselt number

† Laminar flow

Solution for Boundary Layer Equations

Solution for Laminar Boundary Layer Equations

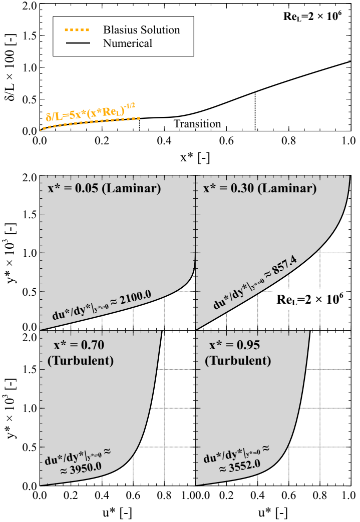
- Blasius Solution (Similarity Solution) – Blasius, 1908:
  - For a parallel flow of constant u_∞ and T_∞ over an isothermal flat plate under steady, incompressible, constant-property fluid conditions, and neglecting viscous dissipation;
  - The solution procedure converts the continuity and momentum equations in a third-order ordinary differential equation with the solution performed by numerical integration techniques or through power-series expansion.
- Integral method (Approximate Analysis) – von Kármán, 1921:
  - The procedure assumes approximate expressions for the velocity and thermal profiles in the integral form of the momentum and energy equations.
- Numerical Methods

Solution for Turbulent Boundary Layer Equations

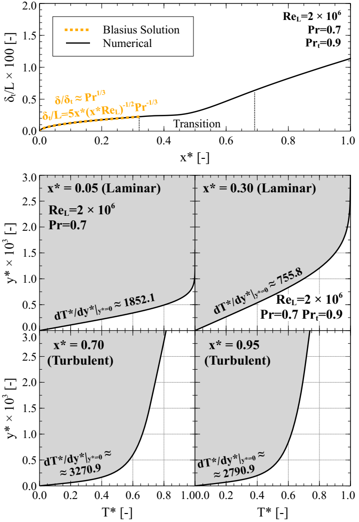
- Integral methods
- Numerical (differential) methods with closure models (for unknown quantities)

Parallel Flow over an Isothermal Plate

Velocity Boundary Layer

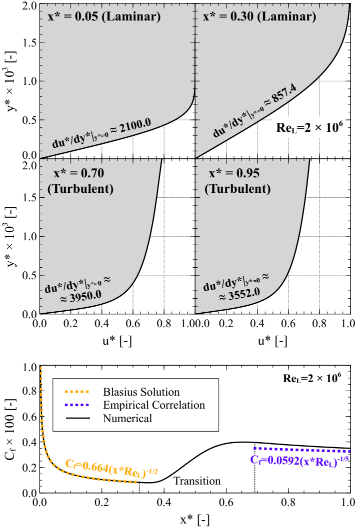


Thermal Boundary Layer

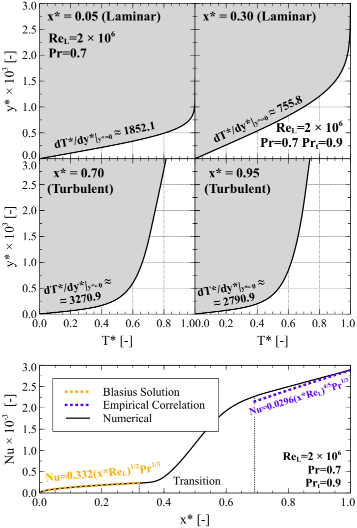


Parallel Flow over an Isothermal Plate

Velocity Boundary Layer



Thermal Boundary Layer



Parallel Flow over an Isothermal Plate

Consider a parallel flow of atmospheric air over an isothermal flat plate with a free stream velocity and temperature equal to $41.85\,\mathrm{m.s^{-1}}$ (u_∞) and $300\,\mathrm{K}$ (T_∞), respectively. The plate is $1\,\mathrm{m}$ (L) long and has a uniform temperature of $400\,\mathrm{K}$ (T_s).

Determine the local surface shear stress (τ_s) and local convection heat transfer coefficient (h) along the plate.

1. Calculation of the film temperature (mean boundary layer temperature) – T_f

$$T_f = \frac{T_\infty + T_s}{2} = 350\,\mathrm{K}$$

2. Evaluation of fluid thermophysical properties at $T_f = 350\,\mathrm{K}$ and $p = 1\,\mathrm{atm}$

Hydrodynamic Properties	Thermal Properties
$\rho = 0.9950\,\mathrm{kg.m^{-3}}$	$c_p = 1.0090\,\mathrm{kJ.kg^{-1}.K^{-1}}$
$\mu = 2.0820 \times 10^{-5}\,\mathrm{N.s.m^{-2}}$	$k_f = 3.0 \times 10^{-2}\,\mathrm{W.m^{-1}.K^{-1}}$

3. Evaluation of Reynolds and Prandtl numbers, Re_L and Pr

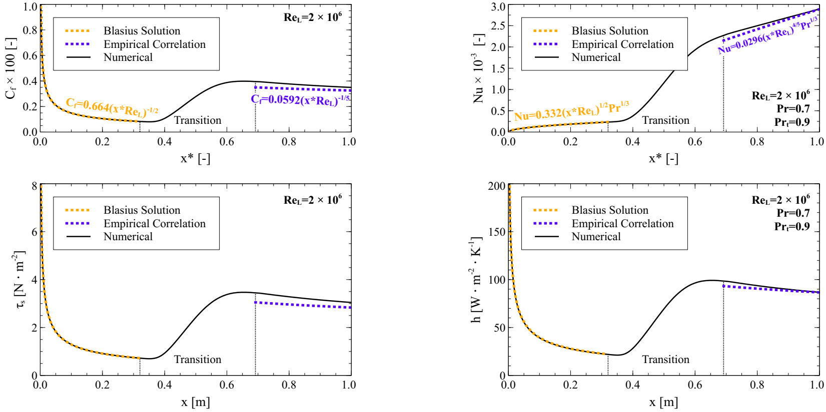
$$Re_L = \frac{\rho u_\infty L}{\mu} = \mathbf{2 \times 10^6} \qquad Pr = \frac{\mu c_p}{k_f} = \mathbf{0.70}$$

Parallel Flow over an Isothermal Plate

3. Evaluation of Reynolds and Prandtl numbers, Re_L and Pr

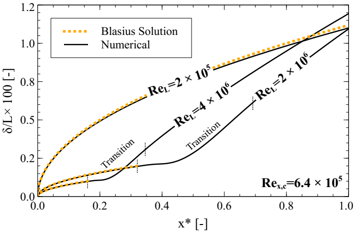
$$Re_L = \frac{\rho u_\infty L}{\mu} = \mathbf{2 \times 10^6} \qquad\qquad Pr = \frac{\mu c_p}{k_f} = \mathbf{0.70}$$

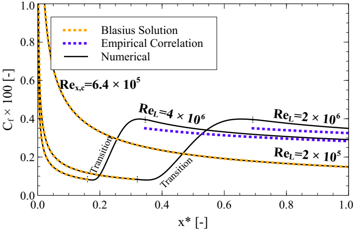
4. Calculation of τ_s and h based on local values of C_f and Nu



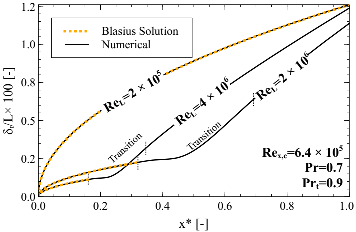
Parallel Flow over an Isothermal Plate: Effect of Re_L

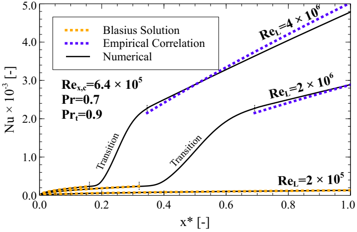
Velocity Boundary Layer





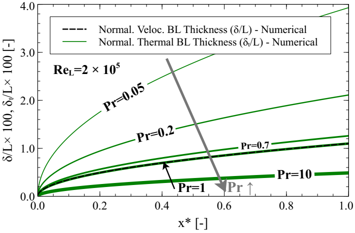
Thermal Boundary Layer

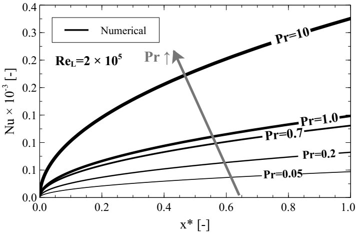




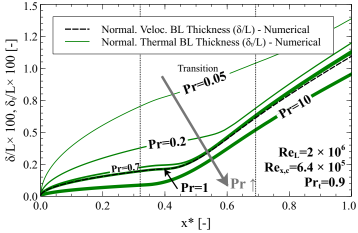
Parallel Flow over an Isothermal Plate: Effect of Pr

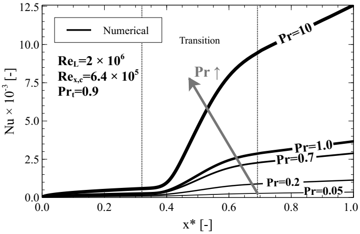
$Re_L = 2 \times 10^5 \ (< Re_{x,c})$





$Re_L = 2 \times 10^6 \ (> Re_{x,c})$





Transition to Turbulent BL and Average BL Parameters

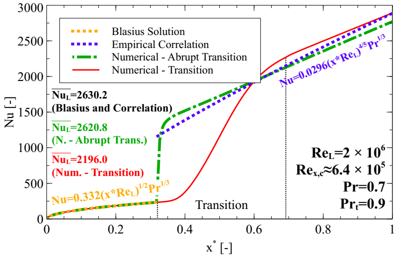
Transition

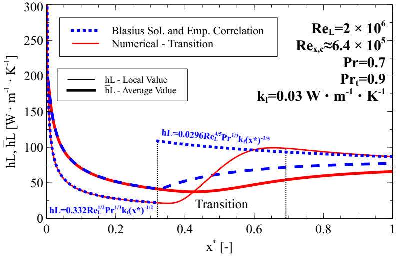
- An abrupt transition is commonly considered, and consequently, the transition region is considered the initial part of the turbulent region.
  - As a result higher local and average values for Nu (C_f), and consequently, for h (τ_s) are predicted.

Average Thermal BL Parameters

$$\bar{h}_{x_2-x_1} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} h_x \, dx =$$

$$= \frac{k_f}{x_2 - x_1} \int_{x_1}^{x_2} \frac{Nu_x}{x} dx = \frac{k_f}{x_2 - x_1} \overline{Nu}_{x_2-x_1}$$





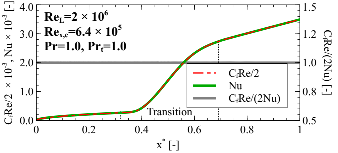
Momentum and Heat Transfer Analogies

Reynolds Analogy

For $\mathbf{dp^*/dx^*} = \mathbf{0}$ (flat plate with constant u_∞), $\mathbf{Pr} = \mathbf{1}$ and $\epsilon_M = \epsilon_H$, *i.e.*, $\mathbf{Pr_t} = \mathbf{1}$ (for turbulent flow) the momentum and energy equations as well as its boundary conditions are mathematically analogous and the solutions for u^* and T^* are equal. Therefore,

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \Rightarrow$$

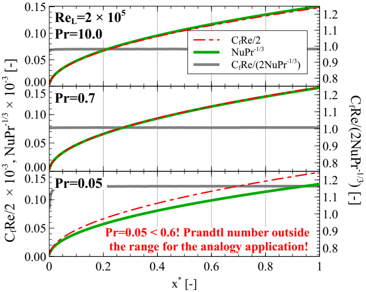
$$\Rightarrow C_{f,x}\frac{Re_x}{2} = Nu_x$$



Chilton-Colburn Analogy

For $\mathbf{dp^*/dx^*} = \mathbf{0}$ the Reynolds analogy can be extended for $\mathbf{0.6} < \mathbf{Pr} < \mathbf{60}$ considering a Prandtl number correction as follows:

$$C_{f,x}\frac{Re_x}{2} = Nu_x Pr^{-1/3}$$

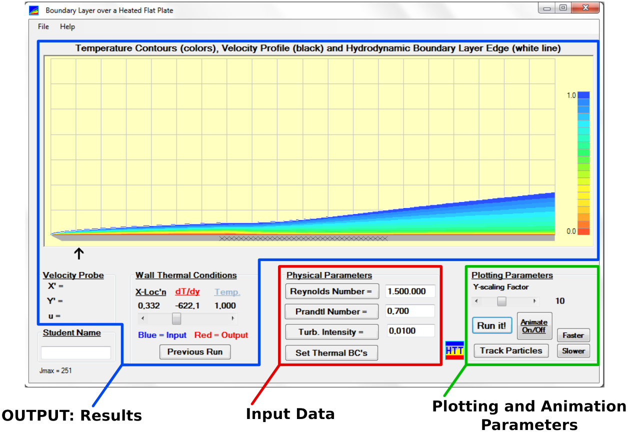


Final Remarks

- Increasing the plate Reynolds number (Re_L) the transition region starts closer to the leading edge $x_c^* = \frac{Re_{x,c}}{Re_L}$.
- For a Prandtl number equal to 1.0 (equal diffusivities for momentum and heat) the velocity and temperature profiles are similar and the velocity and thermal BLs grow along the plate at the same rate, *i.e.*, $\delta = \delta_t$.
- For $Pr > 1$ the momentum diffusivity is higher than the thermal diffusivity and therefore $\delta > \delta_t$ (oils).
- For $Pr < 1$ the thermal diffusivity is higher than the momentum diffusivity and therefore $\delta < \delta_t$ (liquid metals).

Exploring the Module – Interface

Software module – `HTTflatp.exe (Version 2.0.0.3)`



- **The module solves the boundary layer governing equations (continuity, x-momentum, and energy equations) for forced convection over a flat plate.**

Exploring the Module – Boundary Conditions

- The program computes velocity and thermal boundary layer parameters in the non-dimensional form:

$$x^* = \frac{x}{L} \quad y^* = \frac{y}{L} \quad u^* = \frac{u}{u_\infty} \quad v^* = \frac{v}{u_\infty} \quad T^*(x^*, y^*) = \frac{T(x^*, y^*) - T_\infty}{T_s - T_\infty}$$

Boundary Conditions:

Continuity and Momentum Equations

1. At $x^* = 0$ (Upstream):
   - $u^*(0, y^*) = 1$
2. At $y^* = \infty$:
   - $u^*(x^*, \infty) = 1$
3. At $y^* = 0$:
   - $u^*(x^*, 0) = 0$
   - $v^*(x^*, 0) = 0$

Energy Equation

1. At $x^* = 0$ (Upstream the leading edge):
   - $T^*(0, y^*) = 0$
2. At $y^* = \infty$:
   - $T^*(x^*, \infty) = 0$
3. At $y^* = 0$ (**User Input**):
   - $T^*(x^*, 0) = C_1$ or $C_1 + C_2 x^*$

     $C_1 = 1.0$ and $C_2 = 0.0$ (isothermal plate)

Exploring the Module – Laminar-Turbulent Transition

- The program considers the turbulence intensity of the free stream (TU) to evaluate the critical Reynolds number ($Re_{x,c}$) through the equation that follows:

$$Re_{x,c}^{1/2} = \frac{-1.0 + \left(132500\,TU^2\right)^{1/2}}{39.2\,TU^2}$$

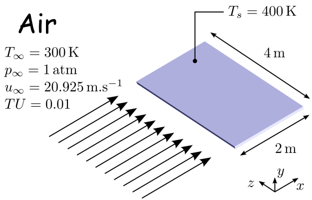
  - A critical Reynolds number ($Re_{x,c}$) of about 5×10^5 is obtained for a free stream turbulence level of 1% .

- The $Re_{x,c}$ is employed to determine the distance from the leading edge of the plate where the flow transition from laminar to turbulent starts (beginning of the transition region).

Exploring the Module – Application Examples

Module Application Example I: Problem Statement

Consider a $2\,\mathrm{m} \times 4\,\mathrm{m}$ isothermal flat plate at $400\,\mathrm{K}$ subjected to air flow parallel to its surfaces along its $2\,\mathrm{m}$ long side. The free stream temperature (T_∞), pressure (p_∞), velocity (u_∞), and turbulence level (TU) are equal to, $300\,\mathrm{K}$, $1\,\mathrm{atm}$, $20.925\,\mathrm{m.s^{-1}}$, and 1%, respectively.



Calculate the following using the module:

1. local Nusselt number values, Nu_x;
2. local convection heat transfer coefficients, h_x;
3. surface average convection heat transfer coefficient over the entire plate, $\overline{h}_L$ and compare with the literature value; and
4. total heat transfer rate from the plate to the fluid, q.

Exploring the Module – Application Examples

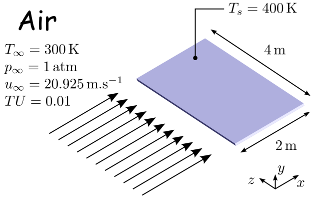
Module Application Example I: Module Application

Preliminary Calculations – Methodology

1. Calculation of the **film temperature** (mean boundary layer temperature) – T_f

$$T_f = \frac{T_\infty + T_s}{2} = 350\,\mathrm{K}$$

2. Evaluation of **fluid properties** at $T_f = 350\,\mathrm{K}$ and $p = 1\,\mathrm{atm}$
   - $\rho = 0.9950\,\mathrm{kg.m^{-3}}$
   - $\mu = 2.0820 \times 10^{-5}\,\mathrm{N.s.m^{-2}}$
   - $c_p = 1.0090\,\mathrm{kJ.kg^{-1}.K^{-1}}$
   - $k_f = 3.0{\times}10^{-2}\,\mathrm{W.m^{-1}.K^{-1}}$

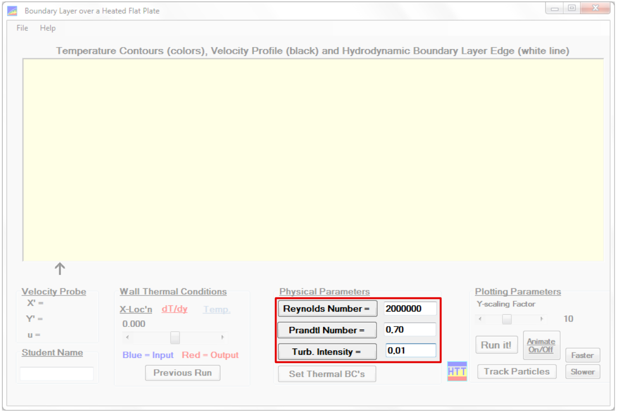


3. Evaluation of **Reynolds and Prandtl numbers**, Re_L and Pr
   - $Re_L = \frac{\rho u_\infty L}{\mu} \approx \mathbf{2 \times 10^6}$
   - $Pr = \frac{\mu c_p}{k_f} \approx \mathbf{0.70}$

Exploring the Module – Application Examples

Module Application Example I: Module Application

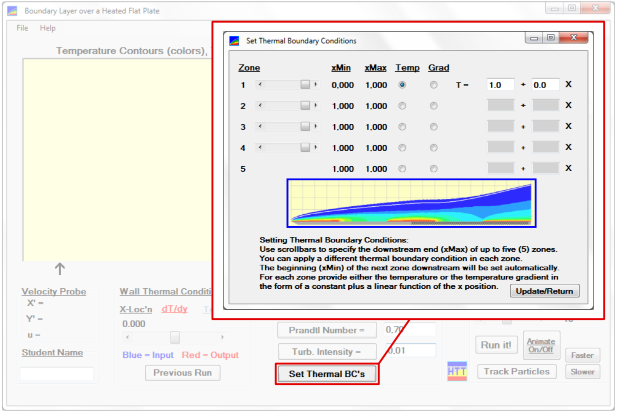
Input Data: Physical Parameters



Exploring the Module – Application Examples

Module Application Example I: Module Application

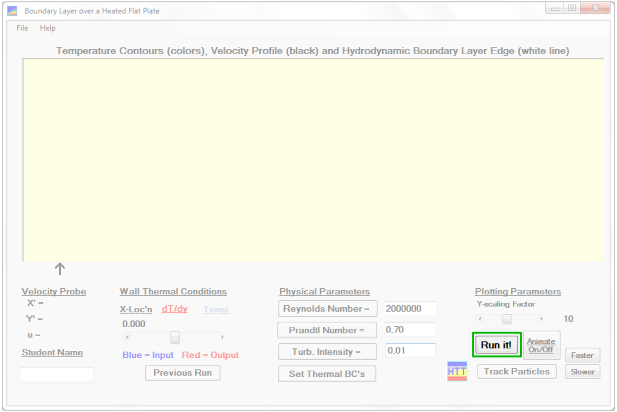
Input Data: Thermal Energy Boundary Condition at the Wall



Exploring the Module – Application Examples

Module Application Example I: Module Application

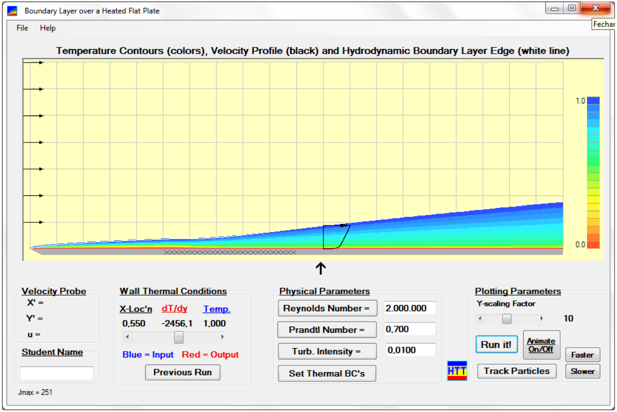
Start Simulation



Exploring the Module – Application Examples

Module Application Example I: Module Results

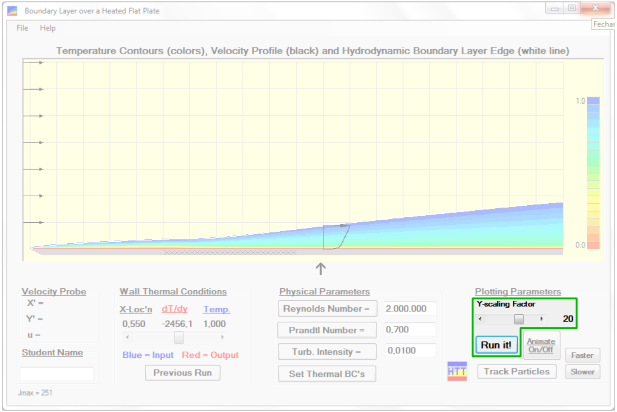
Module Results



Exploring the Module – Application Examples

Module Application Example I: Module Results

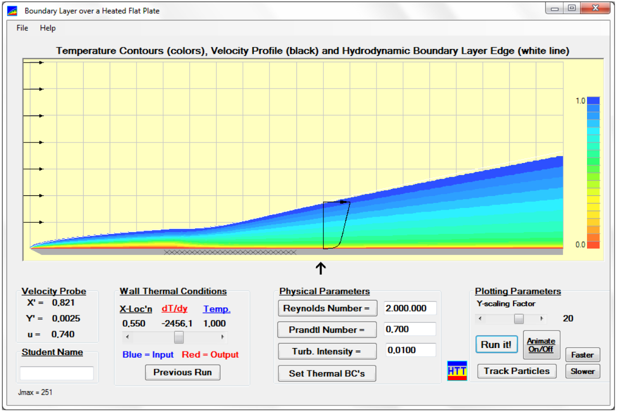
Re-scaling plotted boundary layer parameters in the vertical direction



Exploring the Module – Application Examples

Module Application Example I: Module Results

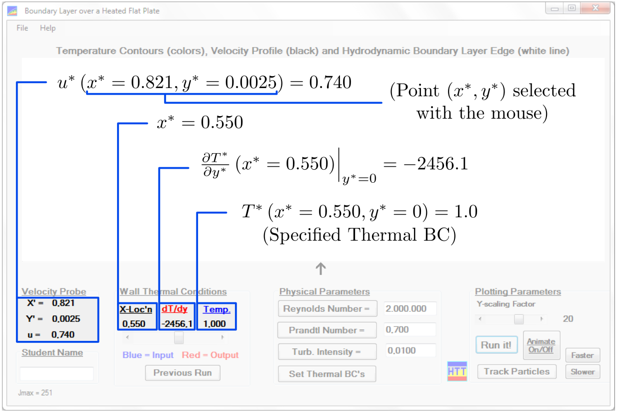
Module Results



Exploring the Module – Application Examples

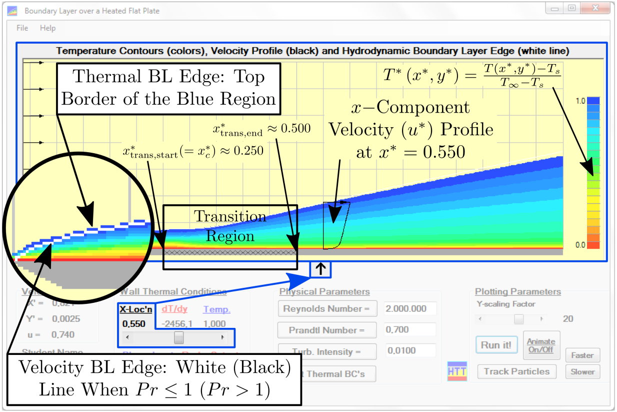
Module Application Example I: Module Results

Module Results: Numerical Output Data



Module Application Example I: Module Results

Module Results: Graphical Output Data



Exploring the Module – Application Examples

Module Application Example I: Results Analysis (1/2)

1. and 2. local Nusselt number and convec. coeff. values, Nu_x and h_x, respectively

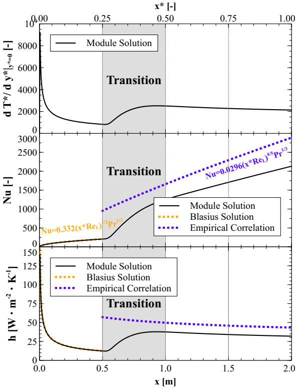
Procedure:

1. Evaluate $\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}(x^*)$ with the module – using the on-screen scrollbar or exporting the data file
2. Calculate $Nu_x(x^*)^\dagger$

$$Nu_x(x^*) = -x^*. \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}(x^*)$$

3. Calculate $h_x(x^*)$

$$h_x(x^*) = \frac{Nu_x(x^*)\,.k_f}{x^* L}$$



†The minus sign in the Nu_x equation is due to the normalization procedure for the temperature (T^*) considered in the program – see Slide 21.

Exploring the Module – Application Examples

Module Application Example I: Results Analysis (2/2)

3. average convection heat transfer coefficient over the entire plate, $\overline{h}_L$

$$\overline{h}_L = \frac{1}{L}\int_0^L h_x(x)\,dx \Rightarrow \overline{h}_L = \int_0^1 h_x(x^*)\,dx^*$$

Where, $h_x(x^*) = -\frac{k_f}{L}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}(x^*)$

Performing the integration: $\boxed{\overline{h}_L \approx 30.3\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}}$

Comparing with literature data

$$\overline{Nu}_L = \left(0.037Re_L^{4/5} - A\right)Pr^{1/3} \quad \text{with} \quad A = 0.037Re_{x,c}^{4/5} - 0.644Re_{x,c}^{1/2}$$

Considering $x_c^* = 0.25 \Rightarrow Re_{x,c} = 5\times10^5 \Rightarrow \boxed{\overline{h}_L \approx 42.5\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}}$

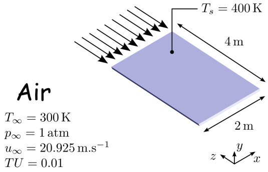
4. total heat transfer rate from the plate to the fluid, q

$$q = A_s.\overline{h}_L.(T_s - T_\infty) = 8.0\times30.3\times(400.0-300.0) \Leftrightarrow \boxed{q = 24.24\,\mathrm{kW}}$$

Exploring the Module – Application Examples

Module Application Example II: Problem Statement

Consider the same plate and the same free stream and thermal conditions of Example I. However, consider the flow aligned with longer plate length.



Calculate the following using the module:

1. position from the plate's leading edge where the laminar regime ends, x_c;
2. local convection heat transfer coefficients, h_x, and compare with the corresponding results of Example I;
3. surface average convection coefficient over the entire plate, $\overline{h}_L$; and
4. total heat transfer rate from the plate to the fluid, q.

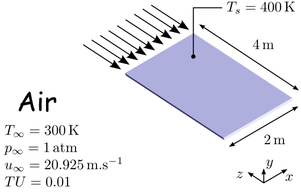
Exploring the Module – Application Examples

Module Application Example II: Module Application

Preliminary Calculations – Methodology

1. Calculation of the **film temperature** (mean boundary layer temperature) – T_f

$$T_f = \frac{T_\infty + T_s}{2} = 350\,\mathrm{K}$$



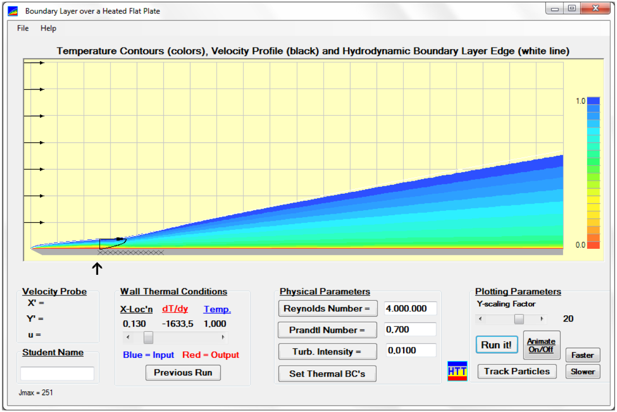
2. Evaluation of **fluid properties** at $T_f = 350\,\mathrm{K}$ and $p = 1\,\mathrm{atm}$
   - $\rho = 0.9950\,\mathrm{kg.m^{-3}}$
   - $\mu = 2.0820 \times 10^{-5}\,\mathrm{N.s.m^{-2}}$
   - $c_p = 1.0090\,\mathrm{kJ.kg^{-1}.K^{-1}}$
   - $k_f = 3.0{\times}10^{-2}\,\mathrm{W.m^{-1}.K^{-1}}$

3. Evaluation of **Reynolds and Prandtl numbers**, Re_L and Pr
   - $Re_L = \frac{\rho u_\infty L}{\mu} \approx \mathbf{4 \times 10^6}$
   - $Pr = \frac{\mu c_p}{k_f} \approx \mathbf{0.70}$

Exploring the Module – Application Examples

Module Application Example II: Module Results

Module Results: Graphical Output Data



Exploring the Module – Application Examples

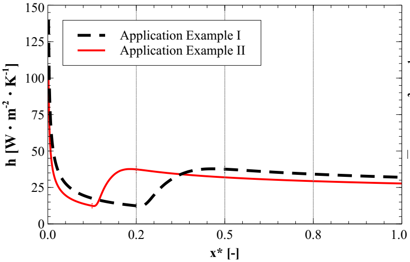
Module Application Example II: Results Analysis (1/2)

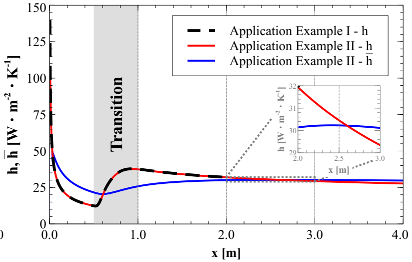
1. position from the plate's leading edge where the laminar regime ends, x_c

$$x_c^* \approx 0.13 \Rightarrow x_c = x_c^* L = 0.13 \times 4 \Leftrightarrow \boxed{x_c = 0.52\,\mathrm{m}}$$

2. local convection heat transfer coefficients, h_x

The procedure to evaluate the local convection heat transfer coefficients is described in Slide 33.





Exploring the Module – Application Examples

Module Application Example II: Results Analysis (2/2)

3. surface average convection coefficient over the entire plate, $\overline{h}_L$

$$\overline{h}_L = \frac{1}{L}\int_0^L h_x(x)\,dx \Rightarrow \overline{h}_L = \int_0^1 h_x(x^*)\,dx^* \Leftrightarrow \boxed{\overline{h}_L \approx 29.5\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}}$$

For the current module application example $\overline{h}_L$ is lower than for the Application Example I (29.5 *vs.* $30.3\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}$). This is observed because at a distance of $2\,\mathrm{m}$ from the plate's leading edge the boundary layer is already fully turbulent and in this regime the local convection heat transfer coefficient decreases as the distance increases.

4. total heat transfer rate from the plate to the fluid, q

$$q = A_s.\overline{h}_L.(T_s - T_\infty) = 8.0 \times 29.5 \times (400.0 - 300.0) \Leftrightarrow \boxed{q = 23.60\,\mathrm{kW}}$$

Since $\left(\overline{h}_L\right)_{\mathrm{App.\,Ex.\,I}} > \left(\overline{h}_L\right)_{\mathrm{App.\,Ex.\,II}}$ then $(q)_{\mathrm{App.\,Ex.\,I}} > (q)_{\mathrm{App.\,Ex.\,II}}$

Useful Relations

Dimensionless Numbers

- $Re_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu} = \frac{\text{Inertia Forces}}{\text{Viscous Forces}}$
- $Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k_f} = \frac{\text{Momentum Diffusivity}}{\text{Thermal Diffusivity}}$
- $Nu_x = -x^*. \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} (x^*) = \frac{x^*.L.h_x}{k_f} = \frac{\text{Convection Heat Transfer}}{\text{Conduction Heat Transfer}}$

Dimensionless variables

- $x^* = \frac{x}{L}$
- $y^* = \frac{y}{L}$
- $u^* = \frac{u}{u_\infty}$
- $v^* = \frac{v}{u_\infty}$
- $T^*(x^*, y^*) = \frac{T(x^*, y^*) - T_\infty}{T_s - T_\infty}$

- Thermal boundary condition for an isothermal surface – $T^* = 1$
- Velocity boundary layer thickness (δ) – distance from the wall ($y-$direction) at which $u(x, \delta)/u_\infty = 0.99$
- Thermal boundary layer thickness (δ_t) – distance from the wall ($y-$direction) at which $[T(x, \delta_t) - T_s]/(T_\infty - T_s) = 0.99$