Comment on "Repulsive Casimir Force in Chiral Metamaterials"

We argue that the proposal of Ref. [1] of Casimir repulsion and nanolevitation based on chiral metamaterials is incompatible with the passivity and the causal response of the materials. As in Ref. [1], we consider an isotropic chiral metamaterial characterized by the constitutive relations: $\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} + \frac{1}{c} i\kappa \mathbf{H}$ and $\mathbf{B} = -i\frac{1}{c}\kappa \mathbf{E} + \mu_0 \mu \mathbf{H}$. By thermodynamical considerations, the heat liberated per unit volume must be positive. For harmonic electromagnetic fields with time variation $\exp(-i\omega t)$, the average heating rate in one cycle is given by (ω is real valued and positive): $q = \frac{1}{2}\,\mathrm{Re}\{-i\omega(\mathbf{E}^* \cdot \mathbf{D} + \mathbf{H}^* \cdot \mathbf{B})\}$. Using the bianisotropic constitutive relations the heating rate may be written as a quadratic form:

$$q = \frac{\omega}{2}(\mathbf{E}^* \quad \mathbf{H}^*) \cdot \begin{pmatrix} \mathrm{Im}\{\varepsilon_0 \varepsilon\} & i\mathrm{Im}\{\frac{\kappa}{c}\} \\ -i\mathrm{Im}\{\frac{\kappa}{c}\} & \mathrm{Im}\{\mu_0 \mu\} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}. \qquad (1)$$

We used the fact that $\mathrm{Re}\{\mathbf{F}^* \cdot \mathbf{M} \cdot \mathbf{F}\} = \mathbf{F}^* \cdot \mathbf{M}' \cdot \mathbf{F}$ for a generic matrix $\mathbf{M}$ and a generic vector $\mathbf{F}$, where $\mathbf{M}' = (\mathbf{M} + \mathbf{M}^\dagger)/2$ and the $\dagger$ represents the adjoint matrix. In order that for arbitrary electric and magnetic fields $q > 0$ at a point, it is necessary that the matrix in Eq. (1) is positive definite. This is ensured by the following set of necessary and sufficient conditions [2,3]:

$$\mathrm{Im}\{\varepsilon\} > 0, \qquad \mathrm{Im}\{\mu\} > 0, \qquad \Delta > 0, \qquad (2)$$

where $\Delta = \frac{1}{c^2}[\mathrm{Im}\{\varepsilon\}\mathrm{Im}\{\mu\} - (\mathrm{Im}\{\kappa\})^2]$ is the determinant of the matrix. The two first inequations in Eq. (2) are the familiar conditions that ensure the passivity of dielectric and magnetic materials. The third condition, $\Delta > 0$, is only relevant in case of a chiral response, since otherwise it is redundant. Clearly, it imposes a limit on the strength of the imaginary part of κ. In Fig. 1(a) we plot Δ as a function of frequency for the parameters used in Ref. [1]. As seen, in the case $\omega_\kappa = 0.7\omega_R$, for which the authors of Ref. [1] predicted a repulsive Casimir force, Δ can be negative, and thus such large values of the chirality parameter are incompatible with the passivity of the material [2]. Indeed, it may be checked that in order that Δ remains positive it is necessary that $\omega_\kappa < 0.032\omega_R$.

One could still try to make sense of the results of Ref. [1], and argue that for $\mathrm{Im}\{\kappa\} \approx 0$ passivity might not be violated. However, even for a lossless material the results of Ref. [1] remain unphysical. Indeed, a bianisotropic reciprocal material (such that the underlying microstructure corresponds to metal-dielectric inclusions) can be as well described by a spatially dispersive dielectric function $\bar{\bar{\varepsilon}}_{\mathrm{eff}}(\omega, \mathbf{k})$ ($\mathbf{k}$ is the wave vector) linked to the parameters of the bianisotropic model ε, μ and κ by formula (6) of Ref. [4] (see also Ref. [5]). For $\mathbf{k} = \mathbf{0}$, the nonlocal dielectric function reduces to a scalar and is given by [6]:

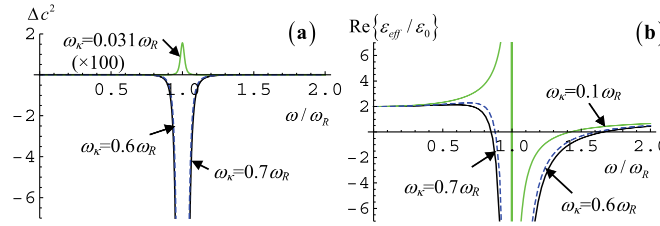

FIG. 1 (color online). (a) Δ as a function of ω/ω_R, for the parameters of Fig. 1 of Ref. [1]. The curves associated with $\omega_\kappa = 0.6\omega_R$ and $\omega_\kappa = 0.7\omega_R$ correspond to the parameters of the square and circle curves of Ref. [1], respectively. (b) $\mathrm{Re}\{\varepsilon_{\mathrm{eff}}\}$ as a function of frequency when all parameters related to loss are set equal to zero.

$$\frac{\varepsilon_{\mathrm{eff}}}{\varepsilon_0}(\omega, \mathbf{k} = 0) = \varepsilon(\omega) - \frac{[\kappa(\omega)]^2}{\mu(\omega)}. \qquad (3)$$

The key point is that because of causality the nonlocal dielectric function satisfies for each fixed wave vector the usual Kramers-Kronig formulas ([7], p. 14). In particular, in case of very low loss it is necessary that $\mathrm{Re}\{\varepsilon_{\mathrm{eff}}\}$ be a strictly increasing function of frequency, i.e., $\mathrm{Re}\{\varepsilon - \kappa^2/\mu\}$ must increase with frequency [2]. However, as shown in Fig. 1(b), when the parameters related to loss considered in Ref. [1] are set equal to zero $\mathrm{Re}\{\varepsilon_{\mathrm{eff}}\}$ has nonmonotonic behavior in case of strong chirality, showing that the corresponding material parameters are nonphysical. In fact, as long as the metamaterials are formed by metal-dielectric particles at the microscopic level, it is not viable to have Casimir repulsion at macroscopic distances, independent of the emergence of strong magnetoelectric coupling [8].

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