2.9 Semicontinuous Variables

The modeling of some situations may require the use of so-called *semicontinuous* variables that either take the value of zero or any number above a given positive threshold. Formally, such a variable will satisfy the conditions $x_j = 0$ or $x_j \geq \ell_j$ where $\ell_j > 0$ is some given threshold value. An example for semicontinuous variables occurs in forestry harvesting where x_j indicates the harvesting effort in a logging area. Either the area is not harvested at all, corresponding to $x_j = 0$, or some minimum prescribed level of harvesting effort ℓ_j is carried out, where ℓ_j is determined by economic or other considerations. Above this minimum, any level of x_j can be chosen.

Semicontinuous variables may be modeled in a number of ways. One possibility is to define zero-one variables y_j that assume a value of 1, if $x_j \geq \ell_j$, and 0 if $x_j = 0$. The constraints are then $x_j \leq My_j$, $x_j \geq \ell_j y_j$, $y_j = 0 \vee 1$, and $x_j \geq 0$, where, as usual, $M >> 0$. Clearly, if $y_j = 0$, then constraints require that $x_j \leq 0$ and $x_j \geq 0$, i.e., $x_j = 0$. If $y_j = 1$, then $x_j \leq M$ (a redundant constraint) and $x_j \geq \ell_j$ which is the desired result. Another possibility to model this situation is to define a zero-one variable in the same way as before and then replace the original semicontinuous variable x_j by a new continuous variable x'_j and a zero-one variable y_j, such that $x_j = \ell_j y_j + x'_j$ with $y_j = 0 \vee 1$ and $x'_j \geq 0$. We then add the contraints $0 \leq x'_j \leq (u_j - \ell_j)\, y_j$ where u_j is some appropriately chosen upper bound on the value of x_j. As an example, if a variable x_j must be either zero or take any value between 3 and 21, we can write $x_j = 3y_j + x'_j$ where $y_j = 0 \vee 1$ and $0 \leq x'_j \leq 18y_j$. If $y_j = 0$, then $x'_j = 0$ and hence $x_j = 0$, whereas if $y_j = 1$, then $x'_j \leq 18$. A value of, say, $x'_j = 7$ then leads to $x_j = 3(1) + 7 = 10$. In this way, any feasible value of x_j can be generated. The modeling of semicontinuous variables is reminiscent of the modeling of fixed charges, described in Section 2.2. In that case, the threshold value ℓ_j corresponds to the fixed cost f_i.

CHAPTER 3 APPLICATIONS AND SPECIAL STRUCTURES

The previous chapter demonstrated how zero-one variables can be used to model logical relationships. In this chapter we present a number of examples involving integer variables. The applications are selected to show the wide range and the versatility of integer programming. However, in order to avoid duplication, we will not describe problems related to network models that are treated in detail in Parts II and III of this volume. Although numerous applications of integer programming are reported in the literature, surprisingly few books devote much attention to applications. Notable exceptions are Taha (1997), Williams (1978), and Rardin (1998). p.498 p 499

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3.1 Applications

3.1.1 A Distribution-Location Problem

Consider a company producing a commodity at each of two factories A and B. The amounts produced can be stored directly at A and/or B, but also in each of the three warehouses D, E, and F. The commodity is delivered free of charge from A, B, D, E and F and sold to eight customers C_1, C_2, ..., C_8. The weekly capacities of the plants and the warehouses A, B, D, E, and F are 3,800; 4,600; 1,100; 2,350;

Table I.2

to / from	D	E	F
A	4.80	5.20	9.90
B	8.50	5.70	4.90