Heat Transfer

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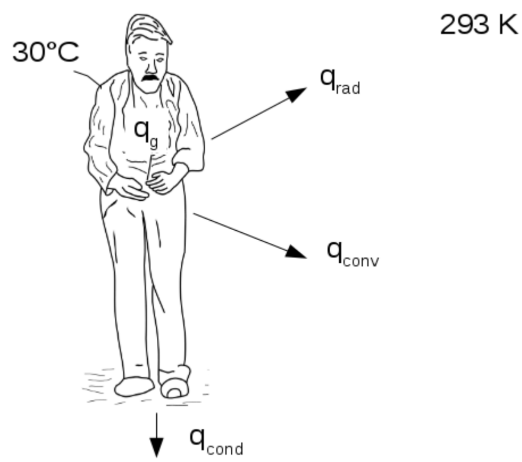
Practical Lecture 1 (Solved Problems)

A. Consider a person standing in a ventilated room at 293 K (including the room walls). The average outer surface of the person is 30°C. The person loses heat by conduction to the floor as well as by convection and radiation to surroundings.



(a) What is the temperature difference between the outer surface of the person and the room environment?

Solution:

The temperature difference is calculated according to Equation (1), where T_{per} and T_{room} correspond to the outer surface temperature of the person and room temperature, respectively.

$$\Delta T = T_{\text{per}} - T_{\text{room}} \tag{1}$$

The temperature values in Equation (1) must be considered in the same units – note that the problem statement gives T_{per} in the Celsius scale (30°C) and T_{room} in a thermodynamic (absolute) scale (293 K). The conversion of a temperature value from the Celsius scale to the Kelvin scale, or *vice-versa*, is determined with Equation (2).

$$T\,[\text{K}] = T\,[^\circ\text{C}] + 273.15 \tag{2}$$

The temperatures values under consideration on both temperature scales read as follows:

- $T_{\text{per}} = 30^\circ\text{C} = 303.15\,\text{K}$; and
- $T_{\text{room}} = 293\,\text{K} = 19.85^\circ\text{C}$

The temperature difference is then computed with Equation (1) considering the temperature values in Kelvin (Equation (3)) or, equivalently, in degrees Celsius (Equation (4)).

$$\Delta T\,[\text{K}] = T_{\text{per}}\,[\text{K}] - T_{\text{room}}\,[\text{K}] = 303.15 - 293 = 10.15\,\text{K} \tag{3}$$

$$\Delta T\,[^\circ\mathrm{C}] = T_{\mathrm{per}}\,[^\circ\mathrm{C}] - T_{\mathrm{room}}\,[^\circ\mathrm{C}] = 30 - 19.85 = 10.15^\circ\mathrm{C} \tag{4}$$

Temperature differences on Celsius and Kelvin scales are identical due to the relation between both temperature scales that is given by Equation (2) – see Equation (5).

$$\begin{aligned}\Delta T\,[\mathrm{K}] &= T_{\mathrm{per}}\,[\mathrm{K}] - T_{\mathrm{room}}\,[\mathrm{K}] = \\ &= (T_{\mathrm{per}}\,[^\circ\mathrm{C}] + 273.15) - (T_{\mathrm{room}}\,[^\circ\mathrm{C}] + 273.15) = \\ &= T_{\mathrm{per}}\,[^\circ\mathrm{C}] - T_{\mathrm{room}}\,[^\circ\mathrm{C}] = \Delta T\,[^\circ\mathrm{C}] \Leftrightarrow \\ &\Leftrightarrow \boxed{\Delta T\,[\mathrm{K}] = \Delta T\,[^\circ\mathrm{C}] \Leftrightarrow 10.15\,\mathrm{K} = 10.15^\circ\mathrm{C}}\end{aligned} \tag{5}$$

(b) Assuming that the person is losing heat to the surrounding at a total rate of 170 W, how much thermal energy (heat) is lost during 1 h? If the outer surface area of the person is $2\,\mathrm{m}^2$, what is the corresponding heat flux?

Solution:

1. How much thermal energy (heat) is lost during 1 h?

The thermal energy transferred (lost), Q, throughout a particular time interval, Δt, during which the total heat transfer rate is constant and equal to q is given by Equation (6). The SI units for each quantity is presented within square brackets in Equation (6).

$$Q\,[\mathrm{J}] = q\left[\mathrm{W} \equiv \mathrm{J\,s^{-1}}\right] \Delta t\,[\mathrm{s}] \tag{6}$$

Applying Equation (6) considering q equal to 170 W and Δt equal to 3600 s ($= 1\,\mathrm{h} \times 3600\,\mathrm{s\,h^{-1}}$), the corresponding thermal energy lost from the person is computed – see Equation (7).

$$Q = q\Delta t = 170 \times 3600 \Leftrightarrow \boxed{Q = 612000\,\mathrm{J}\ (= 612\,\mathrm{kJ})} \tag{7}$$

2. If the outer surface area of the person is $2\,\mathrm{m}^2$, what is the corresponding heat flux?

The relation between the heat transfer rate or, shortly, heat rate, q, and the heat flux, q'' is given by Equation (8), where A corresponds to the area across which the heat transfer is observed.

$$q\,[\mathrm{W}] = q''\left[\mathrm{W\,m^{-2}}\right] A\left[\mathrm{m^2}\right] \tag{8}$$

For the case under consideration, A corresponds to the outer surface area of the person ($= 2\,\mathrm{m}^2$) and q corresponds to the total heat transfer rate from the person ($= 170\,\mathrm{W}$). Therefore, the corresponding heat flux is calculated applying Equation (8) – see Equation (9).

$$q'' = \frac{q}{A} = \frac{170}{2} \Leftrightarrow \boxed{q'' = 85\,\mathrm{W\,m^{-2}}} \tag{9}$$

(c) Assuming that the person generates heat internally (at a rate q_{g} $[W]$) to keep constant the inner body temperature, write the steady-state energy balance for the person.

Solution:

The application of the conservation of energy principle (first law of thermodynamics) – considering only the mechanical and thermal components of the total energy – on a time rate basis leads to the Equation (10). (Note that this equation corresponds to the formulation of first law of thermodynamics convenient for solving heat transfer problems.)

$$\dot{E}_{\mathrm{st}} = \dot{E}_{\mathrm{in}} - \dot{E}_{\mathrm{out}} + \dot{E}_{\mathrm{g}} \tag{10}$$

In this equation, $\dot{E}_{\mathrm{st}}$ corresponds to the storage rate of thermal and mechanical energy, $\dot{E}_{\mathrm{in}}$ and $\dot{E}_{\mathrm{out}}$ correspond to the inflow and outflow, respectively, rates of thermal and mechanical energy through the control surfaces, and $\dot{E}_{\mathrm{g}}$ is the thermal energy generation rate (to account for eventual conversion between other forms of energy not directly considered in the heat transfer formulation of first law of thermodynamics (*e.g.*, chemical, nuclear, electric, electromagnetic,...) and thermal energy.

For the problem under consideration, only thermal energy is relevant – kinetic and potential energy (*i.e.*, mechanical energy) variations are neglected. (Mechanical energy contributions are relevant in heat transfer problems involving fluid flow – convection heat transfer problems.)

The following simplifications are applied to Equation (10):

- $\dot{E}_{\mathrm{st}}\ (\equiv dE_{\mathrm{st}}/dt) = 0$ – the steady-state condition is under consideration;
- $\dot{E}_{\mathrm{in}} = 0$ – no addition of thermal energy to the person is being considered;
- $\dot{E}_{\mathrm{g}} = q_{\mathrm{g}}$; and
- $\dot{E}_{\mathrm{out}} = q_{\mathrm{cond}} + q_{\mathrm{conv}} + q_{\mathrm{rad}}$ – the person is losing heat (thermal energy) through conduction, convection, and radiation modes of heat transfer.

Therefore, the steady-state energy balance equation for the person is given by Equation (11).

$$\dot{E}_{\mathrm{out}} = \dot{E}_{\mathrm{g}} \Leftrightarrow \boxed{q_{\mathrm{cond}} + q_{\mathrm{conv}} + q_{\mathrm{rad}} = q_{\mathrm{g}}} \tag{11}$$

B. (Homework) Consider the process of baking a potato inside an oven that is kept at the constant temperature T_∞. Initially the potato is at a temperature T_i and, since it receives heat at a rate q_{in} [W] (non-constant), its temperature will increase.

(a) What is the energy balance for the potato?

Solution:

The energy balance equation for the potato is derived from Equation (10) – energy balance equation applied to a (finite) control volume and written on an instantaneous time rate form – after applying the adequate simplifying assumptions as described below:

- $\dot{E}_{\text{g}} = 0$ – there is no thermal energy generation within the control volume (potato);
- $\dot{E}_{\text{out}} = 0$ – there is no thermal energy being transferred from the potato the the surroundings;
- $\dot{E}_{\text{in}} = q_{\text{in}}$; and
- $\dot{E}_{\text{st}} = \frac{dE_{\text{st}}}{dt} = \frac{d}{dt}\left(\cancel{KE} + \cancel{PE} + U_t\right) \Leftrightarrow \dot{E}_{\text{st}} = \frac{dU_t}{dt}$, where KE, PE, and U_t are the kinetic energy, potential energy, and thermal energy, respectively.

After these simplifications, the energy balance equation for the potato is given by Equation (12).

$$\boxed{\frac{dU_t}{dt} = q_{\text{in}}} \tag{12}$$

Note that if convection is the only relevant mode of heat transfer for baking the potato, the heat transfer rate is calculated according to Equation (13) – Newton's law of cooling (convection rate equation) –, where A, h, and T_s are the potato surface area, convective heat transfer coefficient, and potato surface temperature, respectively. Even though A, h, and T_∞ are held constant during the baking process, the heat transfer rate decreases over time due to a progressive approach to the thermal equilibrium between the potato and the oven – *i.e.*, $T_s \to T_\infty$ as elapsed baking time increases.

$$q_{\text{in}} = Ah\left(T_\infty - T_s\right) \tag{13}$$

(b) What is the expression for the heat accumulated by the potato over time?

Solution:

The expression for the heat accumulated by the potato from time instants t_1 to t_2 can be obtained by integrating Equation (12) over the corresponding time interval, as shown in Equation (14). (Note that because q_{in} varies along the time interval, $\Delta t\,(= t_2 - t_1)$, the expression cannot be computed as $\Delta U_t = q_{\text{in}}\Delta t$.)

$$dU_t = q_{\text{in}}dt \Rightarrow \boxed{\Delta U_t = U_t\left(t_2\right) - U_t\left(t_1\right) = \int_{t_1}^{t_2} q_{\text{in}}\left(t\right)dt} \tag{14}$$

During the baking process, the thermal energy accumulated by the potato between time instants t_1 and t_2 can be evaluated according to Equation (15) as long as the potato mass (m) and specific heat (c) are constants, the latent energy effects are negligible, and the spatial temperature gradients at any time instant within the control volume are negligible, *i.e.*, $T(\mathbf{x}, t) = T(t)$.

$$dU_t = mcdT \Rightarrow U_t(t_2) - U_t(t_1) = mc[T(t_2) - T(t_1)] \quad (15)$$

Under the stated conditions, by replacing Equation (15) in Equation (14) it is possible to calculate the temperature change between time instants t_1 and t_2 – see Equation (16).

$$T(t_2) - T(t_1) = \frac{1}{mc}\int_{t_1}^{t_2} q_{\text{in}}(t)\,dt \quad (16)$$

(c) Since the oven is kept at constant temperature, will the potato temperature increase forever? What will be the potato temperature when the system reaches steady-state?

Solution:

Since the temperature and thermal energy of the potato increases due to heat transfer, q_{in} – in accordance to Equations (16) and (14), respectively – that is sustained by the temperature difference (driving potential) between the oven and potato, when the potato reaches the same temperature of the oven, the heat transfer ceases (*i.e.*, $q_{\text{in}} = 0$) and steady-state (time-independent) conditions are reached. At these state conditions, the potato temperature becomes constant with time (and equal to the oven temperature – thermal equilibrium) and no temporal changes are observed in the accumulated (stored) thermal energy within the potato. Therefore, at steady-state conditions (conditions reached as $t \to \infty$), the following equation holds.

$$\boxed{\frac{dU_t}{dt} = \frac{dT}{dt} = q_{\text{in}}\,(= T - T_\infty) = 0} \quad (17)$$