Nonresonant structured material with extreme effective parameters

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We demonstrate that a metamaterial formed by crossed metallic wires is characterized by an anomalously high index of refraction. Such property does not rely on a resonant response of the inclusions, but rather is a consequence of the ultrahigh density of the wires and of the strong electromagnetic interaction between crossed wires. It is shown that the bandwidth of such phenomenon can be quite large, and that it is mildly affected by metallic loss in the far-infrared domain. Prospective applications in the miniaturization of waveguides and other devices are discussed.

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The possibility of controlling the electromagnetic response of a material by tailoring its microstructure has been a topic of great interest in recent years. Novel materials with anomalous properties such as negative index of refraction[1] or near-zero permittivity[2] have been the subject of intense theoretical and experimental research. In particular, arrays of metallic wires have received significant attention due to their potentials in manipulating the electromagnetic field in the subwavelength spatial scale, which include magnifying, demagnifying, or repeating a given field distribution.[3–5] In this Brief Report, we propose a different application for structured materials formed by metallic wires. We demonstrate theoretically that a crossed wire mesh of nonconnected metallic wires may be characterized by an anomalously high index of refraction. It is discussed that such property may enable the miniaturization of waveguides and other devices. In this work, it is assumed that the electromagnetic fields have a time dependence of the form $e^{j\omega t}$.

The bulk metamaterial analyzed here is formed by two orthogonal meshes of infinitely long parallel metallic wires with radius r_w. Each mesh of parallel wires is arranged in a square lattice with lattice constant a. One set of wires is oriented along the direction $\hat{\mathbf{u}}_1=(1,0,1)/\sqrt{2}$ and is located in planes of the form $y=la$, where l is a generic integer. The complementary set of wires is oriented along the direction $\hat{\mathbf{u}}_2=(-1,0,1)/\sqrt{2}$ and is located in planes of the form $y=(l+1/2)a$. The wires are embedded in a host medium with relative permittivity ε_h, which for simplicity is taken as $\varepsilon_h=1$, except when noted otherwise. The complex relative permittivity of the metal is denoted by ε_m. For noble metals, it is a good approximation to suppose that at terahertz and infrared frequencies, ε_m follows a Drude dispersion model.[6] The geometry of the structure is similar to that of Fig. 1, except that the material is unbounded (there are no interfaces) and the wires are infinitely long.

In the long-wavelength limit, the described "double wire medium" can be characterized using homogenization techniques.[7,8] The dielectric function of the material is of the form $\bar{\bar{\varepsilon}}=\hat{\mathbf{u}}_y\hat{\mathbf{u}}_y+\varepsilon_{11}\hat{\mathbf{u}}_1\hat{\mathbf{u}}_1+\varepsilon_{22}\hat{\mathbf{u}}_2\hat{\mathbf{u}}_2$. Using the results of Ref. 9 to take into account the effect of the finite conductivity of the metal, it is found that the permittivity components are

$$\varepsilon_{ii}(\omega,k_i) = 1 + \frac{1}{\dfrac{1}{(\varepsilon_m-1)f_V} - \dfrac{(\omega/c)^2-k_i^2}{\beta_p^2}}, \quad i=1,2 \qquad (1)$$

where $\beta_p=\{2\pi/[\ln(a/2\pi r_w)+0.5275]\}^{1/2}/a$ is the plasma wave number, $f_V=\pi(r_w/a)^2$, c is the speed of light in a vacuum, $\mathbf{k}=(k_x,k_y,k_z)$ is the wave vector, and $k_i=\mathbf{k}.\hat{\mathbf{u}}_i$. Notice that since the length of the wires is much larger than the lattice constant, the material is spatially dispersive and the dielectric function depends explicitly on $\mathbf{k}$.[7,8]

Here, we are mainly interested in propagation along the yoz plane with $k_x=0$. In such case, the dielectric function reduces to a scalar in the xoz plane, $\varepsilon(\omega,k_z)=\varepsilon_{ii}(\omega,k_i)$, $i=1,2$, because for $k_x=0$ we have $k_1=k_2=k_z/\sqrt{2}$. Thus, the dispersion characteristic of the plane-wave modes supported by the bulk medium with electric field polarized along the x direction is

$$\varepsilon(\omega,k_z)(\omega/c)^2 = k_y^2+k_z^2, \quad \mathbf{k}=(0,k_y,k_z) \qquad (2)$$

which corresponds to a quadratic equation in the variable k_z^2. Hence, for each ω and k_y fixed, there are two different solu-

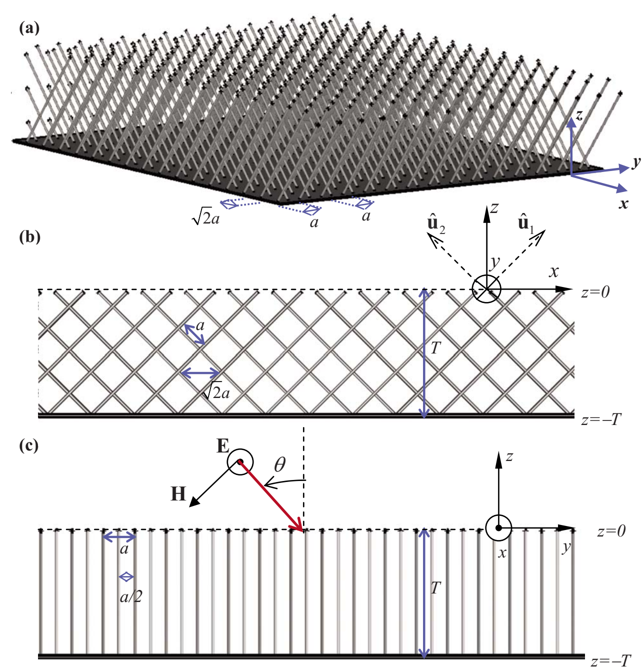

FIG. 1. (Color online) Panel (a): Geometry of a structured substrate formed by a "double wire medium." Panels (b) and (c) show cuts of the structure along the planes xoz and yoz, respectively. At $z=-T$ the wires are connected to a conducting ground plane.

tions for k_z^2. This means that the material supports two distinct plane waves with the same ω, k_y, and electric field, but with different k_z. This anomalous phenomenon is a consequence of strong spatial dispersion effects.[7,8]

Let us analyze first the case of wires made of perfect electric conductors (PEC), i.e., $\varepsilon_m = -\infty$. Using Eq. (2) and assuming propagation along z, it is found that the effective index of refraction $n_{\mathrm{ef}}^{(1,2)}$ is given by

$$n_{\mathrm{ef}}^{(1,2)} \equiv \left. \frac{k_z^{(1,2)} c}{\omega} \right|_{k_y=0} = \sqrt{\frac{3}{2} \pm \frac{1}{2}\left[1 + 8\left(\frac{\beta_p c}{\omega}\right)^2\right]^{1/2}} \quad (3)$$

where $k_z^{(1)}$ and $k_z^{(2)}$ are the propagation constants of the two modes supported by the bulk material, and the superscripts "1" and "2" are associated with the "+" and "−" signs, respectively (notice that n_{ef} depends on the considered mode). In the very long-wavelength regime we have that $a \ll \lambda$, and thus $\omega/c \ll \beta_p$ (this follows from $\beta_p \sim 1/a$). Hence, Eq. (3) implies that the electromagnetic mode associated with the "−" sign is strongly attenuated, whereas the mode associated with "+" sign propagates seeing a very large index of refraction.

It is this latter mode the main subject of study of this work. For a fixed frequency ω, its index of refraction increases more and more as the density of wires increases (a decreases), supposing a constant metal volume fraction (r_w/a=const). This means that n_{ef} may be made arbitrarily large by increasing the density of wires, at least if metallic losses remain negligible. This is a surprising result since intuitively, one might expect that the material would be characterized by a negative permittivity rather than by $n_{\mathrm{ef}} \gg 1$. The large index of refraction stems from the fact that the wires oriented along the direction $\hat{\mathbf{u}}_1$ tend to obstruct the propagation of transmission line modes[9] in the wires oriented along the direction $\hat{\mathbf{u}}_2$, and vice versa.

To illustrate the suggested possibilities and assess the effect of metallic losses and plasmonic effects, we have calculated the index of refraction $n_{\mathrm{ef}} = n'_{\mathrm{ef}} - jn''_{\mathrm{ef}}$ of the propagating mode as a function of the free-space wavelength λ_0 for different lattice constants a and silver (Ag) rods (Fig. 2). We have assumed that Ag follows the Drude dispersion model proposed in Ref. 6 obtained from experimental data at terahertz and infrared frequencies [$\varepsilon_m = \varepsilon_\infty - \omega_p^2/\omega(\omega - j\Gamma)$, with $\varepsilon_\infty = 5.0$, $\omega_p/2\pi = 2175$ [THz] and $\Gamma/(2\pi = 4.35$ [THz]]. Notice that for each curve, the metal fill fraction is constant, as well as the electrical size of the unit cell (a/λ_0=const). For long wavelengths, Ag behaves practically as a PEC material and thus n'_{ef} is described very accurately by Eq. (3). For example, when $a = 0.01\lambda_0/2\pi$ the index of refraction can be as large as 16.6. On the other hand, starting at the far-infrared range, the conducting properties of Ag deteriorate, and plasmonic effects may become important. It can be verified that the PEC approximation is only valid when $r_w > \delta_s$, where δ_s is the skin depth of Ag (shown in Fig. 2 at selected wavelengths). When $r_w < \delta_s$, plasmonic effects dominate, and n'_{ef} becomes even larger. This property can be justified by noting that for a plasmonic material the fields are highly concentrated near the wires, and consist of a collection of (loosely coupled) surface-plasmon polaritons that propagate

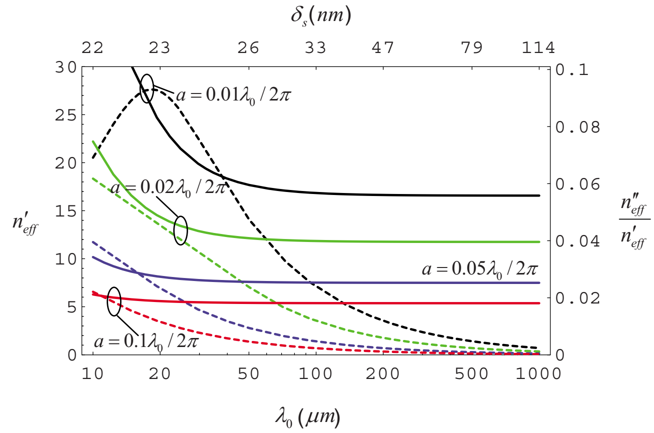

FIG. 2. (Color online) Index of refraction $n_{\mathrm{ef}} = n'_{\mathrm{ef}} - jn''_{\mathrm{ef}}$ as a function of the free-space wavelength for different values of the lattice constant a. The solid lines represent n'_{ef}, whereas the dashed lines represent $n''_{\mathrm{ef}}/n'_{\mathrm{ef}}$. Notice the different vertical scales. The wires are made up of Ag and have a radius $r_w = 0.05a$. The skin depth of Ag, δ_s, is also indicated at selected wavelengths.

strongly attached to the wires, consistently with the results of Refs. 9 and 10. Such regime is less interesting since it is more sensitive to losses; Not to mention that when r_w is less than a few tenths of nanometers, silver may not be adequately described by a continuous material model. It is also remarkable that the effect of losses may be relatively mild in the considered frequency range: $n''_{\mathrm{ef}}/n'_{\mathrm{ef}} \ll 1$. In fact, provided that $r_w > \delta_s$, the electromagnetic fields propagate mainly in the air region, being extruded from the metal. Thus, it is expected that if the radius of the wires is made larger, the effect of losses will be weaker [the model (1) only applies to thin wires $r_w/a \ll 1$].

The extreme optical parameters of the metamaterial may enable the miniaturization of waveguides and other devices. To demonstrate this, we consider a structured substrate formed by an array of crossed PEC wires (Fig. 1) with thickness T. It is supposed here that the substrate is backed by a PEC ground plane since this situation is of potential interest in the design of waveguides in integrated circuits, antenna substrates, and other devices. However, effects qualitatively similar to those described next are still observed when the ground plane is removed. The wires are connected with good Ohmic contact to the ground plane. Let us consider first the problem of reflection of a plane wave by the structured substrate [Fig. 1(c)]. The electric field in all space can be written as (the y variation of the fields is omitted)

$$E_x = E_x^{\mathrm{inc}}(e^{\gamma_0 z} + \rho e^{-\gamma_0 z}), \quad z > 0$$

$$E_x = A_1^+ e^{-jk_z^{(1)}z} + A_1^- e^{+jk_z^{(1)}z} + A_2^+ e^{-jk_z^{(2)}z} + A_2^- e^{+jk_z^{(2)}z}, \quad z < 0 \quad (4)$$

where E_x^{inc} is the incident field, ρ is the reflection coefficient, $\gamma_0 = \sqrt{k_y^2 - \omega^2 \varepsilon_0 \mu_0}$, $k_y = (\omega/c)\sin\theta$ and only depends on the angle of incidence, $k_z^{(1,2)}$ are the propagation constants of the modes excited inside the wire medium [obtained by solving Eq. (2) with respect to k_z], and $A_i^{\pm}$ are the corresponding

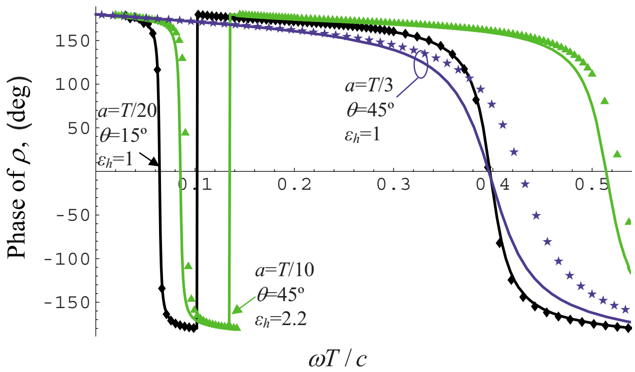

FIG. 3. (Color online) Phase of the reflection coefficient as a function of normalized frequency for various lattice spacings and incident angles θ. The radius of the wires is r_w=0.05a. The discrete symbols and the solid lines correspond to the full wave simulations and to the theoretical model, respectively.

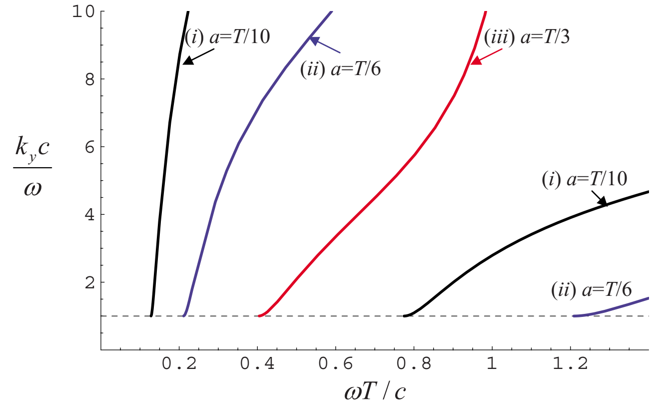

FIG. 4. (Color online) Normalized propagation constant k_y of the TE-guided modes as a function of frequency, for a fixed thickness T of the substrate and different lattice spacings a. The radius of the wires is r_w=0.05a.

amplitudes. The boundary conditions at the interfaces impose that the electric field vanishes, E_x=0, at z=$-T$, and that the tangential electromagnetic fields E_x and H_y are continuous at z=0 (the continuity of H_y is equivalent to the continuity of dE_x/dz). However, these classical boundary conditions are insufficient to solve the scattering problem.[11,12] This is a consequence of spatial dispersion, and in particular of the existence of two waves with the same polarization. To remove the extra degrees of freedom we use a generalization of the additional boundary conditions (ABCs) recently introduced in Refs. 12 and 13.

Using such homogenization concepts, we calculated the phase of the reflection coefficient as a function of the normalized frequency for various geometries (solid lines in Fig. 3). The discrete symbols shown in Fig. 3 correspond to the numerical results obtained using the commercial electromagnetic solver CST MICROWAVE STUDIO™. The agreement between our analytical model and the full wave simulations is excellent, especially for a/T small. Since the structured material is backed by a PEC plane, for very low frequencies the phase of ρ is approximately 180°. For a regular dielectric with permittivity ε_d, the phase crosses zero at a frequency such that $\omega T/c=\pi/(2\sqrt{\varepsilon_d})$ (for normal incidence). Thus, Fig. 3 confirms that the equivalent permittivity of the structured substrate is extremely large, and that when the density of the wires is increased (a is reduced), the effective permittivity increases. The frequency ω_0 at which the phase vanishes is very stable with respect to variations of the angle of incidence. In such regime, the structured substrate behaves in many ways as perfect magnetic conductor with $\rho=+1$ (Ref. 14).

The extreme permittivity of the structured substrate does not rely on the individual resonant response of the metallic wires. In fact, the length of each metallic wire is only $\sqrt{2}T$, which is well below the traditional $\lambda/2$ resonance length at the frequency ω_0. In particular, since each wire is spanned over many unit cells, the size of the inclusions is independent of the lattice constant a. In this respect, the considered substrate is truly unique. Indeed, its effective index of refraction does not saturate as a is made smaller, as would happen in a conventional metamaterial substrate. In fact, in a conventional design, if the size of the inclusions is scaled with the size of the cell, the response of the structure becomes independent of a in the quasistatic limit. Quite differently, when the density of wires is increased in a crossed wire mesh, keeping the metal volume fraction constant, the index of refraction increases with no bounds provided that $r_w > \delta_s$.

Using the developed analytical model, we computed the dispersion characteristic of the surface wave modes supported by the substrate (Fig. 4). The dispersion characteristic is obtained by calculating the values of k_y for which the

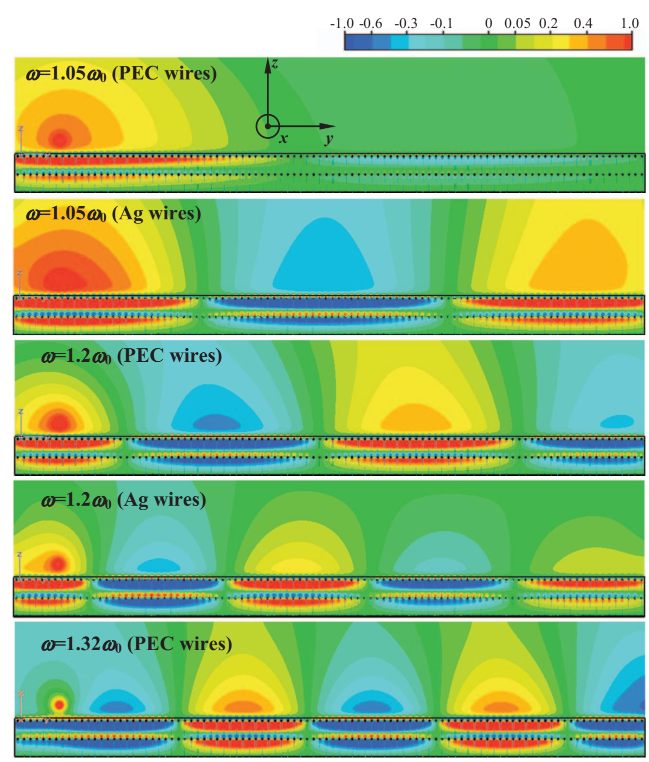

FIG. 5. (Color) Time snapshot of normalized E_x at different frequencies of operation when a finite length substrate with thickness T=5.7 μm is excited by a dipole antenna. The length of the substrate along the y direction is L=16T, the lattice constant is a=T/3, and the radius of the wires is r_w=0.05a.

structured substrate can sustain a propagating wave attached to the interface with no incident field, $E_x^{\text{inc}}=0$. A standard grounded dielectric substrate only supports transverse electric (TE)-guided modes when the thickness T of the substrate is larger than $\lambda_0/4\sqrt{\varepsilon_d-1}$, which corresponds approximately to $\omega>\omega_0$. Figure 4 demonstrates that the equivalent permittivity of the structured substrate can be dramatically large. For example, for the case $a=T/10$ surface waves start propagating for $T>\lambda_0/48.3$, which yields an equivalent permittivity of $\varepsilon_d=147$. In general, the guided modes start propagating at the frequencies where the phase of ρ vanishes for normal incidence. The number of supported guided modes increases with frequency.

In order to confirm the existence of surface waves and demonstrate the good prospects for the realization of very subwavelength waveguides at terahertz frequencies, we have simulated the response of a finite length substrate with thickness $T=5.7\ \mu$m excited by a dipole-type antenna with input impedance 50 Ω. To simplify the numerical modeling, the structure was assumed periodic along the x direction and open boundaries were defined at the remaining interfaces. The amplitude of the E_x component of the electric field is depicted in Fig. 5 for different values of ω. The frequency $\omega_0/(2\pi)=3.27$[THz] corresponds to the theoretical frequency where guided modes start propagating, which, from Fig. 4, verifies $\omega_0 T/c=0.4$ or equivalently $T=\lambda_0/15.7$.

For PEC wires, Fig. 5 shows that at $\omega=1.05\omega_0$, some field concentration starts being visible near the interface, but the mode is still very weakly bounded to the substrate. At this frequency, the length of the substrate is as small as the free-space wavelength (which is approximately equal to the wavelength of the guided mode). As the frequency increases, the surface wave becomes the dominant radiation mechanism, and the wavelength of the guided mode becomes shorter and shorter, consistently with the predictions of Fig. 4. Full wave simulations (not shown here) demonstrate that the return loss of the dipole antenna is -8 dB around $\omega=1.32\omega_0$, despite the proximity of the antenna to the ground plane. To study the effect of metallic loss, we have also analyzed the case in which the wires and the ground plane are made of Ag. At $\omega=\omega_0$ the skin depth of Ag verifies $\delta_s\approx 0.3r_w$. It is seen in Fig. 5 that when metal loss is considered, the wavelength of the guided mode becomes shorter. This is consistent with our theoretical results which predict that n_{ef} increases when losses are added to the system. It is also clear from Fig. 5 that the attenuation resulting from metal loss is relatively mild, and that the anomalously high index of refraction is not due to a resonant phenomenon and may be observed over a wide bandwidth. This property is a consequence of the ultrahigh density of wires.

In conclusion, we have demonstrated that an array of crossed nonconnected wires may behave as material with extremely large effective positive permittivity and low loss. Unlike conventional metamaterials, the response of the considered structure does not saturate in the quasi-static limit when the density of wires is increased and the metal volume fraction constant is kept constant. It was proven that as long as the skin depth of the metal remains smaller than the radius of the wires, the effective index of refraction can be made arbitrarily large by increasing the density of wires. We envision that the considered material may be used in the design of ultra-compact waveguides and other devices. Finally, we note that the proposed structure may be fabricated using lithography techniques such as the superlattice nanowire pattern transfer described in Ref. 15.

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