Morgado and Silveirinha Reply: In the preceding Comment [1], Svintsov and Ryzhii (SR) criticized the conductivity derived in our Letter using the self-consistent field approach [2,3]:

$$\sigma_g^{\rm drift}(\omega, q_x) = (\omega/\tilde{\omega})\sigma_g(\tilde{\omega}, q_x). \quad (1)$$

Here, $\sigma_g(\omega, q_x)$ is the nonlocal conductivity with no drift and $\tilde{\omega} = \omega - q_x v_0$. The drift effect was modeled by the interaction Hamiltonian $\hat{H}_{\rm int,drift} = \mathbf{v}_0 \cdot \hat{\mathbf{p}}$, with $\mathbf{v}_0 = v_0\hat{\mathbf{x}}$ being the drift velocity and $\hat{\mathbf{p}} = -i\hbar\nabla$ [2]. The use of this interaction Hamiltonian was motivated by an analogy with moving media [4]. Equation (1) extends to graphene the well-known result $\varepsilon^{\rm drift}(\omega, q_x) = \varepsilon(\omega - q_x v_0, q_x)$ for a drift-biased plasma [5–7] [for 3D materials $\varepsilon(\omega) = 1 + \sigma/(-i\omega\varepsilon_0)$].

SR argued that, because the electrons in graphene are massless, the Galilean-Doppler shift cannot be used. They rely on the distribution $f_{\rm drift}(\mathbf{k}) = f^0(\mathcal{E}_{\mathbf{k}}^0 - \hbar\mathbf{k}\cdot\mathbf{v}_0)$, which is applicable when the electron-electron (e–e) scattering predominates [8]. Here, $\mathcal{E}_{\mathbf{k}}^0$ is the energy dispersion of the relevant electronic band and $f^0(\mathcal{E})$ is the Fermi-Dirac distribution. SR used $f_{\rm drift}(\mathbf{k})$ in the Lindhard formula. However, they missed a subtle point. In the shifted Fermi distribution, $\hbar\mathbf{k}$ is a kinetic-type momentum rather than a canonical momentum [9]. The canonical momentum is $\mathbf{p} = \hbar\mathbf{k} - e\mathbf{A}$, with $\mathbf{A}$ the vector potential due to the static electric field $\mathbf{E}_0 = E_0\hat{\mathbf{x}}$. The vector potential is $\mathbf{A}(t) = -(t - t_0)\mathbf{E}_0$ in the intervals between e-e collisions ($t = t_0$ is the time instant of a collision).

The Lindhard formalism relies on the time evolution of Bloch states ($\psi_{n\boldsymbol{\kappa}}$). The Bloch wave vector $\boldsymbol{\kappa}$ determines the canonical momentum ($\mathbf{p} = \hbar\boldsymbol{\kappa}$). Thus, the relevant distribution for the Lindhard formula is the canonical-momentum distribution [10]. It is roughly

$$\tilde{f}_{\rm drift}(\boldsymbol{\kappa}) \approx f_{\rm drift}|_{\mathbf{k}=\boldsymbol{\kappa}+\hbar^{-1}e\mathbf{A}} \approx f^0(\mathcal{E}_{\boldsymbol{\kappa}}^0 + e\langle\mathbf{A}\rangle\cdot\mathbf{v}_{\boldsymbol{\kappa}}^0 - \hbar\boldsymbol{\kappa}\cdot\mathbf{v}_0)$$

where $\mathbf{v}_{\boldsymbol{\kappa}}^0 = \hbar^{-1}\partial_{\boldsymbol{\kappa}}\mathcal{E}_{\boldsymbol{\kappa}}^0$. In the second identity, we used a Taylor expansion, replaced $\mathbf{A}(t)$ by its time average $\langle\mathbf{A}\rangle$, and dropped the term $e\mathbf{A}\cdot\mathbf{v}_0$ because it is of second order ($\sim E_0^2$). Moreover, because $\mathbf{E}_0 = E_0\hat{\mathbf{x}}$ is space independent, the canonical momentum of an electron must be preserved by the static field (it is also preserved by the e-e collisions because, on average, they are independent of the space coordinates). This implies that $\tilde{f}_{\rm drift}(\boldsymbol{\kappa}) = f^0(\mathcal{E}_{\boldsymbol{\kappa}}^0)$. Thus, when the e-e collisions predominate, one has $e\langle\mathbf{A}\rangle\cdot\mathbf{v}_{\boldsymbol{\kappa}}^0 = \hbar\boldsymbol{\kappa}\cdot\mathbf{v}_0$.

Substitution of $\tilde{f}_{\rm drift}(\boldsymbol{\kappa}) = f^0(\mathcal{E}_{\boldsymbol{\kappa}}^0)$ in the Lindhard formula yields (taking the band overlap integral $F_{\boldsymbol{\kappa},\boldsymbol{\kappa}+\mathbf{q}} \approx 1$)

$$\sigma_{\omega,q}^{\rm drift} = \frac{i\omega e^2}{q^2}\frac{g_s g_v}{(2\pi)^2}\iint d^2\boldsymbol{\kappa}\frac{f^0(\mathcal{E}_{\boldsymbol{\kappa}}^0) - f^0(\mathcal{E}_{\boldsymbol{\kappa}+\mathbf{q}}^0)}{\hbar\omega + \mathcal{E}_{\boldsymbol{\kappa}} - \mathcal{E}_{\boldsymbol{\kappa}+\mathbf{q}}}. \quad (2)$$

Here,

$$\mathcal{E}_{\boldsymbol{\kappa}} \approx \langle\psi_{\boldsymbol{\kappa}}|\hat{H}_0(\hat{\mathbf{p}} + e\langle\mathbf{A}\rangle)|\psi_{\boldsymbol{\kappa}}\rangle \approx \mathcal{E}_{\boldsymbol{\kappa}}^0 + \langle e\mathbf{A}\rangle\cdot\langle\psi_{\boldsymbol{\kappa}}|\partial_{\mathbf{p}}\hat{H}_0(\hat{\mathbf{p}})|\psi_{\boldsymbol{\kappa}}\rangle$$

is the average electron energy during the interaction with the static field. Combining $\langle\psi_{\boldsymbol{\kappa}}|\partial_{\mathbf{p}}\hat{H}_0(\hat{\mathbf{p}})|\psi_{\boldsymbol{\kappa}}\rangle = \mathbf{v}_{\boldsymbol{\kappa}}^0$ and $e\langle\mathbf{A}\rangle\cdot\mathbf{v}_{\boldsymbol{\kappa}}^0 = \hbar\boldsymbol{\kappa}\cdot\mathbf{v}_0$, it is found that $\mathcal{E}_{\boldsymbol{\kappa}} \approx \mathcal{E}_{\boldsymbol{\kappa}}^0 + \hbar\boldsymbol{\kappa}\cdot\mathbf{v}_0$. Note that $\mathcal{E}_{\boldsymbol{\kappa}} \neq \mathcal{E}_{\boldsymbol{\kappa}}^0$ because the electron is accelerated by $\mathbf{E}_0$. Substituting $\mathcal{E}_{\boldsymbol{\kappa}} \approx \mathcal{E}_{\boldsymbol{\kappa}}^0 + \hbar\boldsymbol{\kappa}\cdot\mathbf{v}_0$ in Eq. (2), we recover Eq. (1) and the Galilean-Doppler shift (see [10] for additional discussion and a derivation with the Boltzmann equation).

Regarding the second point raised by SR about the long-wavelength approximation, we underline that the nonlocality precludes neither the negative Landau damping (NLD) nor the emergence of instabilities in graphene platforms. Indeed, the square root singularity of σ_g is compatible with gain regimes because, when $\tilde{\omega} = \omega - q_x v_0$ is negative, the prefactor $\omega/\tilde{\omega}$ of Eq. (1) is also negative. Thus, the NLD [$\mathrm{Re}\{\sigma_g^{\rm drift}(\omega, q_x)\} < 0$] can occur in the real-frequency axis (Fig. 1(a)) or in the upper-half frequency plane (UHP): $\omega'' = \mathrm{Im}\{\omega\} \geq 0$ with $\omega = \omega' + i\omega''$ [10,11]. The same result is predicted by the collisionless SR, Levitov *et al.*'s [12] and Polini *et al.*'s [13] models (see Fig. 1(a)). Thus, similar to Ref. [2], by coupling the drift-current biased graphene to a resonant system (here, a metal half-space) (Fig. 1(b)), it is possible to spontaneously generate terahertz and infrared radiation (Fig. 1(c)). The collisionless models of SR and Levitov predict quantitatively similar unstable regimes; all the models predict solutions with $f'' = \omega''/(2\pi) > 0$ that grow exponentially with time.

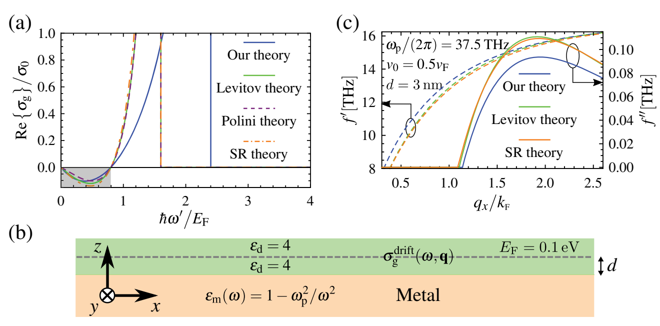

FIG. 1. (a) $\mathrm{Re}\{\sigma_g^{\rm drift}\}$ in the UHP as a function of $\hbar\omega'/E_F$ for $q_x = 1.6k_F$, $E_F = 0.1$ eV, $v_0 = 0.5v_F$, and $\omega'' = 0^+$. The NLD region is shaded in gray. (b) A drift-current biased graphene sheet and a metal half-space are separated by the distance d with a dielectric; (c) $\omega/(2\pi) = f' + if''$ for the unstable mode as a function of q_x.

In summary, the drift-biased conductivity is ruled by a Galilean transformation when the e-e collisions force the electron gas to move with a constant velocity. The instabilities predicted in our Letter may be observed in properly designed drift-current biased graphene platforms.

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Tiago A. Morgado[1,*] and Mário G. Silveirinha[1,2,†]

[1]Instituto de Telecomunicações and Department of Electrical Engineering, University of Coimbra
3030-290 Coimbra, Portugal

[2]Instituto Superior Técnico, University of Lisbon
Avenida Rovisco Pais, 1, 1049-001 Lisboa, Portugal

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*tiago.morgado@co.it.pt

†mario.silveirinha@co.it.pt

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[10] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.219402 for (i) additional discussion of the derivation of Eq. (1) using the Lindhard formula; (ii) derivation of Eq. (1) using the Boltzmann theory; (iii) an overview of the graphene conductivity models in the collisionless regime; (iv) dispersion equation of the graphene-dielectric-metal cavity; (v) analysis of the scattering of an evanescent wave by a drift-current biased graphene sheet; (vi) properties of the conductivity of a passive material in the upper-half frequency plane.

[11] Note that a passive medium has $\mathrm{Re}\{\sigma(\omega, q_x)\} \geq 0$ in the UHP for any real-valued q_x [10]. A response with $\mathrm{Re}\{\sigma(\omega, q_x)\} < 0$ may lead to power flows emerging from the graphene sheet [10].

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