Heat Transfer

Computational Laboratories

Forced Convection on a Flat Plate

(Laboratory III)

Convective heat transfer over a flat plate (external flow) with a parallel flow under steady, incompressible and constant-property fluid conditions

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Convection Heat Transfer - Introduction

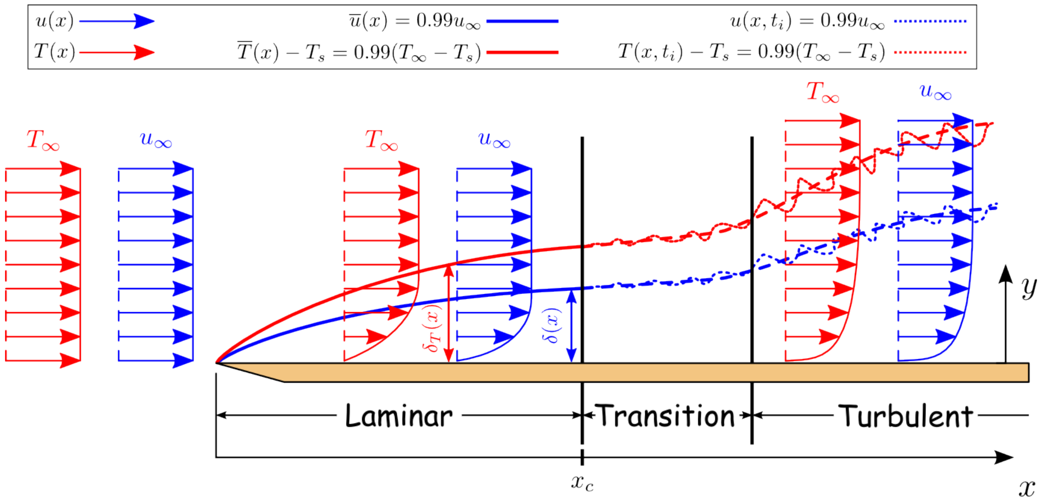
- Convection heat transfer between a fluid and a surface with different temperatures requires simultaneously:
  - energy transfer by bulk fluid motion over the surface (**advection**)
  - energy transfer by microscopic random motion of fluid molecules (**conduction**)
- Convection heat transfer occurs along a fluid region over a surface (thermal boundary layer) where exists temperature gradients (mainly along the surface normal direction), *i.e.*, heat fluxes.

Problem of convection heat transfer

Determination of local or average convection heat transfer coefficients h_x or $\overline{h}$, respectively. This is performed by determining (experimentally or theoretically) local or average Nusselt (Nu_x, $\overline{Nu}$) correlations suitable for a broad range of conditions. **The theoretical fashion requires the solution of the appropriate form of the boundary layer equations.**

Velocity and Thermal Boundary Layers - Convective Momentum and Heat Transfer (1/2)

External flow configuration
Parallel flow over an isothermal flat plate ($T_\infty > T_s$)



Velocity and Thermal Boundary Layers - Convective Momentum and Heat Transfer (2/2)

- The velocity boundary layer starts growing from the beginning of the plate ($x = 0$) - no slip condition;
- The transition to the turbulent boundary layer occurs when small disturbances (perturbations) are no longer damped by viscous forces and are amplified;
  - A critical (transition) Reynolds number ($Re_{x,c}$) is defined based on the flow configuration, surface geometry, free stream turbulence level and on surface roughness.
- The thermal boundary layer develops from a position above the plate where the surface temperature (T_s) differs from the free stream temperature (T_∞) - not necessarily from the leading edge of the plate ($x = 0$).
- The temperature at the surface of the plate may differ of the free stream value as a consequence of an imposed surface temperature or an imposed surface heat flux.

Boundary Layer (BL) Governing Equations

Governing equations in cartesian coordinates for the fluid and heat flow in the BLs

1. Overall Continuity Equation - Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2. x-Momentum Equation - Conservation of Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + (\nu + \varepsilon_M)\frac{\partial^2 u}{\partial y^2}$$

3. Energy Equation - Conservation of Energy

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (\alpha + \varepsilon_H)\frac{\partial^2 T}{\partial y^2} + \frac{\nu + \varepsilon_M}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

- $\vec{V} = (u, v)$ where $u = f(x, y)$ and $v = f(x, y)$.
- $T = f(x, y)$
- ε_M, ε_H - Eddy diffusivities for momentum and heat, respectively.

Assumptions: (1) steady-state conds.; (2) 2D flow; (3) incomp. flow; (4) fluid with constant properties; (5) negligible body forces; (6) no thermal energy generation; (7) $\partial^2 u/\partial y^2 >> \partial^2 u/\partial x^2$; (8) $\partial^2 T/\partial y^2 >> \partial^2 T/\partial x^2$.

Boundary Layer Equations - Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

Simplifications:

- $\frac{dp_\infty}{dx} = 0 \Rightarrow$ since $u_\infty = \mathrm{Cnt}$
- $\left(\frac{\partial u}{\partial y}\right)^2 \simeq 0 \Rightarrow$ for low-speed flows

1. Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2. Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (\nu + \varepsilon_M)\frac{\partial^2 u}{\partial y^2}$$

3. Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (\alpha + \varepsilon_H)\frac{\partial^2 T}{\partial y^2}$$

Boundary Conditions

- Continuity and Momentum Equations
  1. $u(0, y) = u_\infty$
  2. $u(x, 0) = 0$
  3. $v(x, 0) = 0$
  4. $u(x, \infty) = u_\infty$
- Energy Equation
  1. $T(0, y) = T_\infty$
  2. $T(x, 0) = T_s$
  3. $T(x, \infty) = T_\infty$

x - any x position along the plate ($x > 0$)

Boundary Layers Parameters - Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

- For a fluid with constant properties (ρ, μ, c_p, k) the velocity field, $\vec{v}$ is evaluated exclusively with the overall continuity and momentum equations (uncoupled from the energy equation);
- The temperature along the thermal boundary layer is evaluated with the solution for $\vec{v}$ through the energy equation.

Velocity Boundary Layer

$$u = f\left(x, y, u_\infty, \rho, \mu\right)^\dagger$$

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$\tau_s = f\left(x, u_\infty, \rho, \mu\right)^\dagger$$

τ_s - surface shear stress

† Laminar flow

Thermal Boundary Layer

$$T = f\left(x, y, u_\infty, T_\infty, T_s, \rho, \mu, c_p, k_f\right)^\dagger$$

$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad q_s'' = h\left(T_s - T_\infty\right)$$

$$h = \frac{k_f}{\left(T_\infty - T_s\right)} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$h = f\left(x, u_\infty, T_\infty, T_s, \rho, \mu, c_p, k_f\right)^\dagger$$

Normalized BL Equations - Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L}$$

$$u^* = \frac{u}{u_\infty} \qquad v^* = \frac{v}{u_\infty} \qquad T^* = \frac{T - T_s}{T_\infty - T_s}$$

1. Continuity Equation

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

2. Momentum Equation

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L}\left(1 + \frac{\varepsilon_M}{\nu}\right)\frac{\partial^2 u^*}{\partial y^{*2}}$$

3. Energy Equation

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\left(1 + \frac{\varepsilon_H}{\alpha}\right)\frac{\partial^2 T^*}{\partial y^{*2}}$$

Boundary Conditions

- Continuity and Momentum Equations
  1. $u^*\left(0, y^*\right) = 1$
  2. $u^*\left(x^*, 0\right) = 0$
  3. $v^*\left(x^*, 0\right) = 0$
  4. $u^*\left(x^*, \infty\right) = 1$
- Energy Equation
  1. $T^*\left(0, y^*\right) = 1$
  2. $T^*\left(x^*, 0\right) = 0$
  3. $T^*\left(x^*, \infty\right) = 1$

  x^* - any x^* position along the plate ($x^* > 0$)

Normalized BL Parameters - Parallel Flow with Constant u_∞ and T_∞ over an Isothermal Flat Plate

$$x^* = \frac{x}{L} \quad y^* = \frac{y}{L} \quad u^* = \frac{u}{u_\infty} \quad v^* = \frac{v}{u_\infty} \quad T^*(x^*, y^*) = \frac{T(x^*, y^*) - T_s}{T_\infty - T_s}$$

Velocity Boundary Layer

$$u^* = f(x^*, y^*, Re_L)^\dagger$$

$$\tau_{s,L} = \left(\frac{\mu u_\infty}{L}\right) \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0}$$

$$C_{f,L} = \frac{\tau_s}{\frac{1}{2}\rho u_\infty^2} = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0}$$

$$C_f = f(x^*, Re_L)^\dagger$$

C_f - friction coefficient

Thermal Boundary Layer

$$T^* = f(x^*, y^*, Re_L, Pr)^\dagger$$

$$h = \left(\frac{k_f}{L}\right) \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

$$Nu_x = \frac{hx}{k_f} = x^* \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

$$Nu = f(x^*, Re_L, Pr)^\dagger$$

Nu - Nusselt number

† Laminar flow

Solution for Boundary Layer Equations

Solution for Laminar Boundary Layer Equations

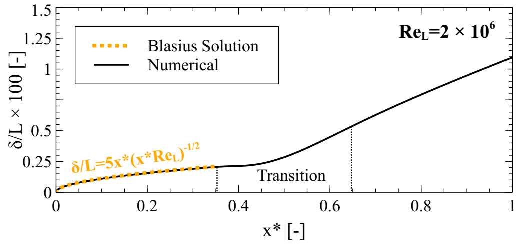
- Blasius Solution (Similarity Solution) - (Blasius - 1908):
  - For a parallel flow of constant u_∞ and T_∞ over an isothermal flat plate under steady, incompressible, constant-property fluid conditions and neglecting viscous dissipation;
  - The solution procedure converts the continuity and momentum equations in a third-order ordinary differential equation with a solution performed with numerical integration techniques or through power-series expansion.
- Integral method (Approximate Analysis) - (von Kármán - 1921):
  - The procedure assume approximate expressions for the velocity and thermal profiles in the integral form of the momentum and energy equations.
- Numerical Methods

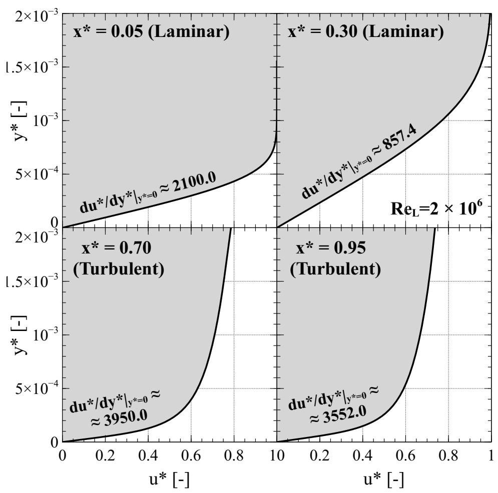
Solution for Turbulent Boundary Layer Equations

- Integral methods
- Numerical (differential) methods with closure models (for unknown quantities)

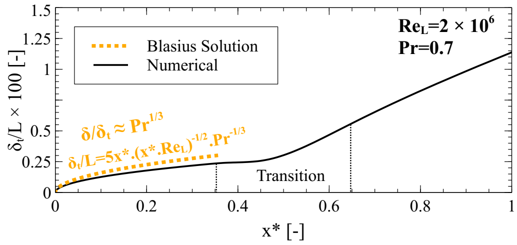
Parallel Flow over an Isothermal Plate

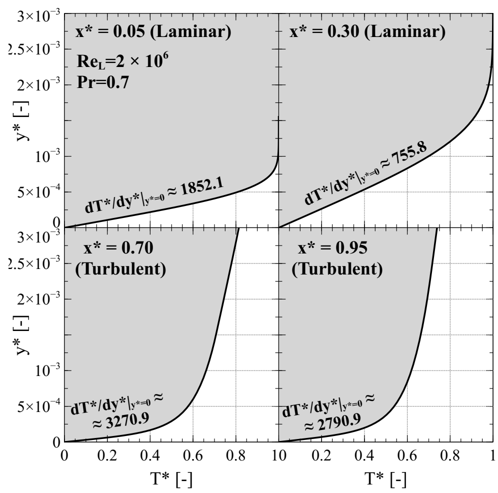
Velocity Boundary Layer





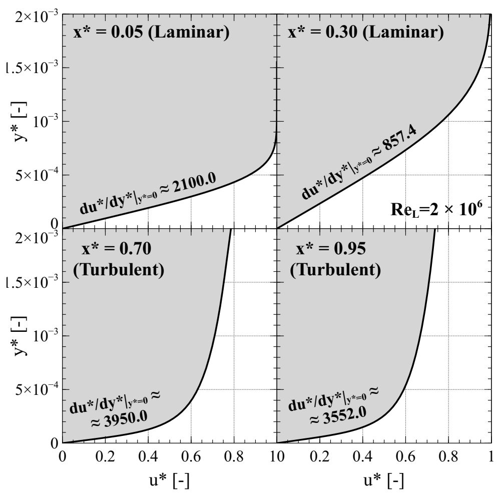
Thermal Boundary Layer

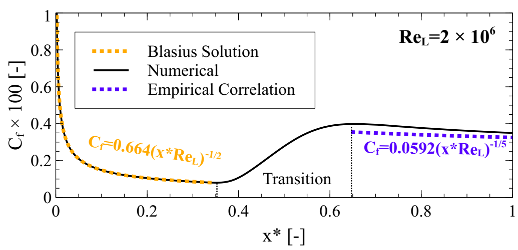




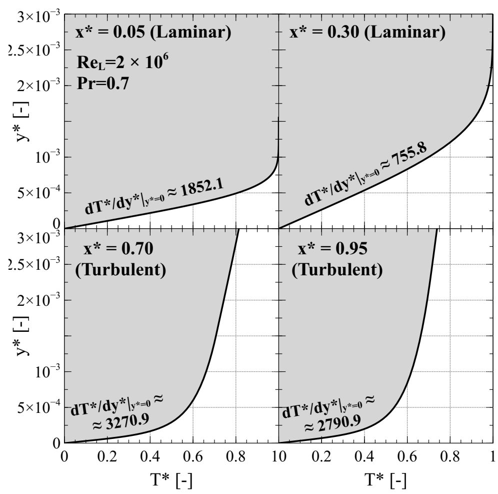
Parallel Flow over an Isothermal Plate

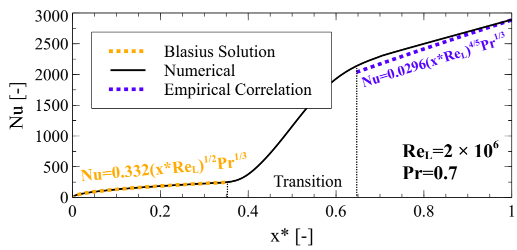
Velocity Boundary Layer





Thermal Boundary Layer





Parallel Flow over an Isothermal Plate

Consider a parallel flow of atmospheric air over an isothermal flat plate with a free stream velocity and temperature equal to $41.85\,\mathrm{m.s^{-1}}$ (u_∞) and $300\,\mathrm{K}$ (T_∞), respectively. The plate is $1\,\mathrm{m}$ (L) long and has a uniform temperature of $400\,\mathrm{K}$ (T_s).

Determine the local surface shear stress (τ_s) and local convection heat transfer coefficient (h) along the plate.

1. Calculation of the film temperature (mean boundary layer temperature) - T_f

$$T_f = \frac{T_\infty + T_s}{2} = 350\,\mathrm{K}$$

2. Evaluation of fluid properties at $T_f = 350\,\mathrm{K}$ and $p = 1\,\mathrm{atm}$

Hydrodynamic Properties	Thermal Properties
$\rho = 0.9950\,\mathrm{kg.m^{-3}}$	$c_p = 1.0090\,\mathrm{kJ.kg^{-1}.K^{-1}}$
$\mu = 2.0820 \times 10^{-5}\,\mathrm{N.s.m^{-2}}$	$k_f = 3.0 \times 10^{-2}\,\mathrm{W.m^{-1}.K^{-1}}$

3. Evaluation of Reynolds and Prandtl numbers, Re_L and Pr

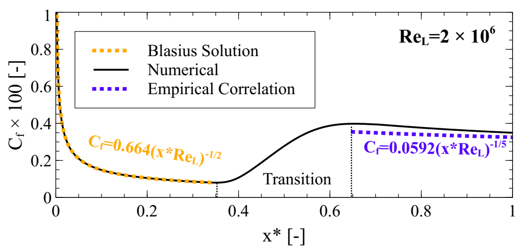
$$Re_L = \frac{\rho u_\infty L}{\mu} = \mathbf{2 \times 10^6} \qquad Pr = \frac{\mu c_p}{k_f} = \mathbf{0.70}$$

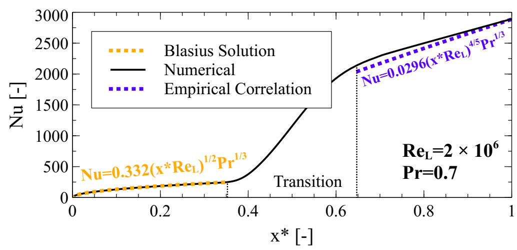
Parallel Flow over an Isothermal Plate

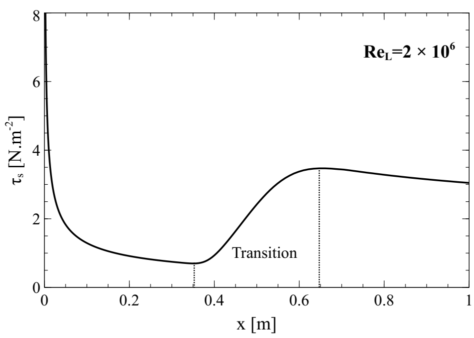
3. Evaluation of Reynolds and Prandtl numbers, Re_L and Pr

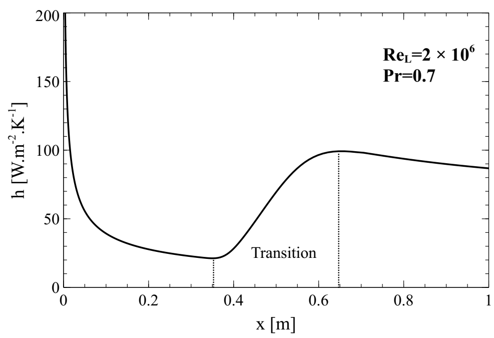
$$Re_L = \frac{\rho u_\infty L}{\mu} = \mathbf{2 \times 10^6} \qquad Pr = \frac{\mu c_p}{k_f} = \mathbf{0.70}$$

4. Calculation of τ_s and h based on local values of C_f and Nu



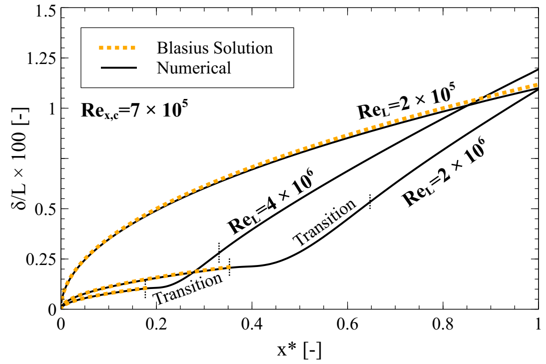


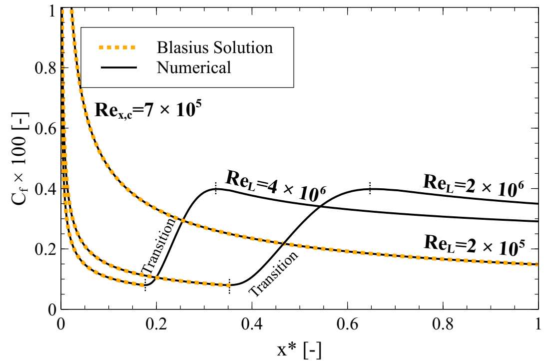




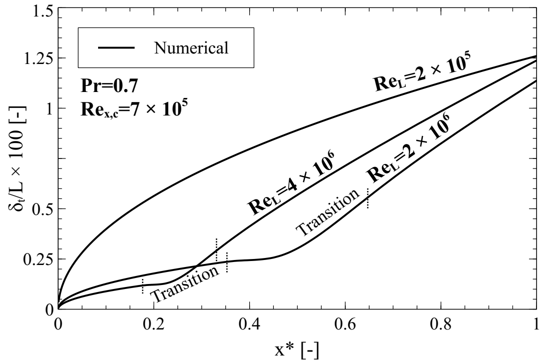
Parallel Flow over an Isothermal Plate: Effect of Re_L

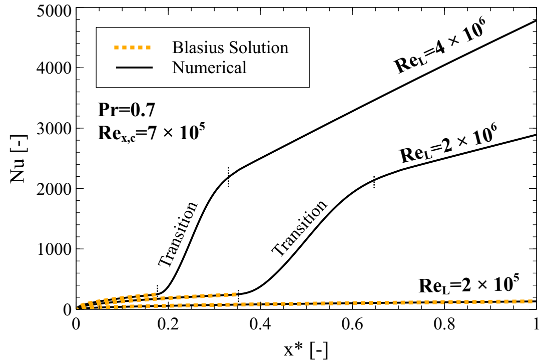
Velocity Boundary Layer





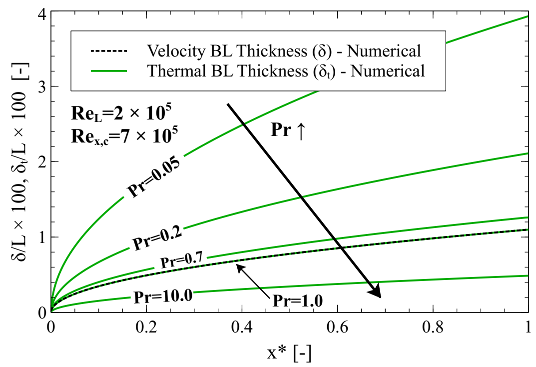
Thermal Boundary Layer

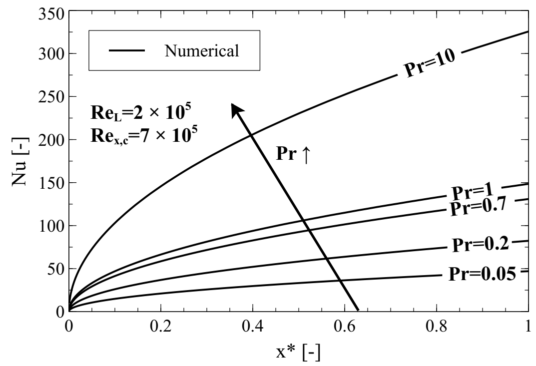




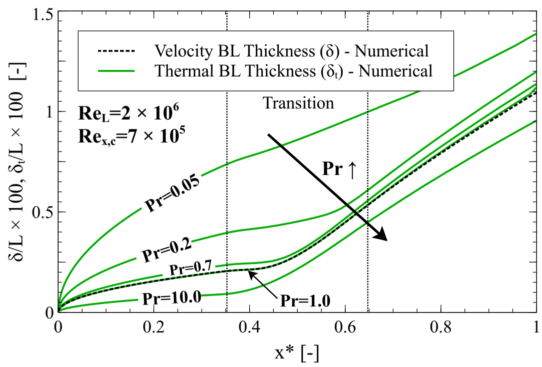
Parallel Flow over an Isothermal Plate: Effect of Pr

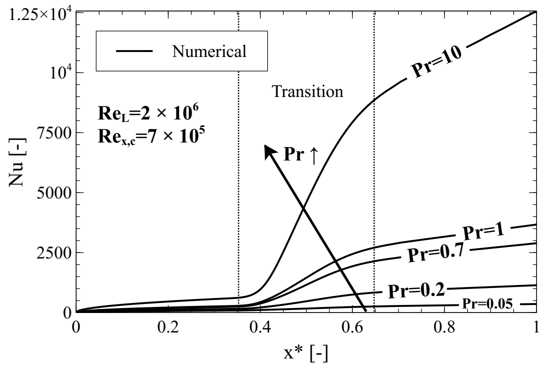
$Re_L = 2 \times 10^5$ ($< Re_{x,c}$)





$Re_L = 2 \times 10^6$ ($> Re_{x,c}$)





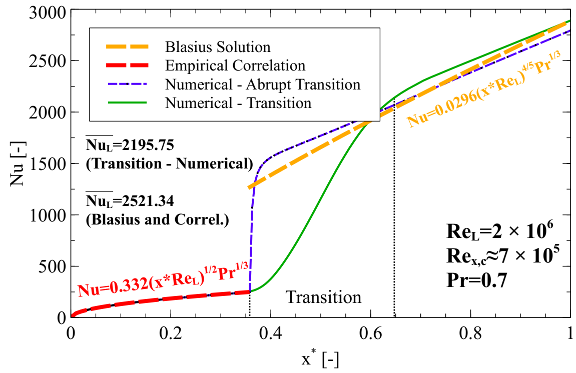
Transition to Turbulent BL and Average BL Parameters

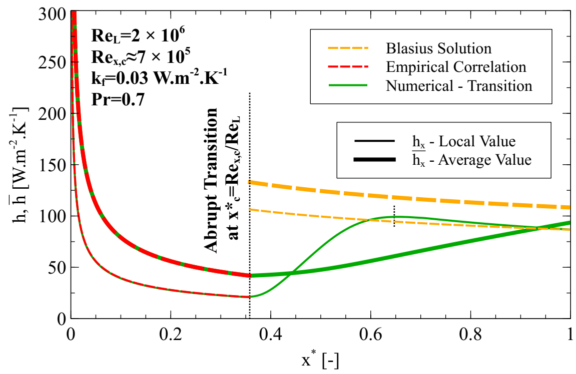
Transition

- An abrupt transition is commonly considered and consequently the transition region is considered the initial part of the turbulent region.
  - As a result higher local and average values for Nu (C_f) and consequently for h (τ_s) are predicted.

Average Thermal BL Parameters

$$\overline{h_x} = \frac{1}{x}\int_0^x h_x\,dx = \frac{k_f}{x}\overline{Nu_x} = \\ = \frac{k_f}{x}\int_0^x \frac{Nu_x}{x}\,dx$$





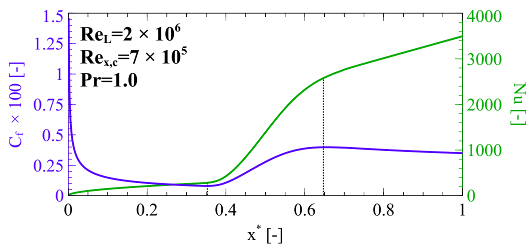
Momentum and Heat Transfer Analogies

Reynolds Analogy

For $\mathbf{dp^*/dx^* = 0}$ (flat plate with constant u_∞), $\mathbf{Pr = 1}$ and $\epsilon_M = \epsilon_H$, *i.e.*, $\mathbf{Pr_t = 1}$ (for turbulent flow) the momentum and energy equations as well as its boundary conditions are mathematically analogous and the solutions for u^* and T^* are equal. Therefore,

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \Rightarrow$$

$$\Rightarrow C_{f,x}\frac{Re_x}{2} = Nu_x$$



Chilton-Colburn Analogy

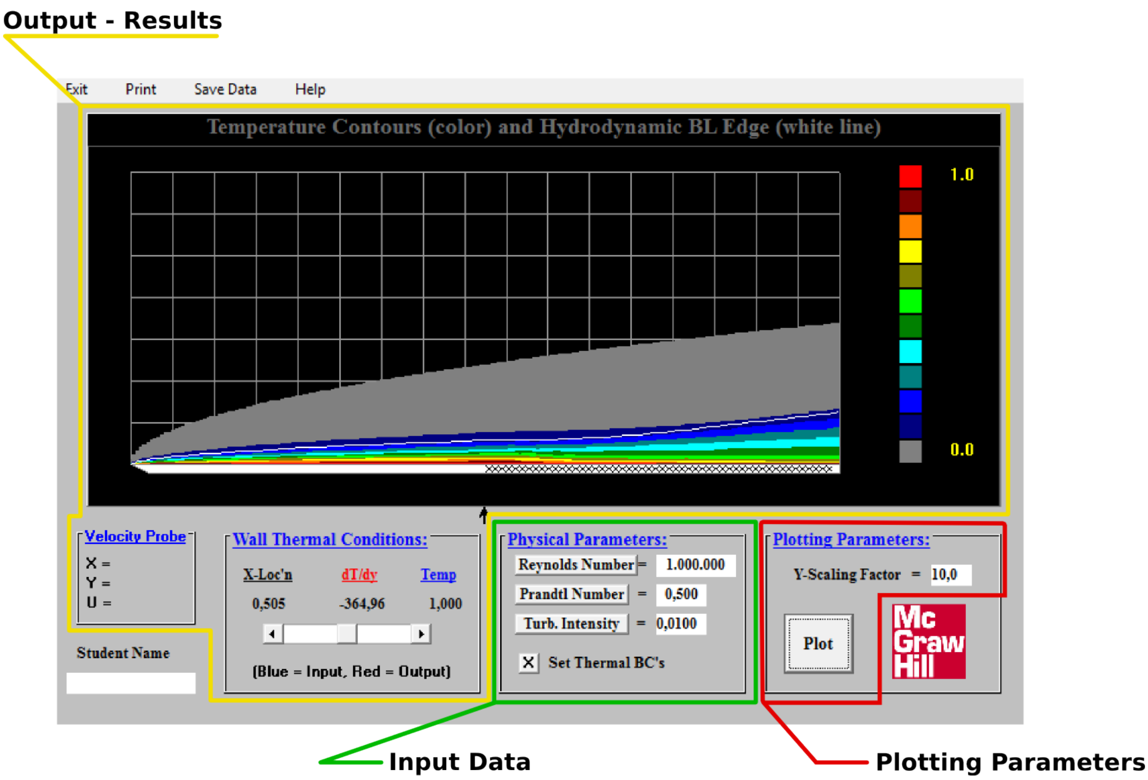
For $\mathbf{dp^*/dx^* = 0}$ the Reynolds analogy can be extended for $\mathbf{0.6 < Pr < 60}$ considering a Prandtl number correction as follows:

$$C_{f,x}\frac{Re_x}{2} = Nu_x Pr^{-1/3}$$

Final Remarks

- Increasing the plate Reynolds number (Re_L) the transition region starts closer to the leading edge $x_c = \frac{Re_{x,c}}{Re_L}$;
- For a Prandtl number equal to 1.0 (equal diffusivities for momentum and heat) the velocity and temperature profiles are similar and the velocity and thermal BLs grow along the plate at the same rate, *i.e.*, $\delta = \delta_t$;
  - For $Pr > 1$ the momentum diffusivity is higher than the thermal diffusivity and therefore $\delta > \delta_t$ (oils);
  - For $Pr < 1$ the thermal diffusivity is higher than the momentum diffusivity and therefore $\delta_t > \delta$ (liquid metals).

Exploring the Software - The Interface



Exploring the Software - Boundary conditions

- The program computes velocity and thermal boundary layer parameters in the non-dimensional form:

$$x^* = \frac{x}{L} \quad y^* = \frac{y}{L} \quad u^* = \frac{u}{u_\infty} \quad v^* = \frac{v}{u_\infty} \quad T^*(x^*, y^*) = \frac{T(x^*, y^*) - T_\infty}{T_s - T_\infty}$$

Boundary Conditions:

Continuity and Momentum Equations

1. At $x^* = 0$ (Upstream)
   - $u^*(0, y^*) = 1$
2. At $y^* = \infty$
   - $u^*(x^*, \infty) = 1$
3. At $y^* = 0$
   - $u^*(x^*, 0) = 0$
   - $v^*(x^*, 0) = 0$

Energy Equation

1. At $x^* = 0$ (Upstream the leading edge)
   - $T^*(0, y^*) = 0$
2. At $y^* = \infty$
   - $T^*(x^*, \infty) = 0$
3. At $y^* = 0$ **(User Input)**
   - $T^*(x^*, 0) = C_1$ or $C_1 + C_2 x^*$

- $C_1 = 1.0$ and $C_2 = 0.0$ (isothermal plate)

Exploring the Software - Laminar-Turbulent Transition

- The program considers the turbulence intensity of the free stream (TU) to evaluate the critical Reynolds number ($Re_{x,c}$) through the equation that follows:

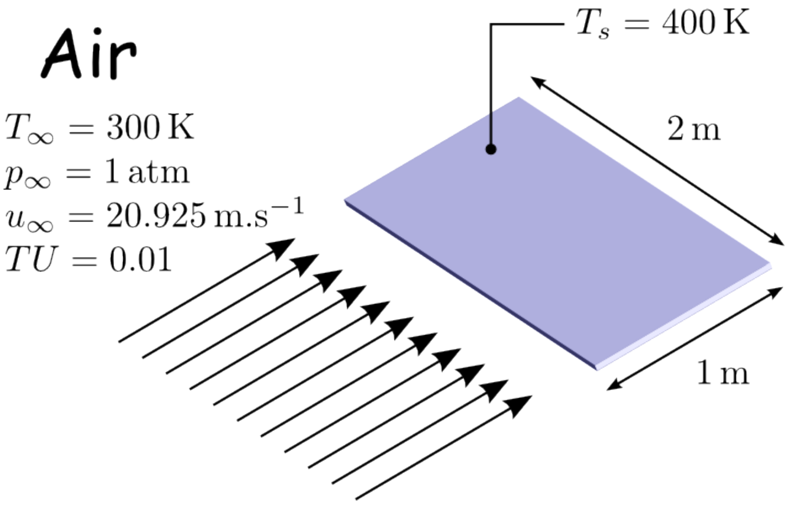
$$Re_{x,c}^{1/2} = \frac{-1.0 + \left(132500\, TU^2\right)^{1/2}}{39.2\, TU^2}$$

  - A critical Reynolds number ($Re_{x,c}$) of about 5×10^5 is obtained for a free stream turbulence level of 1% .

- The $Re_{x,c}$ is employed to determine the distance from the leading edge of the plate where the flow transition from laminar to turbulent starts (beginning of the transition region).

Exploring the Software - Example (1/6)

Consider a $1\,\mathrm{m} \times 2\,\mathrm{m}$ isothermal flat plate at $400\,\mathrm{K}$ subjected to air flow parallel to its surfaces along its $1\,\mathrm{m}$ long side. The free stream temperature (T_∞), pressure (p_∞), velocity (u_∞) and turbulence level (TU) are equal to, $300\,\mathrm{K}$, $1\,\mathrm{atm}$, $20.925\,\mathrm{m.s^{-1}}$ and 1%, respectively.



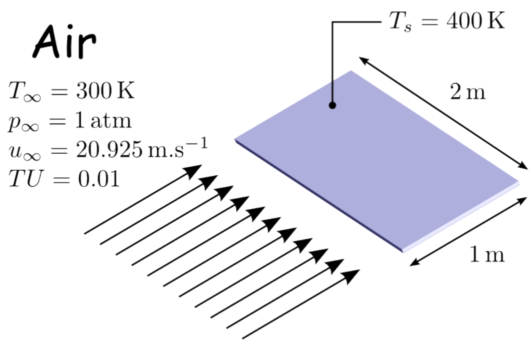
Exploring the Software - Example (2/6)

Methodology:

1. Calculation of the **film temperature** (mean boundary layer temperature) - T_f

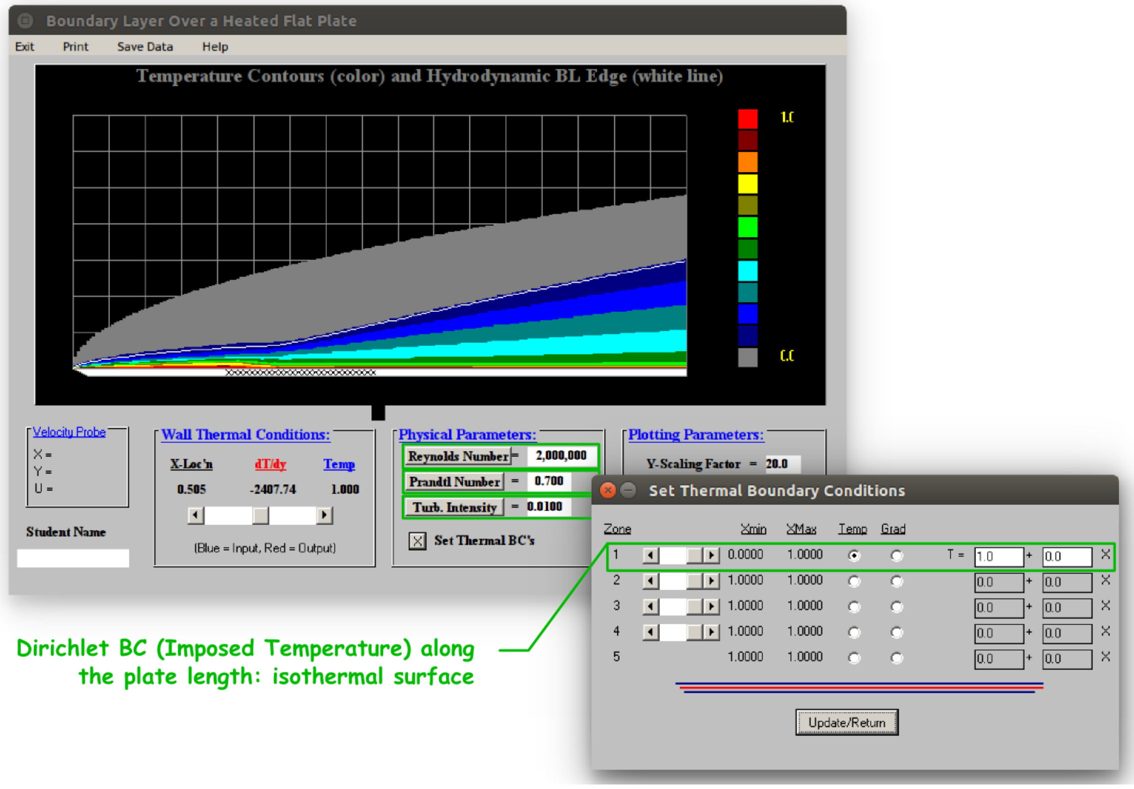
$$T_f = \frac{T_\infty + T_s}{2} = 350\,\mathrm{K}$$

2. Evaluation of **fluid properties** at $T_f = 350\,\mathrm{K}$ and $p = 1\,\mathrm{atm}$
   - $\rho = 0.9950\,\mathrm{kg.m^{-3}}$
   - $\mu = 2.0820 \times 10^{-5}\,\mathrm{N.s.m^{-2}}$
   - $c_p = 1.0090\,\mathrm{kJ.kg^{-1}.K^{-1}}$
   - $k_f = 3.0\times10^{-2}\,\mathrm{W.m^{-1}.K^{-1}}$

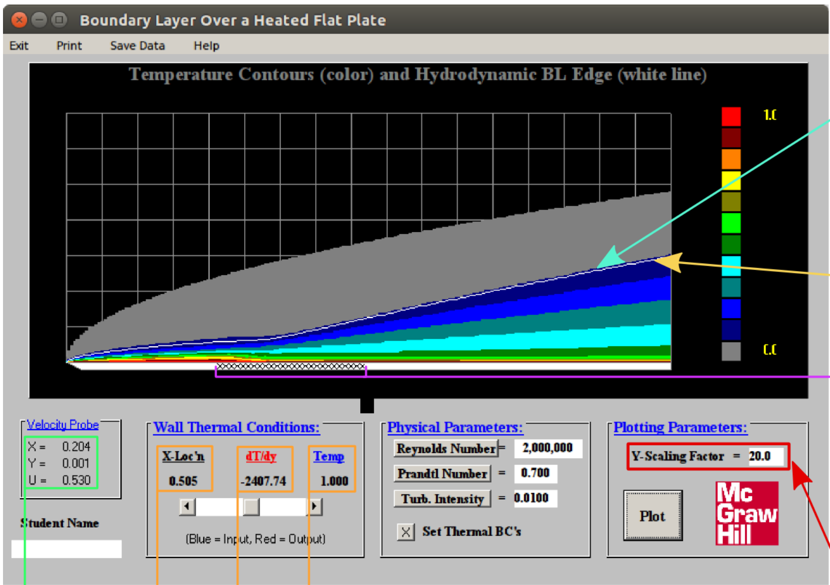


3. Evaluation of **Reynolds and Prandtl numbers**, Re_L and Pr
   - $Re_L = \frac{\rho u_\infty L}{\mu} = \mathbf{2 \times 10^6}$
   - $Pr = \frac{\mu c_p}{k_f} = \mathbf{0.70}$

Exploring the Software - Example (3/6)



Exploring the Software - Example (4/6)



Top border of the blue region: Thermal BL Edge

White Line: Velocity BL Edge

Transition Region

$x_c^* \approx 0.26$

$Re_{x,c} = 5.2 \times 10^5$

Plot enlargement factor in the y direction

$T^* (x^* = 0.505, y^* = 0) = 1.0$ (Specified Thermal BC)

$$\frac{\partial T^*}{\partial y^*} (x^* = 0.505)\bigg|_{y^*=0} = -2407.74$$

$x^* (= x/L) = 0.505$

$u^* (x^* = 0.204, y^* = 0.001) = 0.530$

Exploring the Software - Example (5/6)

1. Calculate the local Nusselt values (Nu_x) and convection heat transfer coefficients (h_x) at $x = 0.25\,\mathrm{m}$, $x = 0.35\,\mathrm{m}$ and $x = 0.55\,\mathrm{m}$ from the leading edge of the plate.

Methodology

1. Calculate $x^* \left(= \frac{x}{L}\right)$
2. Determine $\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$ at x^*
3. Calculate $Nu_x^\dagger$. Recall that:

$$Nu_x = -x^* . \left.\frac{\partial T^*}{\partial y^*}(x^*)\right|_{y^*=0}$$

4. Calculate h_x

$$h_x = \frac{Nu_x . k_f}{x}$$

Results

x [m]	0.25	0.35	0.55
x^* [−]	0.25	0.35	0.55
$\left.\frac{\partial T^*}{\partial y^*}\right\|_{y^*=0}$ [−]	-827.91	-1973.10	-2364.21
Nu_x [−]	**206.98**	**690.59**	**1300.32**
h_x [$\mathrm{W.m^{-2}.K^{-1}}$]	**24.84**	**59.19**	**70.93**

†The minus sign in the Nu_x equation is due to the normalization procedure for the temperature (T^*) considered in the program:

$$T^*(x^*, y^*) = \frac{T(x^*, y^*) - T_\infty}{T_s - T_\infty}$$

Exploring the Software - Example (6/6)

2. Estimate the average convection heat transfer coefficient ($\overline{h}$) over the entire length of the plate.

$$\overline{h} = \frac{1}{L}\int_0^L h_x(x)\,dx \Rightarrow \overline{h} = \int_0^1 h_x(x^*)\,dx^*$$

Where, $h_x(x^*) = -\frac{k_f}{L}\left.\frac{\partial T^*}{\partial y^*}(x^*)\right|_{y^*=0}$

Performing the integration: $\overline{h} \approx 58.2\,\mathrm{W.m^{-2}.K^{-1}}$.

3. Estimate the total heat transfer rate (q) from the plate to the air to maintain the surface of the plate at the constant temperature of $400\,\mathrm{K}$.

$$q = \overline{h}.A_s.(T_s - T_\infty) = 58.2 \times 2.0 \times (400.0 - 300.0) = 11.66\,\mathrm{kW}$$

Useful Relations

Dimensionless Numbers

- $Re_L = \frac{\rho u_\infty L}{\mu} = \frac{\text{Inertia Forces}}{\text{Viscous Forces}}$
- $Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k_f} = \frac{\text{Momentum Diffusivity}}{\text{Thermal Diffusivity}}$
- $Nu_x = -x^*. \frac{\partial T^*}{\partial y^*}(x^*)\Big|_{y^*=0} = \frac{x^*.L.h_x}{k_f} = \frac{\text{Convection Heat Transfer}}{\text{Conduction Heat Transfer}}$

Dimensionless variables

- $x^* = \frac{x}{L}$
- $y^* = \frac{y}{L}$
- $u^* = \frac{u}{u_\infty}$
- $v^* = \frac{v}{u_\infty}$
- $T^*(x^*, y^*) = \frac{T(x^*,y^*)-T_\infty}{T_s - T_\infty}$

- Thermal BC for an Isothermal Surface: $T^* = 1$
- Velocity BL Thickness (δ) - distance from the wall (y-direction) at which $u(x,\delta)/u_\infty = 0.99$
- Thermal BL Thickness (δ_t) - distance from the wall (y-direction) at which $[T(x,\delta_t) - T_s]/(T_\infty - T_s) = 0.99$