Heat Transfer

Exam 1 – Problem 2

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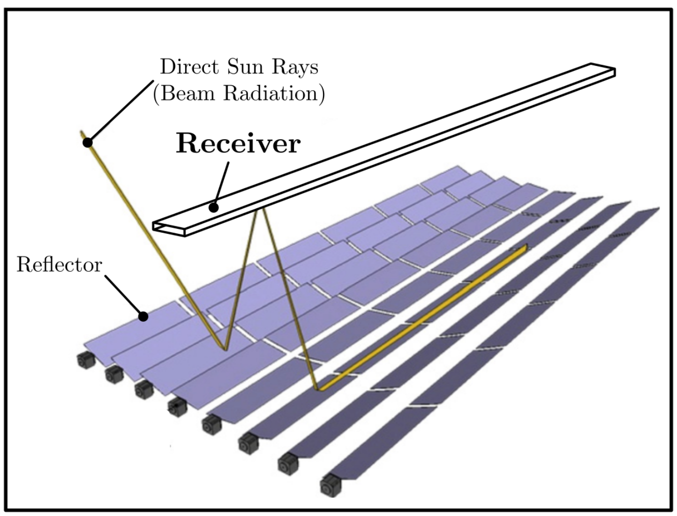
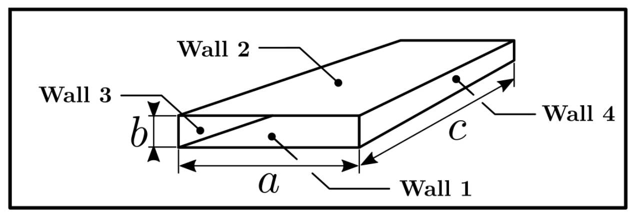
Consider an experimental campaign designed to evaluate the performance of an innovative solar receiver. The solar receiver is composed by a rectangular duct – within which a fluid circulates – that is placed horizontally as shown in the figure (see left inset). The experiments were conducted in a demo-site near Évora (location with high beam radiation intensity potential). In all experiments, concentrated solar radiation (solar radiation reflected and augmented by optical concentration devices (reflectors) placed on the ground) reached Surface 1-ext (external surface of Wall 1 – see the top-right inset) of the receiver with a uniform irradiation. Surface 1-ext is diffuse, opaque, and selective with a spectral reflectivity, ρ_λ, provided in the figure – see the bottom-right inset. Walls 3 and 4 (vertical walls) are adiabatic. All receiver external surfaces are surrounded by air at 20°C and only Surface 1-ext is surrounded by quiescent air. Consider the duct width (a) and height (b) equal to 0.4 m and 0.05 m, respectively, and the duct length (c) much higher than the duct width, *i.e.*, $c >> a$ – see the dimensions in the figure top-right inset. Neglect the following: (i) end (vertical side wall) effects for convection and radiation heat transfer calculations; (ii) receiver wall thicknesses and wall thermal resistances; and (iii) radiative emission losses (long-wave radiative heat transfer) from the irradiated receiver surface – Surface 1-ext.

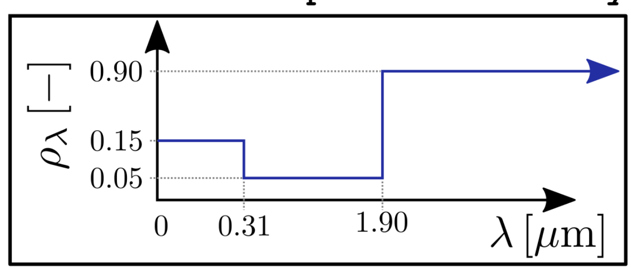
Solar Collector System



Receiver



Surface 1-ext: Spec. Reflectivity



Surface i-ext (i-int): external (internal) suface of Wall i

External (internal) surface: surface facing the exterior (interior) duct region

(a) (2.0 v.) With a particular solar insolation, a concentrated solar irradiation value equal to 20 kW m^{-2} is measured on Surface 1-ext. In such conditions, determine the absorbed solar heat flux.

Solution:

According to the figure, the spectral reflectivity distribution for Surface 1-ext is given in Equation (1).

$$\rho_\lambda = \begin{cases} \rho_{\lambda,1} = 0.15 & 0 < \lambda \leq 0.31\,\mu\text{m} \\ \rho_{\lambda,2} = 0.05 & 0.31 < \lambda \leq 1.90\,\mu\text{m} \\ \rho_{\lambda,3} = 0.90 & 1.90\,\mu\text{m} < \lambda < \infty \end{cases} \quad (1)$$

The total reflectivity is calculated according to Equation (2).

$$\rho = \frac{\int_0^\infty \rho_\lambda(\lambda)\, G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} \Leftrightarrow \rho = \frac{\int_0^\infty \rho_\lambda(\lambda)\, E_{\lambda,\mathrm{b}}(\lambda, 5800\,\mathrm{K})\, d\lambda}{\int_0^\infty E_{\lambda,\mathrm{b}}(\lambda, 5800\,\mathrm{K})\, d\lambda} \Leftrightarrow$$
$$\Leftrightarrow \rho = \rho_{\lambda,1} \frac{\int_0^{0.31} E_{\lambda,\mathrm{b}}(\lambda, 5800\,\mathrm{K})\, d\lambda}{E_\mathrm{b}(5800\,\mathrm{K})} + \rho_{\lambda,2} \frac{\int_{0.31}^{1.90} E_{\lambda,\mathrm{b}}(\lambda, 5800\,\mathrm{K})\, d\lambda}{E_\mathrm{b}(5800\,\mathrm{K})} + \tag{2}$$
$$\rho_{\lambda,3} \frac{\int_{1.90}^{\infty} E_{\lambda,\mathrm{b}}(\lambda, 5800\,\mathrm{K})\, d\lambda}{E_\mathrm{b}(5800\,\mathrm{K})} \Leftrightarrow \rho = \rho_{\lambda,1} F_{0\to\lambda_1} + \rho_{\lambda,2}\left(F_{\lambda_1\to\lambda_2}\right) + \rho_{\lambda,3}\left(F_{\lambda_2\to\lambda_3}\right) \Leftrightarrow$$
$$\Leftrightarrow \rho = \rho_{\lambda,1} F_{0\to\lambda_1} + \rho_{\lambda,2}\left(F_{0\to\lambda_2} - F_{0\to\lambda_1}\right) + \rho_{\lambda,3}\left(1 - F_{0\to\lambda_2}\right)$$

The fractions $F_{0\to\lambda_1}$ and $F_{0\to\lambda_2}$ are obtained from Table 12.2 ("Fundamentals of Heat and Mass Transfer", 7$^{\mathrm{th}}$) considering $\lambda_1 T = 0.31 \times 5800 = 1798\,\mu\mathrm{mK}$ and $\lambda_2 T = 1.90 \times 5800 = 11020\,\mu\mathrm{mK}$, respectively. Consequently, $F_{0\to\lambda_1} = 0.039341$ and $F_{0\to\lambda_2} = 0.93189$.
Replacing the variables in Equation (2), the total reflectivity for Surface 1-ext when exposed to the concentrated solar irradiation is obtained – see Equation (3).

$$\rho = \rho_{\lambda,1} F_{0\to\lambda_1} + \rho_{\lambda,2}\left(F_{0\to\lambda_2} - F_{0\to\lambda_1}\right) + \rho_{\lambda,3}\left(1 - F_{0\to\lambda_2}\right) \Leftrightarrow$$
$$\rho = 0.15 \times 0.039341 + 0.05 \times (0.93189 - 0.039341) + 0.9 \times (1 - 0.93189) \Leftrightarrow \tag{3}$$
$$\rho \approx 0.112$$

Since Surface 1-ext is opaque, the corresponding total absorptivity is given as shown in Equation (4).

$$\rho + \alpha = 1 \Leftrightarrow \alpha = 1 - 0.112 \Leftrightarrow \alpha = 0.888 \tag{4}$$

Finally, the absorbed solar heat flux is computed in Equation (5).

$$q''_{\mathrm{sol,abs}} = \alpha G_{\mathrm{sol,inc}} \Leftrightarrow q''_{\mathrm{sol,abs}} = 0.888 \times 20 \Leftrightarrow \boxed{q''_{\mathrm{sol,abs}} = 17.760\,\mathrm{kW\,m^{-2}}} \tag{5}$$

(b) (2.5 v.) For a specific operating condition and under a steady-state regime, the absorbed solar heat flux is equal to $15\,\mathrm{kW\,m^{-2}}$, the temperatures of Walls 1 and 2 are equal to 133.7°C, and the convection heat transfer coefficient over Surface 2-ext (external surface of Wall 2) is equal to $30\,\mathrm{W\,m^{-2}\,K^{-1}}$. Calculate the thermal power transferred to the fluid circulating inside the duct per unit duct length.

Solution:
The power transferred to the heat transfer fluid per unit duct length, q'_{rec}, is obtained applying an energy balance to the control surface surrounding the boundaries (wall) of the receiver – see Equation (6).

$$\dot{E}_{\mathrm{in}} = \dot{E}_{\mathrm{out}} \Leftrightarrow q''_{\mathrm{sol,abs}} = q''_{\mathrm{conv},1} + q''_{\mathrm{conv},2} + q''_{\mathrm{rec}} \Leftrightarrow$$
$$\Leftrightarrow q'_{\mathrm{rec}} = a \cdot \left[q''_{\mathrm{sol,abs}} - (h_1 + h_2) \cdot (T_{\mathrm{rec}} - T_\infty)\right] \tag{6}$$

h_2 corresponds to the convection heat transfer coefficient for Surface 2-ext – $h_2 = 30\,\mathrm{W\,m^{-2}\,K^{-1}}$. The (average) convection heat transfer coefficient for Surface 1-ext (h_1) is evaluated considering a suitable (average) Nusselt number correlation for free convection – since Surface 1-ext is surrounded by quiescent air. The Rayleigh number is computed with Equation (7). The thermophysical properties required for computing h_1) are evaluated at a mean fluid temperature, T_f.

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha} \tag{7}$$

For air at atmospheric pressure and at the mean temperature $T_f = 0.5 \times (T_{\text{rec}} + T_\infty) = 0.5 \times (133.7 + 20) + 273.15 \Leftrightarrow T_f = 350\,\text{K}$, the following properties are gathered from Table A.4 "Fundamentals of Heat and Mass Transfer", 7th): $\nu = 2.092 \times 10^{-5}\,\text{m}^2\,\text{s}^{-1}$; $k = 0.03\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$; $\alpha = 2.99 \times 10^{-5}\,\text{m}^2\,\text{s}^{-1}$; and $Pr = 0.7$. Furthermore, $\beta = 1/T_f = 0.002857\,\text{K}^{-1}$ and $g = 9.8\,\text{m}\,\text{s}^{-2}$. The characteristic length is calculated according to Equation (8).

$$\begin{aligned} L = \frac{A_s}{p} \Leftrightarrow L = \frac{a \cdot c}{2\left(a + c\right)} \Leftrightarrow L \approx \frac{a \cdot c}{2c} \Rightarrow L = \frac{a}{2} \Leftrightarrow \\ \Leftrightarrow L = \frac{0.4}{2} \Leftrightarrow L = 0.2\,\text{m} \end{aligned} \tag{8}$$

$$\begin{aligned} Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha} \Leftrightarrow Ra_L = \frac{9.8 \times 0.002857 \times (133.7 - 20) \times 0.2^3}{2.092 \times 10^{-5} \times 2.99 \times 10^{-5}} \Leftrightarrow \\ \Leftrightarrow Ra_L \approx 40714949.385 \end{aligned} \tag{9}$$

Two (average) Nusselt number correlations are available for the current heat transfer problem in textbooks. Consequently, two different solutions to this question are acceptable – see below Correlations A and B.

- Correlation A

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \Leftrightarrow \overline{Nu}_L = 0.27 \times 40714949.385^{1/4} \Leftrightarrow \overline{Nu}_L \approx 21.568 \tag{10}$$

$$\overline{Nu}_L = \frac{h_1 L}{k} \Leftrightarrow h_1 = \frac{k}{L}\overline{Nu}_L \Leftrightarrow h_1 = \frac{0.03}{0.2} \times 21.568 \Leftrightarrow h_1 \approx 3.235\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1} \tag{11}$$

$$\begin{aligned} q'_{\text{rec}} = a \cdot \left[q''_{\text{sol,abs}} - (h_1 + h_2) \cdot (T_{\text{rec}} - T_\infty)\right] \Leftrightarrow \\ \Leftrightarrow q'_{\text{rec}} = 0.4 \times [15000 - (3.235 + 30) \times (133.7 - 20)] \Leftrightarrow \\ \Leftrightarrow \boxed{q'_{\text{rec}} = 4488.472\,\text{W}\,\text{m}^{-1}} \end{aligned} \tag{12}$$

- Correlation B

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \Leftrightarrow \overline{Nu}_L = 0.52 \times 40714949.385^{1/5} \Leftrightarrow \overline{Nu}_L \approx 17.296 \tag{13}$$

$$\overline{Nu}_L = \frac{h_1 L}{k} \Leftrightarrow h_1 = \frac{k}{L}\overline{Nu}_L \Leftrightarrow h_1 = \frac{0.03}{0.2} \times 17.296 \Leftrightarrow h_1 \approx 2.594\,\mathrm{W\,m^{-2}\,K^{-1}} \tag{14}$$

$$\begin{aligned} q'_{\text{rec}} &= a \cdot \left[q''_{\text{sol,abs}} - (h_1 + h_2) \cdot (T_{\text{rec}} - T_\infty)\right] \Leftrightarrow \\ \Leftrightarrow q'_{\text{rec}} &= 0.4 \times [15000 - (2.594 + 30) \times (133.7 - 20)] \Leftrightarrow \\ &\Leftrightarrow \boxed{q'_{\text{rec}} \approx 4517.625\,\mathrm{W\,m^{-1}}} \end{aligned} \tag{15}$$

(c) (2.5 v.) At a particular moment, the fluid circulating inside the duct was replaced by air at atmospheric pressure without providing any momentum source – promoted, for instance, by the application of a ventilator. At steady-state conditions and with an unknown value for the concentrated solar irradiation on Surface 1-ext, the temperatures of Wall 1 and Wall 2 are equal to 200°C and 60°C, respectively. Under such conditions, the radiation heat transfer coefficient, h_r, between Surface 1-int and Surface 2-int (internal surfaces of Walls 1 and 2, respectively) is equal to $3.8\,\mathrm{W\,m^{-2}\,K^{-1}}$. Estimate the convection heat transfer coefficient over Surface 2-ext. Consider the following air thermophysical properties: $\nu = 2.679 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$; $k = 3.402 \times 10^{-2}\,\mathrm{W\,m^{-1}\,K^{-1}}$; $\alpha = 3.886 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$; and $Pr = 0.69$.

Solution:

At steady-state conditions and according to the energy conservation principle, the heat flux from Wall 1 to Wall 2 is equal to the heat flux from Wall 2 to the surrounding (external) air. Consequently, the convection heat transfer coefficient over Surface 2-ext, $h_{2-\text{ext}}$, can be obtained through Equation (16).

$$\begin{aligned} \frac{T_1 - T_2}{R''_{t,\text{tot}}} = \frac{T_2 - T_\infty}{R''_{t,\text{conv}_{2-\text{ext}}}} \Leftrightarrow \frac{T_1 - T_2}{\frac{1}{h_{\text{int}} + h_r}} = \frac{T_2 - T_\infty}{\frac{1}{h_{2-\text{ext}}}} \Leftrightarrow \\ \Leftrightarrow h_{2-\text{ext}} = (h_{\text{int}} + h_r) \cdot \frac{T_1 - T_2}{T_2 - T_\infty} \end{aligned} \tag{16}$$

To obtain $h_{2-\text{ext}}$ through Equation (16) only the convection heat transfer coefficient between Surfaces 1-int and 2-int, h_{int} (internal convection heat transfer coefficient), is unknown. This transport property is obtained applying a suitable correlation as follows.

For a horizontal enclosure filled with a quiescent fluid and heated from below the following correlation (average) Nusselt number correlation applies – see Equation (17). Equation (17) is valid for $a >> b$ and $c >> b$ – for the current receiver geometry $a/b = 8$ – and for the following Rayleigh number range: $3 \times 10^5 \lesssim Ra_L \lesssim 7 \times 10^9$.

$$\overline{Nu}_L = 0.069 Ra_L^{1/3} Pr^{0.074} \tag{17}$$

The Rayleigh number is computed with Equation (18) considering $L = b = 0.05\,\mathrm{m}$ and $g = 9.8\,\mathrm{m\,s^{-2}}$. The fluid (air) thermophysical properties required in Equations (17) and (18) are evaluated at the average temperature between the temperatures of Wall 1 (T_1) and Wall 2 T_2.

$$Ra_L = \frac{g\beta\,(T_1 - T_2)\,L^3}{\nu\alpha} \tag{18}$$

For air at atmospheric pressure and at the mean temperature $T_f = 0.5 \times (T_1 + T_2) = 0.5 \times (200 + 60) + 273.15 \Leftrightarrow T_f = 403.15\,\mathrm{K}$, the following properties are gathered from Table A.4 "Fundamentals of Heat and Mass Transfer", 7^{th}): $\nu \approx 2.679 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$; $k \approx 0.03402\,\mathrm{W\,m^{-1}\,K^{-1}}$; $\alpha \approx 3.886 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$; and $Pr \approx 0.690$. Furthermore, $\beta = 1/T_f \approx 0.002480\,\mathrm{K^{-1}}$.

The Rayleigh number is calculated replacing the thermophysical properties and geometrical parameters in Equation (18) – see Equation (19). Note that the obtained Rayleigh number is in the range of applicability of the Nusset number correlation.

$$Ra_L = \frac{g\beta\,(T_1 - T_2)\,L^3}{\nu\alpha} \Leftrightarrow Ra_L = \frac{9.8 \times 0.002480 \times (200 - 60) \times 0.05^3}{(2.679 \times 10^{-5}) \times (3.886 \times 10^{-5})} \Leftrightarrow \Leftrightarrow Ra_L \approx 408545.372 \tag{19}$$

The Nusselt number is computed replacing the corresponding variables in Equation (17) – see Equation (20).

$$\overline{Nu}_L = 0.069 Ra_L^{1/3} Pr^{0.074} \Leftrightarrow \overline{Nu}_L = 0.069 \times 408545.372^{1/3} \times 0.690^{0.074} \Leftrightarrow \overline{Nu}_L \approx 4.981 \tag{20}$$

The internal convection heat transfer coefficient is obtained from the definition of the Nusselt number as shown in Equation (21).

$$\overline{Nu}_L = \frac{h_{\text{int}} L}{k} \Leftrightarrow h_{\text{int}} = \frac{k}{L}\overline{Nu}_L \Leftrightarrow h_{\text{int}} = \frac{0.03402}{0.05} \times 4.981 \Leftrightarrow h_{\text{int}} \approx 3.389\,\mathrm{W\,m^{-2}\,K^{-1}} \tag{21}$$

Finally, replacing T_1, T_2, h_{rad} (provided in the statement) and h_{int} (computed as shown above) in Equation (16) the convection heat transfer coefficient over Surface 2-ext is obtained – see Equation (22).

$$h_{2-\text{ext}} = \frac{(h_{\text{int}} + h_r) \cdot (T_1 - T_2)}{T_2 - T_\infty} \Leftrightarrow h_{2-\text{ext}} = \frac{(3.389 + 3.8) \times (200 - 60)}{60 - 20} \Leftrightarrow \Leftrightarrow \boxed{h_{2-\text{ext}} \approx 25.162\,\mathrm{W\,m^{-2}\,K^{-1}}} \tag{22}$$