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Two cases of spatial transformations

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Here, we give an overview of our work on two topics related to the theme of spatial transformations in wave theory, namely the concepts of transformation electronics and 'digital' metamaterials. In the first topic, we show that the notion of transformation optics can be extended to other physical phenomena such as tailoring the effective mass of charged carriers, e.g. electrons, in specially designed semiconductor superlattices. We discuss how the combination of thin layers of electronic materials with different effective mass of electrons may lead to bulk composite structures in which the effective mass of electrons may exhibit extreme anisotropy. For the second case, we show that any desired electromagnetic permittivity can, in principle, be engineered with proper combinations of two deeply subwavelength building blocks with relative permittivity values whose real parts have opposite signs. Owing to the presence of a plasmonic resonance between the two building blocks with oppositely signed dielectric constants, the achieved effective relative permittivity for the bulk composite may have values outside the range defined by the two permittivity values of the building blocks. We discuss some of the salient features of these two spatial transformation phenomena.

1. Introduction

In recent years, there have been exciting developments in the fields of metamaterials and metasurfaces due to their unique characteristics in electromagnetic and optical interaction [1–9]. Numerous studies and a variety of potential applications have been suggested and explored for bulk three-dimensional metamaterials and two-dimensional metasurfaces [1–9]. One of the interesting tools in the design of volumetric metamaterials and flat metasurfaces is the methodology of transformation optics [10–12], in which, by proper design of permittivity and/or permeability at the subwavelength scale, one can tailor the local electromagnetic fields to achieve the desired electromagnetic wave interaction with structures. This has opened doors to a set of exciting research topics such as cloaking [10], optical illusion [13,14] and light concentrators [15], to name a few. We have recently investigated how the notion of transformation optics can be transplanted into other physical phenomena such as superlattices of electronic materials with engineered effective mass of electrons, particularly when such an effective mass is extremely anisotropic, leading to the exciting possibility of tailoring and enhancing conductivity of such composites [16]. In another effort, we have also explored how the concept of metamaterials can be 'digitized', providing an interesting route to spatial transformation optics [17]. Here, we briefly review these two concepts.

2. Transformation electronics

The mobility of charged carriers, i.e. electrons and holes, in electronic materials such as semiconductors depends strongly on the effective mass of such carriers. The effective mass can be evaluated from the energy-momentum diagram using the relation $m^* = \hbar^2(\partial^2 E_{\mathrm{n}}/\partial k^2)^{-1}$, where E_{n} and k are the energy and momentum, and $\hbar \equiv h/2\pi$ is the reduced Planck constant. Engineering the effective mass of electrons may have several interesting consequences: (i) it may lead to higher mobility and conductivity, which may directly lead to a faster response in electronic and optical systems; (ii) in materials with free electrons the effective mass of the charged carriers, e.g. electrons, plays an important role in determining the relative permittivity. For example, the plasma frequency in the Drude model of an electron gas, $\varepsilon = \varepsilon_o(\varepsilon_\infty - \omega_p^2/\omega(\omega + \mathrm{i}\gamma))$, represented as $\omega_p^2 = Ne^2/m^*\,\varepsilon_o$, depends inversely on the effective mass. Therefore, the smaller effective mass of the electrons may lead to a higher plasma frequency, and therefore for a given operating frequency below the plasma frequency one may achieve higher negative values for the relative permittivity of the composite. For optical signals, this may imply that the optical composite material may have higher negative permittivity (so they may get closer to a 'perfect conductor' in higher frequencies as we do have nearly perfectly conducting materials in the radio frequency and microwave domains), thus less field can penetrate into the material, and consequently, under certain conditions, one may achieve lower loss, which will be a desirable outcome; and (iii) if one could design an electronic material with small effective mass of electrons, could this lead to materials with enhanced nonlinearities suitable for diodes for higher frequencies? If this could be possible, would we be able to consider the possibility of optical antennas equipped with such high-speed diodes? These are some possible interesting research directions and questions, which may benefit from the possibility of engineering the effective mass of electrons with extreme anisotropy.

In bringing the notion of transformation optics from electromagnetic metamaterials into the field of semiconductor electronics (thus coining the term 'transformation electronics' [16]), we were also inspired by the field of optical metatronics [18–23]. In optical metatronics, which deals with nanocircuits with light, collections of nanostructures designed with proper shapes, sizes and materials function as 'lumped optical circuit elements', thus providing us with an interesting platform to perform optical information processing at the nanoscale. Indeed, the field of optical metatronics was developed based on inspiration from electronic circuits, i.e. bringing the circuit concepts into the materials concepts. In other words, in optical metatronics, materials become circuits. In transformation electronics, however, this inspiration is in the other direction, i.e. we bring the notion of spatial transformations from optical materials back into the field of electronics and semiconductor design. In our recent work [16], we have explored a methodology for designing the effective mass of electrons using properly designed superlattices of properly selected semiconductors. We also got inspiration from the 'extreme metamaterials' concept, in particular the epsilon-near-zero (ENZ) metamaterials [24–27]. It is well known that stacks of thin layers of pairs of epsilon-positive and epsilon-negative materials can provide us with effective permittivity values that are highly anisotropic [28]. Figure 1*a* shows a sketch of such layered metamaterials. It is known that such a structure may exhibit the following effective relative permittivity values:

$$\varepsilon_{\text{eff}}^{\parallel} = \frac{d_1}{d_1 + d_2}\varepsilon_1 + \frac{d_2}{d_1 + d_2}\varepsilon_2 \tag{2.1}$$

and

$$\varepsilon_{\text{eff}}^{\perp} = \frac{(d_1 + d_2)\varepsilon_1\varepsilon_2}{d_2\varepsilon_1 + d_1\varepsilon_2}, \tag{2.2}$$

where ε_1, d_1 and ε_2, d_2 are the relative permittivity and thickness of the two layers, respectively, and $\perp$, $\parallel$ superscripts represent the orientation of the electric field perpendicular and parallel to the stack. One can easily note that if the real parts of ε_1 and ε_2 have opposite signs, then, under proper conditions, one can achieve the limiting cases of $\varepsilon_{\text{eff}}^{\parallel} \to 0$ and $\varepsilon_{\text{eff}}^{\perp} \to \infty$. Therefore, the ENZ materials can be constructed using such composites. Can this ENZ concept be applied to the effective mass of semiconductor materials? Indeed, this is possible, as we showed in [16]. By analogy, let us consider a stack of thin layers of semiconductor materials with oppositely signed effective mass of both electrons and holes. Examples of such materials can be mercury telluride, HgTe, which has an inverted band diagram with a negative energy gap and negative effective mass, and the ternary compound $Hg_{1-x}Cd_xTe$, which has a regular band structure and a positive effective mass for sufficiently large values of the mole fraction x [29,30]. Figure 1*b* shows the sketch of a superlattice, analogous to the ENZ metamaterials shown in figure 1*a*. Analysing the electron quantum mechanics in this semiconductor superlattice, details of which can be found in our work [16], we found that the dispersive mass of the entire superlattice is anisotropic and can be expressed as

$$M_{zz} = \frac{d_1}{d_1 + d_2}m_1 + \frac{d_2}{d_1 + d_2}m_2 \tag{2.3}$$

and

$$M_{xx} = M_{yy} = \frac{(d_1 + d_2)m_1 m_2}{d_2 m_1 + d_1 m_2}, \tag{2.4}$$

where, by analogy, m_1, d_1 and m_2, d_2 are the dispersive masses of the semiconductor alloys and the thickness of the two layers, respectively, and the subscripts zz, xx and yy denote the directions of motion of electrons in the z (normal to the stack) and in the transverse plane xy (parallel with the stack), respectively [16]. The dispersive mass is a parameter that depends on the electronic structure of the semiconductor and also on the energy [16]. In general, it may differ from the effective mass (the parameter that determines the electron transport properties) calculated from the curvature of the band structure. Here, we can also note some interesting limiting scenarios for the dispersive mass of the superlattice, analogous to the ENZ case, i.e. when one of m_1 and m_2 is negative, under certain conditions (at some specific energy level), we can have $M_{zz} \to 0$, $M_{xx} \to \infty$ and $M_{yy} \to \infty$, which represent extremely anisotropic values for the mass of electrons in such a properly designed superlattice shown in figure 1*c*. According to our analysis [16], the relationship between the energy and momentum in such a superlattice can be written as

$$E_{\text{n}} = \pm \frac{\hbar\,|k_z|\,v_P}{\sqrt{1 + k_{\parallel}^2(4\hbar^2 v_P^2/\Lambda^2)}}, \tag{2.5}$$

which, for electron waves with small momentum, simplifies to $E_{\text{n}} = \pm\hbar|k_z|\text{v}_P$ showing a linear variation. Here $\text{v}_P = \sqrt{E_P/(3m_0)}$ is Kane's velocity, m_0 is the free electron mass, $E_P = 2P^2 m_0/\hbar^2$, P being Kane's parameter [31], and $\Lambda = E_{\text{v},2} - E_{\text{v},1}$ is the difference in the valence band energy of the two layers. Hence, we see that, analogous to the dispersive mass, the effective mass of the superlattice is extremely anisotropic, being zero along the z-direction and infinitely large along the x- and y-directions. Figure 2*a*, adapted from our work first reported in [16], shows the energy–momentum diagram for the specific parameters chosen for HgTe and $Hg_{0.65}Cd_{0.35}Te$

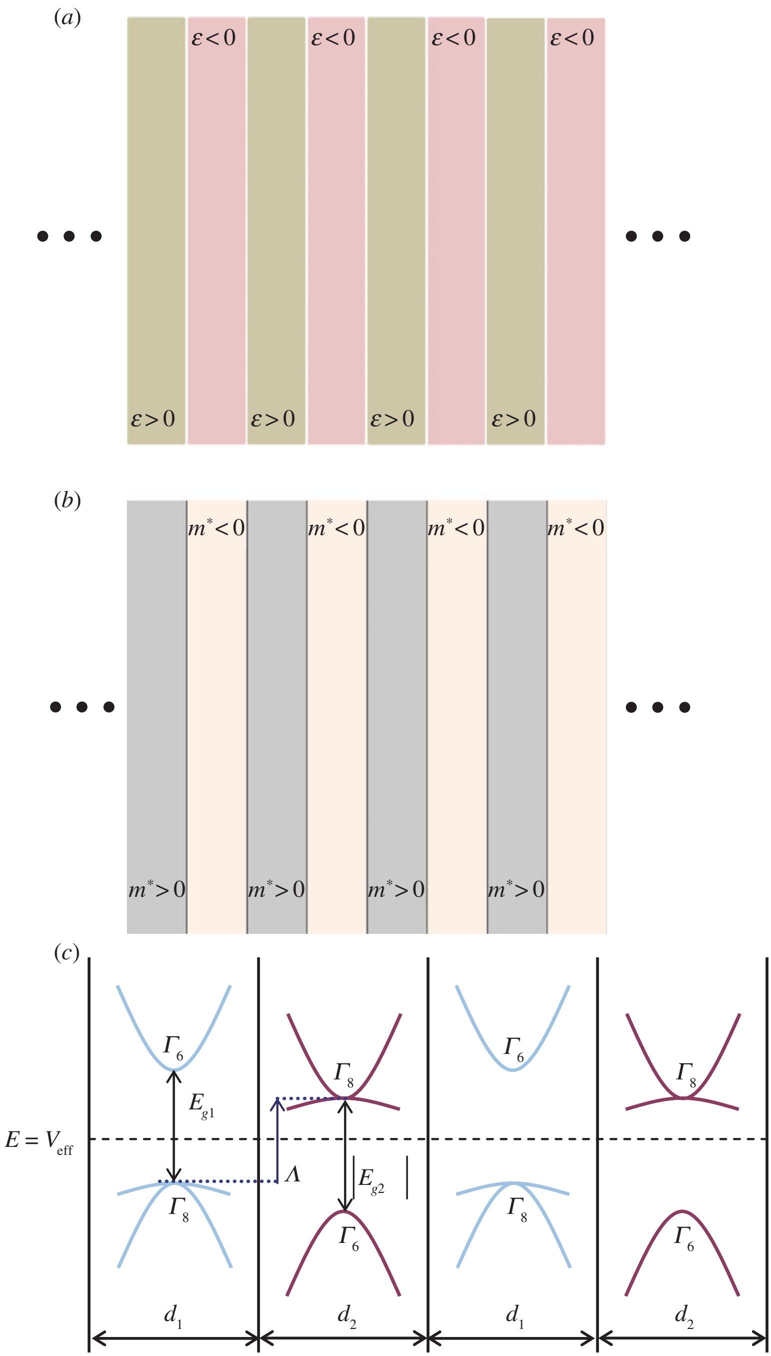

Figure 1. (*a*) An ENZ electromagnetic structure formed of a stack of pairs of thin layers of epsilon-positive and epsilon-negative materials. (*b*) Analogous to (*a*), here we consider an electronic composite material formed of a stack of effective mass-positive and effective mass-negative semiconductor materials. (*c*) Proposed semiconductor superlattice made of pairs of materials with positive effective mass (and positive band gap) and negative effective mass (and negative band gap). (Adapted from [16]. Copyright © American Physical Society.)

with $\Lambda = 0.40|E_g| = 0.12\,\text{eV}$, $v_P = \sqrt{E_P/3m_0} = 1.06 \times 10^6\,\text{m s}^{-1}$, $a = 6a_s = 3.9\,\text{nm}$, and the band gap energies $E_{g1} = -E_{g2} = 0.30\,\text{eV}$. We can clearly see that this diagram is significantly different from the conventional parabolic variation of the energy versus momentum. Consequently, the effective

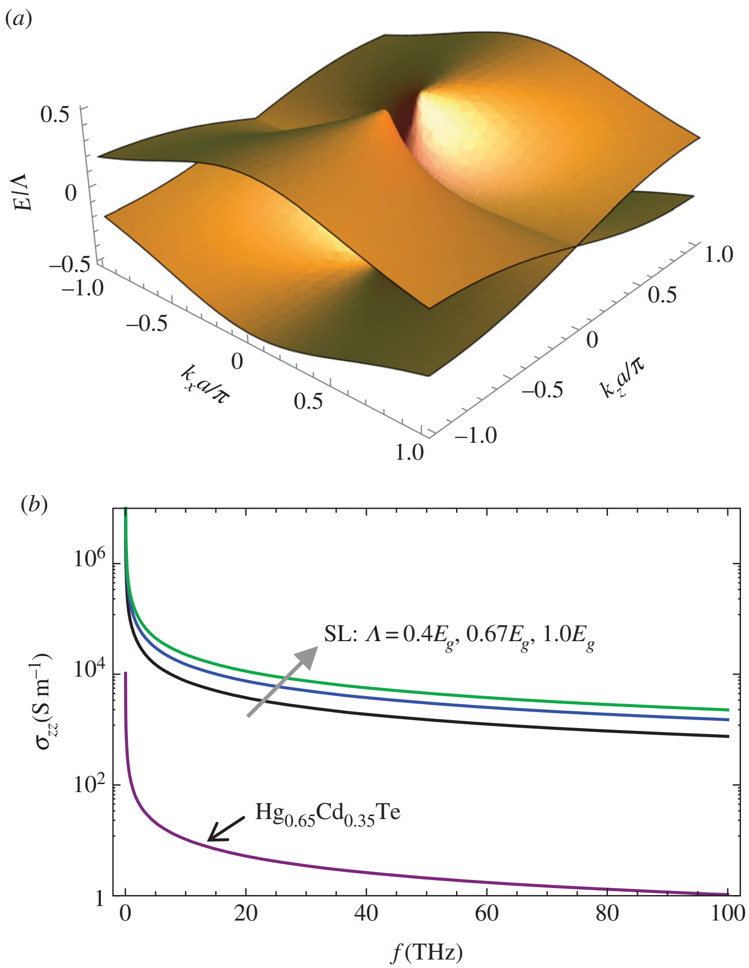

Figure 2. (*a*) Energy–momentum dispersion of the semiconductor superlattice formed by HgTe and $Hg_{0.65}Cd_{0.35}Te$ with $\Lambda = 0.12$ eV, $v_P = 1.06 \times 10^6$ m s^{-1}, $a = 6a_s = 3.9$ nm, $d_1 = d_2$ and $E_{g1} = -E_{g2} = 0.30$ eV. (*b*) Conductivity of our proposed semiconductor superlattice versus frequency for different values of the valence band offset at 300 K. (Adapted from [16]. Copyright © American Physical Society.)

mass, which is inversely proportional to the second derivative of energy versus momentum, is vastly different for electrons moving in the z-direction as compared with the electrons moving in the x or y directions. Ignoring collisions, we calculated the effective conductivity of such a semiconductor superlattice at room temperature, which is an imaginary quantity since the collision is ignored, as a function of frequency, and we compared it with the corresponding quantity for one of the building blocks, i.e. $Hg_{0.65}Cd_{0.35}Te$. We note significant enhancement (about 3 orders of magnitude) in the conductivity of electrons in the z-direction in this superlattice. Importantly, at room temperature the electron transport, and thereby the conductivity, is determined by a wide range of electronic states with energies separated by a value of the order of $k_B T$. Thus, the fact that conductivity enhancement is large demonstrates that the metamaterial effect is 'broadband' on the scale of the energy fluctuations determined by the thermal agitation, and hence is not due to a resonant effect. In subsequent works, we have further demonstrated that the paradigm of transformation optics may allow bulk materials to be designed with an isotropic zero mass and a giant nonlinear response [32], and to have a 'perfect lens' for electron waves [33]. Furthermore, the proposed ideas, in particular the extreme anisotropy regime, can be extended to graphene electronics [34,35]. Our theoretical study of tailoring the effective mass of electrons shows that the transformation electronics can indeed provide an exciting route to tailor the electronic transport in semiconductors and graphene and in general in 'electronic metamaterials'.

3. 'Digital' metamaterials

In science and engineering, the balance between simplicity and complexity is an important factor. Often simplifying a concept may lead to clearer intuition, and sometimes to better and more relevant applications. A good example is the concept of binary Boolean algebra and digital systems. This concept led to groundbreaking developments in the field of electronics and the birth of digital electronics. Motivated by the notion of simplicity versus complexity, we explored how the concept of metamaterials can be 'digitized'. In other words, if one could have access to only two materials, say materials A and B with two relevant material parameters (e.g. permittivity), would it be possible to combine them such that the resulting structure would exhibit an effective parameter that can be vastly different from those of materials A and B? If by analogy we call the two elemental material building blocks metamaterial 'bits', then the resulting structure, which can still be subwavelength, may be called a metamaterial 'byte' [17]. So our motivation to explore this topic was to show that it would be possible to design a metamaterial byte with an effective parameter (e.g. effective permittivity) outside the range of the permittivity of the two metamaterial bits. This is analogous to the binary system of '0's and '1's, where with proper combination one can get any number.

In our recent work [17], we indeed showed, analytically and numerically, that this would be possible. The condition to apply such an approach is that the permittivity of the two metamaterial bits should have oppositely signed real parts. This is justifiable, since in order to obtain an effective relative permittivity that is much larger than the permittivity of the bits, one needs to achieve plasmonic resonance, and this would require mixing an epsilon-positive material with an epsilon-negative material. Figure 3 shows the schematic of our concept. To obtain a desired permittivity as a function of spatial coordinates (in the figure we assume two coordinates x and y, for the sake of simplicity), we should first sample the area, and at each sample point we discretize the desired value of permittivity. Then each of these discretized values can be obtained by proper combination of two metamaterial building blocks. The orientation, volume fraction and relative locations of these two blocks should be taken into account for achieving the desired values of the permittivity in that cell. In our recent work [17], we analysed two categories of building blocks: (i) planar layers suitable for planar nanofabrication and (ii) core–shell two-dimensional cylindrical structures, suitable for self-assembly of nanorods and nanostructures. Here, we briefly review the second scenario, i.e. core–shell two-dimensional cylinders, which are currently being extended to the three-dimensional core–shell spherical particles. Let us consider a two-dimensional circular cylindrical structure made of two material bits, one with relative permittivity having positive real part (e.g. a dielectric with ε_d) and the other with relative permittivity having negative real part (e.g. a metal with ε_m). Figure 4 shows our theoretical results for the effective permittivity of a single two-dimensional core–shell cylinder when we choose $\varepsilon_d = 2.42$ for silica [36] and $\varepsilon_m = -4.70 + i0.22$ [37] for silver at the wavelength of 405 nm. In figure 4*a*, we show the case where the core is SiO_2 and the shell is silver with the complex effective relative permittivity of the core–shell structure shown as a function of the ratio of radii a/b. In this case, the effective relative permittivity attains values that are in between the permittivity values of the two bits SiO_2 and Ag. In figure 4*b*, we show the other scenario in which the locations of the metal and dielectric have been swapped. In this case, the complex effective relative permittivity of the core–shell structure behaves vastly differently, achieving values outside the range of ε_d and ε_m. This example reveals the point that, by combining two material bits, one can achieve a wide range of values of permittivity that can be significantly different from those of the original building blocks. The intrinsic metal loss in the frequency range where the relative permittivity is negative sets a limit on the accessible range of the effective permittivity that the two bits can synthesize. Furthermore, the frequency dispersion of the metal affects the bandwidth of the digital bytes since the effective permittivity of the byte becomes frequency-dependent as well.

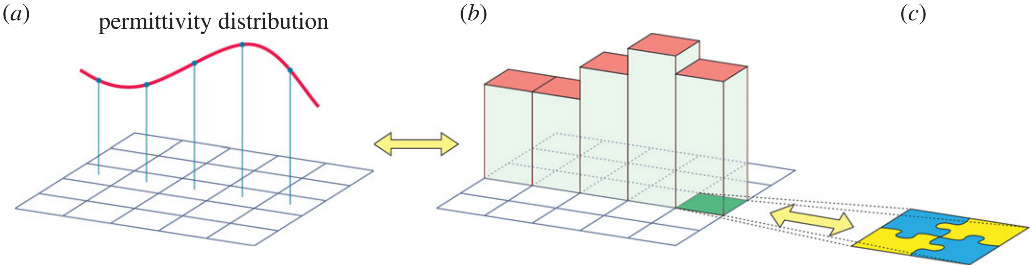

Figure 3. 'Digitizing' metamaterials: (*a*) distribution of desired relative permittivity; (*b*) sampling and discretizing desired values of desired permittivity; (*c*) proper combination of two metamaterial building blocks ('bits') taking into account orientation, volume fraction and relative locations of these material bits. (Inspired by fig. 1b of [17].) (Online version in colour.)

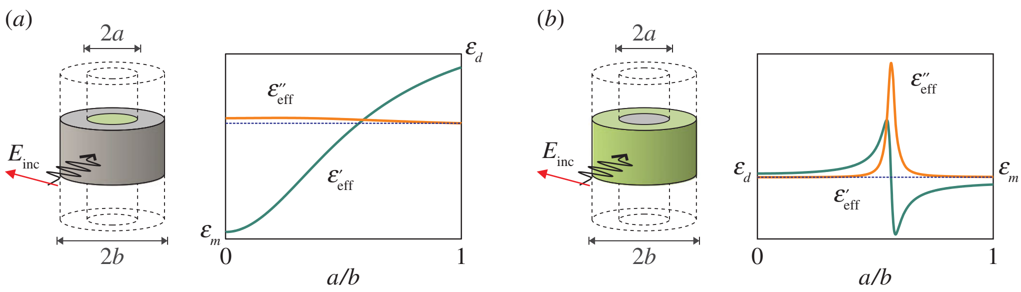

Figure 4. Metamaterial 'bytes': (*a*) SiO_2 as the core and silver as the shell of a two-dimensional circular cylinder of infinite length with deeply subwavelength diameter, and its effective relative permittivity as a function of the ratio of radii a/b. In this case, the complex values of the effective permittivity are in 'between' the two permittivity values of the bits. (*b*) Similar to (*a*), except here the core is silver and the shell is SiO_2. In this case, the complex values of the effective relative permittivity are highly different from (*a*), i.e. being outside the range defined by the two permittivity values of the bits. (Inspired by fig. 2b of [17].)

This aspect was not discussed in [17]. In our work in [17], we have shown several examples that illustrate how this concept of digital metamaterials can be applied to the design of optical devices and components. Particularly, we have shown how different types of lenses of different geometries (convex lenses, graded-index lenses, hyperlenses) and ENZ supercoupling, which require either uniform or graded permittivity distributions, can be designed using only SiO_2 and Ag metamaterial bits with proper spatial arrangements and filling fractions of one material with respect to the other one.

4. Summary

In conclusion, in this brief review we have highlighted some of the features of two cases of spatial transformations, one related to transformation electronics, in which the notion of metamaterials was transplanted into semiconductor materials with engineered effective mass of electrons, and the other related to digital metamaterials, in which with two building blocks one can construct material bytes with vastly different permittivity values. Both cases offer exciting possibilities for the design of novel platforms with rich and unusual light–matter interactions.

Authors' contributions. M.G.S. and N.E. contributed to the development of transformation electronics. C.D.G. and N.E. contributed to the development of digital metamaterials. N.E. supervised the projects. All authors contributed to the writing of the manuscript.

Competing interests. The authors declare no competing interests.

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