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Hawking-type radiation in transluminal gratings

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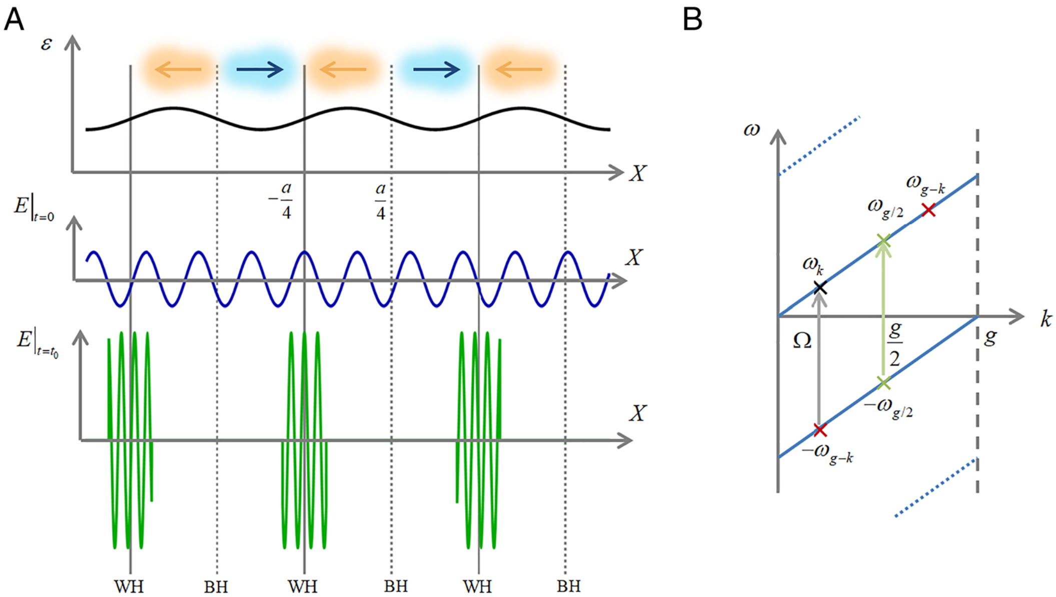

Fig. 1. (*A*, *Top*) Permittivity profile of a matched transluminal grating. The arrows indicate the direction of the relative velocity of the wave that propagates toward $+x$ with respect to the grating. The dashed vertical lines delimit the subluminal and superluminal regions and indicate the location of the event horizons (WH and BH). (*Middle* and *Bottom*) Electric field profile along the grating right after the modulation is switched on (*Middle*) and after some time interval has elapsed so that the amplification factor is $1/\gamma = 4$. The initial field distribution is drained toward the WH singularities. (*B*) Illustration of the allowed interband transitions in a transluminal grating when $\alpha << 1$. The figure shows the band structure of the modes that copropagate with the matched grating in the absence of the time modulation. The transitions triggered by the time modulation resonantly couple the modes with frequencies ω_k and ω_{k-g} leading to the spontaneous emission of correlated photon pairs. The mode with frequency $\omega_{g/2} = \Omega/2$ is resonantly coupled to itself and is the primary radiation channel in the transluminal grating.

In recent times, there has been a significant surge in interest toward time-varying optical platforms (1). These platforms hold a dual allure: Not only do they promise novel nonreciprocal and active electromagnetic devices, but they also open doors to explore the intriguing landscapes of wave physics (1–7). The interaction of light with spacetime gratings stands out as particularly captivating, as it mimics the way waves engage with moving systems, a subject that has been explored in depth (4–7). Now, a new and remarkable proposition comes from Horsley and Pendry, introducing a theoretical concept that pushes the boundaries even further (8). Their proposal envisions spacetime gratings functioning within a transluminal regime, giving rise to an optical analogy of the Schwarzschild singularity found in Einstein's field equations.

The theoretical model of Horsley and Pendry relies on a matched spacetime grating described by identical permittivity and permeability with a sinusoidal-type profile:

$$\varepsilon(X) = \mu(X) = 1 + 2\alpha\cos(gX),\ X = x - c_g t\cdot \qquad [1]$$

The permittivity and permeability are defined with respect to some background where light propagates with speed c_0. In the above, α determines the modulation strength, $a = 2\pi/g$ is the grating spatial period, and $c_g > 0$ is the grating modulation speed. The parameters of the grating are dynamically modulated in time with frequency $\Omega = c_g g$. The spacetime modulation of the grating parameters is reminiscent of a linear physical motion with a constant speed c_g. Therefore, the optical response of the spacetime grating shares some similarities with the response of a moving system, albeit the two cases are not in general equivalent (6).

The local speed of light in the grating is governed by $c(X) = c_0/\varepsilon(X)$. This speed varies within the range $c_- \le c(X) \le c_+$ with $c_\pm = c_0/(1 \mp 2\alpha)$. In their proposition, Horsley and Pendry venture into the peculiar transluminal regime where the grating speed is $c_g = c_0$. In such transluminal setting, the grating propagates faster than light in some segments ($c_g > c(X)$, superluminal region), while it

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propagates slower than light in other segments ($c_g < c(X)$, subluminal region).

Remarkably, the coexistence of subluminal and superluminal regions within the same grating forges spacetime pockets from which light waves copropagating with the grating cannot escape: the singularity region ($c_g > c(X)$). Each of these spacetime pockets is demarcated by two-event horizons, where the local speed of light matches the grating speed, $c = c_g$ —an optical analogue of the Schwarzschild singularity. Event horizons wherein $dc/dX < 0$ ($dc/dX > 0$) are optical analogues of black (white) holes (BH and WH). Correspondingly, light copropagating with the grating finds itself trapped within the singularity region, unable to traverse beyond the BH-like event horizon. Conversely, light waves that traverse the subluminal regions cannot breach the confines of these spacetime pockets, defined by the WH-like event horizon. Over time, the energy of the wave copropagating with the grating undertakes a fateful process and is inexorably drawn toward the WH event horizons (Fig. 1*A*). This accumulation arises due to the opposite signs of the relative velocity of the wave with respect to the grating within and outside the singularity pockets (9, 10). Roughly speaking, each parcel of the wave in a unit cell is simultaneously amplified and compressed by a factor $1/\gamma$, where $1/\gamma = e^{2\alpha\Omega t}$ grows exponentially with the interaction time (8) (Fig. 1*A*). It is noteworthy that light waves counterpropagating with the grating can pass through the singularity region without being drawn toward the event horizons.

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Horsley and Pendry delve into the interaction of the quantum vacuum with a transluminal grating. Interestingly, they demonstrate that the coupling between positive and negative frequency oscillators can lead to the emission of correlated photon pairs, bearing resemblance to the radiation predicted by Hawking to arise from a BH event horizon.

It is enlightening to examine the significance of negative frequencies in the Hawking-like spontaneous emission process identified in ref. 8. Due to the reality of the electromagnetic fields, positive and negative frequencies inevitably blur together, describing the same physical process like a shared oscillation that cannot be distinguished separately. However, the presence of temporal modulation allows for a clear differentiation of the interactions involving positive and negative frequencies.

To further expand on this point, consider the interband transition process in a spacetime-modulated grating (11): If one begins with a mode with frequency ω, it may resonate with other modes with frequencies $\omega \pm \Omega$. The transition $\omega \rightarrow \omega + \Omega$ is due to the interaction of oscillators with the same frequency sign ($e^{\pm i\omega t}$ interacts with $e^{\pm i\Omega t}$; it is implicit that $\omega, \Omega > 0$), whereas the transition $\omega \rightarrow \omega - \Omega$ is due to an interaction of oscillators with the different frequency signs ($e^{\pm i\omega t}$ interacts with $e^{\mp i\Omega t}$). The reason why the two processes are fundamentally different is that the transition $\omega \rightarrow \omega + \Omega$ always describes the coupling between two physically distinct oscillators, whereas the transition $\omega \rightarrow \omega - \Omega$ may describe the coupling of a mode with itself. Such a situation occurs when $\omega - \Omega = -\omega$ because positive and negative frequencies of the same mode describe the same physical oscillation. Thus, through the interaction between negative and positive frequencies, it may be possible to resonantly extract energy from the time modulation to the system, leading to an amplification of the modes with oscillation frequency $\omega = \Omega/2$. The electromagnetic instabilities that may arise from this process have been previously discussed in different contexts (12–17).

The above discussion can be readily extended to the quantum realm. For example, consider the case of a finite-length spacetime grating terminated with periodic boundary conditions. This is a slightly modified version of the setup considered in ref. 8. Such a system may be implemented, for example, in the form of a circular resonator. Suppose that the time modulation is switched on at time $t = 0$. A linear time-variant system can always be modeled by a quadratic time-dependent Hamiltonian written in terms of the creation and annihilation operators associated with the grating modes of the static system. Thereby, the system Hamiltonian can always be expressed as $\hat{\mathcal{H}}(t) = \hat{\mathcal{H}}_{c}(t) + \hat{\mathcal{H}}_{nc}(t)$, with

$$\begin{aligned} \hat{\mathcal{H}}_{c}(t) &= \sum_k h_{c,k}(t)\left(\hat{a}_k \hat{a}_k^{\dagger} + h.c.\right), \\ \hat{\mathcal{H}}_{nc}(t) &= \sum_k h_{nc,k}(t)\left(\hat{a}_k \hat{a}_{g-k} + h.c.\right), \end{aligned} \qquad [2]$$

where "h.c." stands for the Hermitian conjugate term. For simplicity, I consider a single band model, so that k labels the Bloch wave vector of the 1st positive dispersion branch of the static grating (solid blue line in Fig. 1*B*). Furthermore, it was taken into account that the time modulation cannot change the wave vector k. Due to this reason, interactions of the type $\left(\hat{a}_k \hat{a}_{k'}^{\dagger} + h.c.\right)$ are only feasible for Bloch waves with $k' = k$, whereas interactions of the type $\left(\hat{a}_k \hat{a}_{k'} + h.c.\right)$ are only feasible when $k' = -k$ modulo g. For convenience, I take the Brillouin zone as $0 < k < g$. Note that at initial time $\hat{\mathcal{H}}_c$ is the Hamiltonian of the static grating and $\hat{\mathcal{H}}_{nc}$ vanishes. For $t > 0$, the two operators depend on the spacetime modulation.

As explained by Horsley and Pendry, the term $\hat{\mathcal{H}}_c(t)$ is associated with conservative interactions as it preserves the total particle number. Thereby, when it interacts with the quantum vacuum, it cannot generate light quanta (8). In contrast, the second component of the Hamiltonian, $\hat{\mathcal{H}}_{nc}(t)$, does not preserve the total particle number and describes a squeezing process that generates correlated photon pairs.

For a weakly modulated system, ($\alpha << 1$) the band structure of the static grating is roughly identical to the light dispersion in a dielectric with $\varepsilon(X) = \mu(X) \approx 1$. Thus, the light dispersion in the static grating can be approximated by $\omega_k \approx c_0 k$, so that $\hat{\mathcal{H}}_{nc}$ models the interaction of modes with frequencies $\omega_k \approx c_0 k$ and $\omega_{g-k} \approx c_0 \left(g - k\right)$ (Fig. 1*B*). As $\omega_k + \omega_{g-k} \approx c_0 g \approx \Omega$, it follows that $\mathcal{H}_{nc}$ describes the interband transition $-\omega_{g-k} \rightarrow \omega_k$ (or alternatively $-\omega_k \rightarrow \omega_{g-k}$), i.e., it describes a transition between frequencies with opposite signs.

Importantly, Horsley and Pendry show that the number density of the emitted quanta peaks near frequency $\omega = \Omega/2$. This corresponds to the situation discussed before, where a mode is resonantly coupled to itself through interband transitions pumped by the time modulation (green arrow in Fig. 1*B*). Thus, the spectral peak at $\omega = \Omega/2$ confirms the role of the negative frequencies in the emission process analyzed in ref. 8. Note that the Hamiltonian $\mathcal{H}_{nc}$ generates components of the field state of the type $\left|1_k, 1_{g-k}\right\rangle$, which in the resonant case ($\sqrt{2}\left|2_{g/2}\right\rangle$) is boosted by a factor of $\sqrt{2}$.

Interestingly, the analysis in ref. 8 reveals a notable trend: The spectrum of the radiation emitted by the transluminal grating asymptotically adopts a quasi-thermal distribution, akin to $e^{-\hbar\omega/k_B T_H}$ with T_H an equivalent Hawking temperature. Importantly, this Hawking temperature undergoes exponential growth in tandem with the interaction time. Consequently, even with a relatively modest modulation strength α, the prospect of experimentally observing Hawking-type radiation emanating spontaneously from the transluminal grating becomes conceivable. However, realizing this requires the modulation of the system for an adequately extended time period at a large modulation speed. Although this poses a significant challenge at optical frequencies, it could potentially be achievable at lower frequencies. Additionally, it is worth noting that the interaction of the grating with thermal light could potentially lead to a related spontaneous emission process. Therefore, exploring this avenue in the terahertz regime offers an interesting alternative direction to consider.

In conclusion, spacetime gratings offer an intriguing setting for investigating extreme wave phenomena. The insights from Horsley and Pendry's research in ref. 8 suggest the possible application of transluminal gratings to compress and amplify electromagnetic waves at synthetic event-horizon singularities. This is likely to stimulate additional studies in this thought-provoking area.

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1. E. Galiffi *et al.*, Photonics of time-varying media. *Adv. Photon.* **4**, 014002 (2022).
2. D. L. Sounas, C. Caloz, A. Alù, Giant non-reciprocity at the subwavelength scale using angular momentum-biased metamaterials. *Nat. Commun.* **4**, 1–7 (2013).
3. Z. Deck-Léger, N. Chamanara, M. Skorobogatiy, M. G. Silveirinha, C. Caloz, Uniform-velocity spacetime crystals. *Adv. Photon.* **1**, 1–26 (2019).
4. P. A. Huidobro, E. Galiffi, S. Guenneau, R. V. Craster, J. B. Pendry, Fresnel drag in space–time-modulated metamaterials. *Proc. Natl. Acad. Sci. U.S.A.* **116**, 24943 (2019).
5. P. Huidobro, M. G. Silveirinha, E. Galiffi, J. B. P. Pendry, Homogenization theory of space-time metamaterials. *Phys. Rev. Appl.* **16**, 014044 (2021).
6. F. R. Prudêncio, M. G. Silveirinha, Replicating physical motion with minkowskian isorefractive spacetime crystals. *Nanophotonics* **12**, 3007–3017 (2023), 10.1515/nanoph-2023-0144.
7. D. Oue, K. Ding, J. B. Pendry, Čerenkov radiation in vacuum from a superluminal grating. *Phys. Rev. Res.* **4**, 013064 (2022).
8. S. A. R. Horsley, J. B. Pendry, Quantum electrodynamics of time-varying gratings. *Proc. Natl. Acad. Sci. U.S.A.* **120**, e2302652120 (2023).
9. J. B. Pendry, E. Galiffi, P. A. Huidobro, Gain mechanism in time-dependent media. *Optica* **8**, 636 (2021).
10. E. Galiffi, M. G. Silveirinha, P. A. Huidobro, J. B. Pendry, Photon localisation and Bloch symmetry breaking in luminal gratings. *Phys. Rev. B* **103**, 214303 (2021).
11. J. N. Winn, S. Fan, J. D. Joannopoulos, E. P. Ippen, Interband transitions in photonic crystals. *Phys. Rev. B* **59**, 1551 (1999).
12. Y. B. Zeldovich, Generation of waves by a rotating body. *JETP Lett.* **14**, 180 (1971).
13. M. G. Silveirinha, Quantization of the electromagnetic field in non-dispersive polarizable moving media above the cherenkov threshold. *Phys. Rev. A* **88**, 043846 (2013).
14. M. G. Silveirinha, Optical instabilities and spontaneous light emission by polarizable moving matter. *Phys. Rev. X* **4**, 031013 (2014).
15. S. A. R. Horsley, S. Bugler-Lamb, Negative frequencies in wave propagation: A microscopic model. *Phys. Rev. A* **93**, 063828 (2016).
16. V. K. O. Sydoruk, E. Shamonina, L. Solymar, Terahertz instability of surface optical-phonon polaritons that interact with surface plasmon polaritons in the presence of electron drift. *Phys. Plasmas* **17**, 102103 (2010).
17. T. A. Morgado, M. G. Silveirinha, Negative Landau damping in bilayer graphene. *Phys. Rev. Lett.* **119**, 133901 (2017).