Parallel Branch and Bound to solve Mixed Integer Linear Programs

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Introduction
Linear Programming

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Introduction

Linear Programming

\[
\text{maximize } \sum_{1 \leq j \leq n} c_j x_j \quad (1)
\]

subject to

\[
\sum_{1 \leq j \leq n} a_{ij} x_j \leq b_j \text{ for } i = 1, 2, \ldots, m \quad (2)
\]

\[
x_j \geq 0 \text{ for } j = 1, 2, \ldots, n \quad (3)
\]

General form of a Linear Program.
A simplex is solved using pivot points to adjust the coefficients of the optimization function and constraints.
Consider the following:

\textit{maximize:}

\[ f = 4x_1 + 11x_2 \]

\textit{subject to:}

\[ 2x_1 - x_2 + x_3 = 4 \quad (4) \]
\[ 2x_1 + 5x_2 + x_4 = 16 \quad (5) \]
\[ -x_1 + 2x_3 + x_5 = 4 \quad (6) \]
\[ x_1, x_2, x_3, x_4, x_5 \geq 0 \quad (7) \]
After the first application of the simplex solver to the problem we get:

\[ f = 34\frac{2}{3}, x_1 = \frac{4}{3}, x_2 = \frac{8}{3} \]

To bound the search to the integers, we conduct two sub-searches, one where \( x_1 \leq 1 \), and the other where \( x_1 \geq 2 \).

These two searches cover the entire valid solution region, but are not necessarily symmetric.
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Introduction

Branch and Bound

Parallel Search

Results

References

Branch and Bound

A complete search tree

The search would look like like:

\[ f_L^1 = 0 \ (x_1 = x_2 = 0) \]
\[ f_L^2 = 34 \ (x_1 = 3, x_2 = 2) \]
\[ f_u = 34 \frac{1}{2} \ (x_1 = 1, x_2 = \frac{5}{2}) \]
\[ f_u = 34 \frac{2}{5} \ (x_1 = 2, x_2 = \frac{12}{5}) \]

Integer: \( x_1 = 3, x_2 = 2 \)

Not feasible
In a single threaded program, each branch is searched in sequence:

**Figure**: View of a node in the ILP search space.
In a multi threaded program, two branches can be searched simultaneously:

Figure: Branching and joining of parallel searches.
The following times were observed from a problem with 40 variables and 20 constraints, running on an 8-core Linux machine:
The following times were observed from a problem with 150 variables and 75 constraints, running on an 8-core Linux machine:
References