



**UNIVERSIDADE TÉCNICA DE LISBOA**  
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**Electromagnetic Waves in Artificial Media**  
**with Application to Lens Antennas**

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# *Abstract*

In recent years there has been an increasing interest in the propagation of electromagnetic waves in artificial materials. Artificial materials consist of a mixture of metallic or dielectric particles in a host medium. It has been shown that artificial materials may significantly improve the performance of antennas, microwave filters and other devices. The characterization and study of these “engineered” materials is thus of increasing relevance.

This thesis comprises a systematic study of electromagnetic waves in artificial materials, and its application to dielectric lens antennas.

We examine the homogenization problem in periodic materials. A new method is proposed to compute the effective parameters of artificial crystals at the long wavelength limit.

The band structure problem in metallic-dielectric crystals is studied. We propose a new hybrid formalism to compute the band structure of periodic media.

The radiation from point sources in metallic crystals is examined. The asymptotic form of the radiated fields in the far field region is derived.

The artificial dielectric concept is put into practice to design artificial material lenses. Several lens configurations with inhomogeneous anisotropic substrate are studied. A new formalism is presented to compute the lens permittivity profile that ensures that the output beam matches a specified target.

**Key Words:** artificial media, PBG material, homogenization theory, periodic Green functions, radiation in periodic media, lens antennas.



# *Resumo*

Nos últimos anos houve um interesse crescente na propagação de ondas electromagnéticas em meios artificiais. Estes meios consistem numa mistura de partículas metálicas ou dieléctricas embebidas num meio ambiente homogéneo. Este tipo de estruturas pode ter propriedades notáveis que geralmente não são observáveis na natureza. Vários resultados mostram que os materiais artificiais podem melhorar significativamente o desempenho de antenas, filtros para microondas, e outros dispositivos. O estudo e caracterização deste tipo de materiais é pois de crescente importância.

Nesta tese realiza-se um estudo sistemático sobre meios artificiais e investiga-se a sua aplicação em antenas baseadas em lentes.

Primeiro estuda-se o problema de homogeneização em materiais periódicos. É proposto um método para calcular os parâmetros efectivos de cristais artificiais no limite de grandes comprimentos de onda.

Em seguida, estuda-se a estrutura de bandas de cristais dieléctricos ou metálicos. Propõe-se um método híbrido para o cálculo numérico da estrutura de bandas.

A radiação de fontes pontuais elementares é examinada. É deduzida a forma assintótica dos campos radiados na zona distante.

O conceito de dieléctrico artificial é posto em prática para projectar lentes cujo substrato é um material artificial. São examinadas várias configurações de lentes com substrato não uniforme e anisotrópico.

**Palavras-chave:** meios artificiais, materiais PBG, teoria de homogeneização, funções de Green periódicas, radiação em meios periódicos, antenas com lentes.

To my parents

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# Symbol List

$a$	Lattice constant
$a_w$	Wire radius
$\mathbf{a}_1, \dots, \mathbf{a}_3$	Primitive vectors
$\mathbf{b}_1, \dots, \mathbf{b}_3$	Reciprocal lattice primitive vectors
$A_{\text{cell}}$	Area of the transversal lattice
$\mathcal{B}$	Induction field
BZ	Brillouin zone
$c$	Velocity of light in vacuum
$C$	Generic constant
$D$	Domain in the 3D-space
$\partial D$	Boundary of domain $D$
$\mathcal{D}$	Dielectric displacement
$\mathbf{e}_j$	Generic plane wave
$\mathbf{E}$	Electric field
$f$	Frequency
$f_A$	Area fraction
$f_V$	Volume fraction
$g_j$	Scalar plane wave
$\overline{\mathbf{G}}_0$	Electric Green dyadic
$\mathbf{H}$	Magnetic field
$\overline{\mathbf{I}}_d$	Identity dyadic
$\mathbf{I} = (i_1, i_2, \dots)$	Multi-index of integers
$\mathbf{J} = (j_1, j_2, \dots)$	Multi-index of integers
$j$	$\sqrt{-1}$
$\mathbf{J}_c$	Density of current over a metallic surface
$\mathbf{k}$	Wave vector
$\mathbf{k}_j$	$\mathbf{k}_j = \mathbf{k} + j_1 \mathbf{b}_1 + j_2 \mathbf{b}_2 + \dots$
$K_1, K_2$	Principal curvatures

$\overline{\mathbf{L}}$	Depolarization dyadic
$n$	Generic integer
$n$	Refraction index
$\mathbf{m}$	Magnetic dipole moment
$\mathcal{M}$	Magnetization vector
$\mathbf{p}$	Wave normal or “momentum”
$\mathbf{p}$	Electric dipole moment
$\mathcal{P}$	Polarization vector
$\mathbf{r}$	Observation point
$\mathbf{r}'$	Source point
$\mathbf{r}_1$	Lattice point
$s$	Radiation vector
$\mathbf{S}$	Poynting vector
$\mathcal{S}^{(n)}$	$n$ -th wave normal surface
$\text{sgn}(\ )$	Sign of a real number
$\mathcal{T}$	Transmissivity
$\mathbf{u}$	$\mathbf{r} - \mathbf{r}'$
$\hat{\mathbf{u}}_n$	Unit vector along the $x_n$ -direction
$U$	Radiation intensity
$V_{\text{cell}}$	Volume of the unit cell
$V_{\text{B}}$	Volume of the Brillouin zone
$x_1, \dots, x_3$	Cartesian coordinates
$Z_0$	Impedance in vacuum
$\beta$	Free-space wave number
$\delta$	Dirac’s distribution
$\delta_{n,m}$	Kronecker’s delta symbol
$\varepsilon$	Permittivity
$\varepsilon_h$	Permittivity of the host material
$\phi$	Electric potential
$\Phi_0$	Free-space Green function
$\Phi_{p0}$	Periodic Green function

$\Phi_{p0,s}$	Periodic Green function for the stratified case
$\Phi_p$	Lattice Green function
$\varphi$	Spherical coordinate
$\gamma_f$	Exit angle
$\lambda$	Wavelength
$\mu$	Permeability
$\hat{\mathbf{v}}$	Outward unit vector
$\theta$	Spherical coordinate
$\sigma_c$	Density of electric charge
$\omega$	Angular frequency
$\Omega$	Unit cell
$\psi$	Wave function
$\nabla$	Gradient
$\nabla_s$	Surface gradient

