HEAT TRANSFER
EXTRAORDINARY EXAM
(September 4, 2021)

PROBLEM 2 (5 Pts)

Consider a horizontal tube with a negligible thickness and with length (L) and diameter (D) equal to $2.0\,\mathrm{m}$ and $2.0\,\mathrm{cm}$, respectively. A mass flow rate ($\dot{m}$) equal to $0.5\,\mathrm{kg\,s^{-1}}$ of water circulates within the tube which enters the tube at the temperature of $80℃$ ($T_{m,i}$). The tube is completely immersed in a tank with quiescent water at the temperature of $20℃$ (T_∞). Estimate the thermal power loss by the water stream that circulates in the tube to the water in the tank (q) and the water tube outlet temperature ($T_{m,o}$). Assume the tube surface temperature (T_s) constant along the entire tube length. Neglect radiative heat transfer and consider steady-state conditions. Consider the following thermophysical properties for the water:

$$\rho = 982\,\mathrm{kg\,m^{-3}};\ \ c_p = 4186\,\mathrm{J\,kg^{-1}\,K^{-1}};\ \ \mu = 4.54\times10^{-4}\,\mathrm{N\,s\,m^{-2}};$$
$$k = 0.656\,\mathrm{W\,m^{-1}\,K^{-1}};\ \ Pr = 2.89;\ \text{and}\ \beta = 5.34\times10^{-4}\,\mathrm{K^{-1}}.$$

Solution Procedure:

1. Calculate Re_D

$$Re_D = \frac{4\dot{m}}{\pi D\mu} \Leftrightarrow Re_D = \frac{4\times0.5}{\pi\times0.02\times4.54\times10^{-4}} \Leftrightarrow Re_D \approx 70112.310\ \Rightarrow \mathrm{Turb.Flw}$$

2. Apply a suitable Nusselt number correlation and compute the average internal convection heat transfer coefficient, $\bar{h}_i$

$$\overline{Nu}_D \equiv \frac{\bar{h}_i D}{k} \approx Nu_{D,\mathrm{fd}} = 0.023 Re_D^{4/5} Pr^{0.3} \Leftrightarrow \bar{h}_i \approx 7807.400\,\mathrm{W\,m^{-2}\,K^{-1}}$$

3. Estimate the water mean outlet temperature, $T_{m,o}$ – Note that $T_\infty < T_s < T_{m,o} < T_{m,i}$

4. For a particular tube surface temperature (T_s), calculate the heat transfer rate from the water to the tube wall (q_i) and ensuring that T_s and q_i is consistent with the heat transfer rate from the tube to the water in the tank (q_e) – note that at steady-state conditions $q_i = q_e$ and the tube

surface temperature (T_s) must be the same for internal and external convection heat transfer calculations.

a. Estimate the tube surface temperature, T_s – Note that $T_\infty < T_s < T_{m,o} < T_{m,i}$

b. Calculate the log mean temperature difference, ΔT_{lm}

$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}$$

where, $\Delta T_o = T_s - T_{m,o}$ and $\Delta T_i = T_s - T_{m,i}$

c. Calculate the heat transfer rate, q_i

$$q_i = \pi D L \bar{h}_i \Delta T_{\mathrm{lm}}$$

d. Calculate $\bar{h}_e$ considering the estimated tube surface temperature (T_s) at 4.a

i. Calculate film temperature

$$T_f = \frac{(T_s + T_\infty)}{2}$$

ii. Calculate Ra_D

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

iii. Calculate $\bar{h}_e$ applying a suitable correlation for a horizontal cylinder (tube), such as the Churchill and Chu correlation.

$$\overline{Nu}_D \equiv \frac{\bar{h}_e D}{k} = \left\{0.60 + \frac{0.387 {Ra_D}^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$$

e. Calculate a new estimate to T_s (${T_s}^{\mathrm{new}}$) considering the (internal) heat transfer rate (q_i) and the average convection heat transfer coefficient at the tube outer surface ($\bar{h}_e$)

$${T_s}^{\mathrm{new}} = \frac{-q_i}{\pi D L \bar{h}_e} + T_\infty$$

f. Check if T_s assumed at 4.a allows a consistent calculation of q_i and q_e – the same value for T_s must lead to equal q_i and q_e values. If the value calculated at 4.e is significantly different than the assumed previously, a new iteration is required. The iteration procedure ends when $T_s^{\text{new}} \approx T_s^{\text{old}}$.

5. Check the consistency of q (evaluated at 4.c) with the energy balance applied to the internal water flow. Since $T_{m,o}$ was estimated (see Step 3.), the energy balance applying this value must agree with the heat transfer rate computed at 4.c.

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \Rightarrow T_{m,o}{}^{\text{new}} = \frac{q}{\dot{m}c_p} + T_{m,i}$$

If $T_{m,o}{}^{\text{new}}$ is significantly different than the value considered at 3. ($T_{m,o}{}^{\text{old}}$), the procedure (Steps 3. and 4.) should be repeated. The final values for q and $T_{m,o}$ are obtained when $T_{m,o}$ does not vary significantly with the iteration number.

The converged solution for q should verify the following:

$$q \equiv \pi D L \bar{h}_i \Delta T_{\text{lm}} = \pi D L \bar{h}_e (T_\infty - T_s) = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

In the equation above, ΔT_{lm} and $\bar{h}_e$ are calculated with the same surface temperature value and ΔT_{lm} and $\left(T_{m,o} - T_{m,i}\right)$ with the same water outlet temperature.

Although a fully converged solution may require several inner and outer iterations, the procedure is always the same.

For a very small (tight) stopping criterion for the iteration cycles, the following solution is obtained:

$$q \approx 8962.057\ \text{W},\quad T_{m,o} \approx 75.781\text{°C},\quad \text{and}\quad T_s \approx 68.558\text{°C}$$

SUPPLEMENTARY MATERIAL:

In the following a script is presented to solve (very accurately) the problem. As it is the script considers the thermophysical properties of water dependent of the corresponding temperatures – saturated (liquid) water thermophysical properties at atmospheric pressure gathered from the textbook: "Fundamentals of Heat and Mass Transfer", 7th Edition. (According to the problem statement the water thermophysical properties were considered as constants.)

After the script, the corresponding script results are presented. Finally, a brief parametric study on the influence of the water mass flow rate in the tube is presented.

Problem 2 – Detailed Solution

GNU OCTAVE (MATLAB) SCRIPT

```
clear all
clc
####################THERMOPHYSICAL PROPERTIES - INCROPERA#####################
Temp=[290 295 300 305 310 315 320 325 330 335 340 345 350 355 360 365 370 375];
DynVisc=[0.00108 9.59E-04 8.55E-04 7.69E-04 6.95E-04 6.31E-04 5.77E-04 ...
5.28E-04 4.89E-04 4.53E-04 4.20E-04 3.89E-04 3.65E-04 3.43E-04 3.24E-04 ...
3.06E-04 2.89E-04 2.74E-04];
ThermCond=[5.98E-01 6.06E-01 6.13E-01 6.20E-01 6.28E-01 6.34E-01 6.40E-01 ...
6.45E-01 6.50E-01 6.56E-01 6.60E-01 6.64E-01 6.68E-01 6.71E-01 6.74E-01 ...
6.77E-01 6.79E-01 6.81E-01];
SpecHeat=[4184 4181 4179 4178 4178 4179 4180 4182 4184 4186 4188 4191 4195 ...
4199 4203 4209 4214 4220];
Pr=[7.56 6.62 5.83 5.2 4.62 4.16 3.77 3.42 3.15 2.88 2.66 2.45 2.29 2.14 ...
2.02 1.91 1.8 1.7];
VolThermExpCoef=[1.740E-04 2.275E-04 2.761E-04 3.206E-04 3.619E-04 4.004E-04 ...
4.367E-04 4.712E-04 5.040E-04 5.355E-04 5.660E-04 5.954E-04 6.242E-04 ...
6.523E-04 6.979E-04 7.071E-04 7.287E-04 7.610E-04];
specVolume=[1.001E-03 1.002E-03 1.003E-03 1.005E-03 1.007E-03 1.009E-03 ...
1.011E-03 1.013E-03 1.016E-03 1.018E-03 1.021E-03 1.024E-03 1.027E-03 ...
1.030E-03 1.034E-03 1.038E-03 1.041E-03 1.045E-03];
################################################################################

################THERMOPHYSICAL PROPERTIES - CONST. SET#########################
%Considerando T=(Tf+Tm)/2=61.658 C (mean temp. calculated for the base case)
%Temp=[290 375];
%DynVisc=[4.5400e-04 4.5400e-04];
%ThermCond=[0.6560 0.6560];
%SpecHeat=[4186.0 4186.0];
%Pr=[2.8900 2.8900];
%VolThermExpCoef=[5.3400e-04 5.3400e-04];
%specVolume=[1.0183e-03 1.0183e-03];
################################################################################


################################INPUT DATA######################################
Leng=2;% Tube length [m]
Diam=0.02;% Tube diameter [m]

MFRT=0.5;% Mass flow rate [kg/s]
inTemp=80;% Inlet water temperature [C]
extWaterTemp=20;% Water temperature in the tank [C]

accelGravity=9.8; % Gravity acceleration [m/(s^2)]
%Note: while varying these parameters please make sure that the underlying
%assumptions (flow regime, Nusselt number correlations, ...) still hold.
################################################################################

#############################INITIAL ESTIMATES#################################
outTemp=70;% Outlet water temperature [C]
surfTemp=60;% Tube surface temperature [C]
################################################################################

############################NUMERICAL PARAMETERS################################
rlxFactSurfTemp=0.1; % Relaxation factor to compute surface temperature
rlxFactOutTemp=0.5;  % Relaxation factor to compute outlet temperature
maxrelErrSurfTemp=1E-10; % Stopping criterion 1
maxrelErrOutTemp=1E-10; % Stopping criterion 2
################################################################################
```

```
%OUTER ITERATION CYLE
outerIt=0;
do
disp(['Outer Iteration # ',num2str(outerIt)])

%Calculate mean water temperature
meanTemp=(inTemp+outTemp)/2;

%Calculate Reynolds number of the internal flow
ReD=4*MFRT/(pi()*Diam*interp1(Temp, DynVisc, meanTemp+273.15));

%Internal average Nusselt number
Nui=0.023*(ReD^(4/5))*(interp1(Temp, Pr, meanTemp+273.15)^(0.3));
%Calculate the average internal convection heat transfer coefficient,
%Fully Turbulent Flow, HTCi
HTCi=(interp1(Temp, ThermCond, meanTemp+273.15)*Nui)/Diam;

%INNER ITERATION CYLE
innerIt=0;
do
disp(['Inner Iteration # ',num2str(innerIt)])

%Calculate log mean temperature difference
LMTD=((surfTemp-outTemp)-(surfTemp-inTemp))/...
(log((surfTemp-outTemp)/(surfTemp-inTemp)));

%Calculate the heat transfer rate from the water flow in the tube to the tube
%surface, HTRi
HTRi=pi()*Diam*Leng*HTCi*LMTD;


%Calculate the average external convection heat transfer coefficient,
%natural convection over a long horizontal cylinder, HTCe
FilmTemp=(surfTemp+extWaterTemp)/2;

RaD=(accelGravity*interp1(Temp, VolThermExpCoef, FilmTemp+273.15)*...
(surfTemp-extWaterTemp)*(Diam^3))/((interp1(Temp, DynVisc, FilmTemp+273.15)*...
interp1(Temp, specVolume, FilmTemp+273.15))*...
((interp1(Temp, ThermCond, FilmTemp+273.15)*...
interp1(Temp, specVolume, FilmTemp+273.15))/...
interp1(Temp, SpecHeat, FilmTemp+273.15)));

Nue=(0.6+((0.387*(RaD^(1/6)))/((1+((0.559/...
interp1(Temp, Pr, FilmTemp+273.15))^(9/16)))^(8/27))))^2;

HTCe=(interp1(Temp, ThermCond, FilmTemp+273.15)*Nue)/Diam;

%New estimate for surfTemp

surfTempNew=-HTRi/(pi()*Diam*Leng*HTCe)+extWaterTemp;
relErrSurfTemp=abs(surfTempNew-surfTemp)/surfTempNew;

if (relErrSurfTemp >= maxrelErrSurfTemp)
  surfTemp=surfTemp*(1-rlxFactSurfTemp)+rlxFactSurfTemp*surfTempNew;
  if (surfTemp>outTemp)
    surfTemp=outTemp;
  endif
endif
innerIt=innerIt+1;
until (relErrSurfTemp<maxrelErrSurfTemp);

relErrSurfTemp
surfTemp

outTempNew=HTRi/(MFRT*interp1(Temp, SpecHeat, meanTemp+273.15))+inTemp;
relErrOutTemp=abs(outTempNew-outTemp)/outTempNew;

if (relErrOutTemp >= maxrelErrOutTemp)
```

```
  outTemp=outTemp*(1-rlxFactOutTemp)+rlxFactOutTemp*outTempNew;
endif

relErrOutTemp
outTemp

outerIt=outerIt+1;
until (relErrOutTemp<maxrelErrOutTemp);

disp([''])
disp([''])
disp(['#########CONVERGED SOLUTION OBTAINED##########'])
disp([''])
disp(['INPUT DATA'])
disp(['Tube length: ', num2str(Leng), ' m'])
disp(['Tube diameter: ', num2str(Diam), ' m'])
disp(['Mass flow rate: ', num2str(MFRT), ' kg/s'])
disp(['Inlet tube temperature: ', num2str(inTemp), ' C'])
disp(['External (tank) water temperature: ', num2str(extWaterTemp), ' C'])
disp([''])
disp([''])
disp(['RESULTS SUMMARY:'])
disp(['Heat Transfer Rates'])
disp(['Thermal power loss from the water flowing inside the tube (calculated'...
' with HTCi) :', num2str(HTRi), ' W'])
disp(['Thermal power loss from the water flowing inside the tube (calculate'...
' with Ener. Balance): ', num2str(MFRT*interp1(Temp, SpecHeat, meanTemp+273.15)*...
 (outTemp-inTemp)), ' W'])
disp(['Thermal power gain to the water in the tank (calculated with HTCe): ',...
 num2str(pi()*Diam*Leng*HTCe*(surfTemp-extWaterTemp)), ' W'])
disp([''])
disp(['Temperatures'])
disp(['Outlet water tube temperature: ', num2str(outTemp), ' C'])
disp(['Tube surface temperature: ', num2str(surfTemp), ' C'])
disp(['Film temperature: ', num2str(FilmTemp), ' C'])
disp(['Mean (bulk) fluid temperature: ', num2str(meanTemp), ' C'])
disp([''])
disp(['Miscellaneous'])
disp(['Reynolds number: ', num2str(ReD)])
disp(['Rayleigh number: ', num2str(RaD)])
disp(['Internal average Nusselt number: ', num2str(Nui)])
disp(['External average Nusselt number: ', num2str(Nue)])
disp(['Internal average convection heat transfer coefficient, HTCi: ',...
 num2str(HTCi), ' W/(m^2K)'])
disp(['External average convection heat transfer coefficient, HTCe: ',...
 num2str(HTCe), ' W/(m^2K)'])
```

SCRIPT OUTPUT

```
#########CONVERGED SOLUTION OBTAINED##########

INPUT DATA
Tube length: 2 m
Tube diameter: 0.02 m
Mass flow rate: 0.5 kg/s
Inlet tube temperature: 80 C
External (tank) water temperature: 20 C


RESULTS SUMMARY:
Heat Transfer Rates
Thermal power loss from the water flowing inside the tube (calculated with HTCi) :-
8198.5864 W
Thermal power loss from the water flowing inside the tube (calculate with Ener.
Balance): -8198.5864 W
Thermal power gain to the water in the tank (calculated with HTCe): 8198.5864 W

Temperatures
Outlet water tube temperature: 76.0921 C
Tube surface temperature: 70.5416 C
Film temperature: 45.2708 C
Mean (bulk) fluid temperature: 78.0461 C

Miscellaneous
Reynolds number: 88483.9886
Rayleigh number: 18199502.6315
Internal average Nusselt number: 266.1415
External average Nusselt number: 40.4592
Internal average convection heat transfer coefficient, HTCi: 8898.6752 W/(m^2K)
External average convection heat transfer coefficient, HTCe: 1290.8621 W/(m^2K)
```

BRIEF RESULTS ANALYSIS

The procedure described above can be extensively applied to evaluate the heat transfer performance of the system. For instance, the effect of the water mass flow rate in the tube on the heat loss to the water in the tank and tube surface and outlet tube temperatures – see figure below. The heat transfer rate from the water circulating inside the tube increases as the water mass flow rate increases – see left figure. (This is due to the fact that the internal convection heat transfer coefficient increases upon increasing the mass flow rate (not shown) – i.e., the thermal resistance across the confined boundary layer decreases.)

As the water mass flow rate increase the tube surface and outlet tube temperatures also increase – see right figure. The outlet temperature is always higher than the surface temperature, but the difference decreases as the mass flow rate increases. The outlet temperature (and consequently, the surface temperature) increases towards the inlet tube temperature (80°C) as the mass flow rate increases.