A Graphical Aid for the Complex Permittivity Measurement at Microwave and Millimeter Wavelengths

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***Abstract*—We introduce a novel procedure to retrieve the complex permittivity $\epsilon' - j\epsilon''$ of dielectric materials. It is a variant of the well-known waveguide method, and uses as input the one-port reflection data from a vector network analyzer connected to a short-circuited rectangular waveguide filled with a dielectric sample of known length. Here, it is shown that for low to moderate loss materials, the locus of the reflection coefficient in the complex plane versus frequency is approximately a circumference arc with curvature radius that depends mainly on ϵ'' and such that the swept angle depends mostly on ϵ'. It is proven that fitting the theoretical circumference arc with the measured data not only allows identifying possible measurement errors but also enables estimating the complex permittivity with good accuracy. A graphical based implementation of the method is described and validated experimentally.**

***Index Terms*—Measurement of complex dielectric permittivity, microwave and millimeter wave measurements, waveguide graphical method.**

I. Introduction

THE knowledge of the complex permittivity $\epsilon_{\rm r} = \epsilon' - j\epsilon''$ of a dielectric material is of key importance for the design of microwave and millimeter-wave components, such as printed circuits, filters, and antennas. For the past two decades, the authors have been working on dielectric lens antennas [1]. An accurate antenna design and characterization requires a precise knowledge of the material complex permittivity within the operating frequency band. For lens antennas applications, it is also essential to confirm that the dielectric material is to a good approximation isotropic and homogeneous before the lens fabrication. Typical materials used in dielectric lenses have low to moderate permittivity, $\epsilon' = 1 - 10$, and loss values of the order of $\tan(\delta) = \epsilon''/\epsilon' < 0.1$.

There are several well-known techniques for the measurement of the complex permittivity at microwaves and millimeter-waves. A very complete revision is presented in [2]. The waveguide based method allows using smaller size samples than open air methods. This is quite useful because it permits cutting samples from different parts of the bulk material, eventually with different orientations, and investigate the homogeneity and isotropy of the material. Here we propose a graphical based approach that allows measuring the complex permittivity of a material and quickly diagnoses the occurrence of common sources of error such as relatively wide air gaps or a misplacement of the sample. These imperfections can be detected during the course of measurements, allowing immediate corrective actions. The method allows for an unambiguous determination of the complex permittivity from a single material sample.

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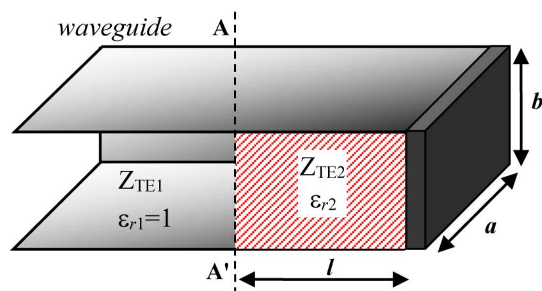

Fig. 1. Experimental setup (open view).

II. Formulation

The experimental setup is shown in Fig. 1. The TE_{10} single mode waveguide operation is assumed. The dimensions of the waveguide cross-section are a in the H-plane, and b in the E-plane. The sample under-test is non-magnetic and assumed non-dispersive within the frequency band of interest. For now, it is supposed that the sample with length l completely fills the waveguide cross-section, without air-gaps. The rectangular waveguide is terminated with a short circuit. A vector network analyzer (VNA) is used to obtain the reflection coefficient referred to the $\mathbf{AA}'$ plane, as the frequency is swept in the interval $[\omega_{\rm c} - 0.5\Delta\omega, \omega_c + 0.5\Delta\omega]$, where $\omega_{\rm c}$ is the central frequency and $\Delta\omega$ is the frequency span.

The reflection coefficient at AA' is calculated with transmission line theory. The waveguide transverse impedance in air (subscript $n = 1$) or in the dielectric $(n = 2)$ is given by

$$Z_{\mathrm{TE}n} = \omega\mu_0/\beta_n \tag{1}$$

where $\beta_{\rm n}$ is the longitudinal wave number in each medium

$$\beta_n = \sqrt{\omega^2\mu_0\varepsilon_0\varepsilon_{r\,n} - \left(\frac{\pi}{a}\right)^2}. \tag{2}$$

The reflection coefficient at the dielectric side of the $\mathbf{AA}'$ interface is

$$\rho_2 = -\exp\left(-j2\beta_2 l\right). \tag{3}$$

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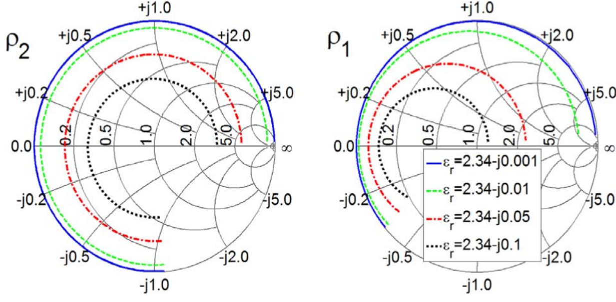

Fig. 2. Locus of the reflection coefficient from $f = 55$ to 65 GHz on the Smith Chart (or complex plane) for different permittivity values of a sample with $l = 5$ mm in a waveguide with width $a = 3.759$ mm: (a) reflection coefficient ρ_2 at the dielectric side of the $\mathbf{AA}'$ interface; (b) reflection coefficient ρ_1 at the air side of the $\mathbf{AA}'$ interface.

On the other hand, the reflection coefficient ρ_1 at the air side of interface $\mathbf{AA}'$, can be expressed in terms of ρ_2 as follows:

$$\rho_1 = \frac{(Z_{TE2} + Z_{TE1})\rho_2 + (Z_{TE2} - Z_{TE1})}{(Z_{TE2} + Z_{TE1}) + (Z_{TE2} - Z_{TE1})\rho_2}. \quad (4)$$

This can be rewritten in a more convenient way as

$$\rho_1 = \frac{\xi\rho_2 + 1}{\xi + \rho_2} = \xi + \frac{1 - \xi^2}{\rho_2 + \xi} \quad (5)$$

where

$$\xi = \frac{\beta_1 + \beta_2}{\beta_1 - \beta_2}. \quad (6)$$

For weak material loss, $\tan(\delta) = \epsilon''/\epsilon' < 0.1$, and a small frequency span $\Delta\omega$, (3) represents approximately a circumference arc in the Smith Chart (or complex plane). This is confirmed in Fig. 2, for a set of simulated curves in the frequency interval $[55, 65]$ GHz. Next, we note that the derivative of the parameter ξ with frequency is

$$\frac{\partial\xi}{\partial\omega} = 2\left(\frac{\pi}{a}\right)^2 \frac{1}{\beta_1\beta_2}\frac{\xi}{\omega}. \quad (7)$$

But typically we have

$$\frac{1}{\xi}\left|\frac{\partial\xi}{\partial\omega}\right|\Delta\omega = 2\left(\frac{\pi}{a}\right)^2\left|\frac{1}{\beta_1\beta_2}\right|\frac{\Delta\omega}{\omega_c} << 1 \quad (8)$$

hence, ξ may be considered approximately constant within the swept frequency band. Within this assumption the reflection coefficient ρ_1 given by (5) is a bilinear (Möbius) transformation

$$g(z) = \frac{c_1 z + c_2}{c_3 z + c_4} \quad (9)$$

with $c_1 = \xi, c_2 = 1, c_3 = 1$ and $c_4 = \xi$ complex constants. It is well known that this transformation maps circles and lines into circles and lines [3]. Thus, because in case of low loss ρ_2 lies in a circumference centered at the origin, we conclude that ρ_1 is mapped by (5) into another circumference with radius R

$$R = \frac{|\rho_2|}{\left||\xi|^2 - |\rho_2|^2\right|}\left|1 - \xi^2\right| \quad (10)$$

and whose center is shifted away from the origin to (x_0, y_0)

$$x_0 + j\,y_0 = \xi\frac{1 - |\rho_2|^2}{|\xi|^2 - |\rho_2|^2}. \quad (11)$$

Calculated examples are presented in Fig. 2.

Consistent with these results it can be checked that ρ_1 can be written as

$$\rho_1 = x_0 + j\,y_0 + R\,\exp(j\phi) \quad (12)$$

where the phase ϕ is given by

$$\phi = \pi - 2l\beta_2 - 2\text{atan}\left[\frac{\sin(2l\beta_2)}{\xi - \cos(2l\beta_2)}\right]. \quad (13)$$

The swept arc associated with ρ_1, $\Delta\phi$, is related to the swept frequency range $\omega_{\min} = \omega_c - 0.5\Delta\omega < \omega_c + 0.5\Delta\omega = \omega_{\max}$ as

$$\Delta\phi = \phi(\omega_{\max}, \varepsilon') - \phi(\omega_{\min}, \varepsilon'). \quad (14)$$

Thus, from the length (in radians) of the swept arc of circumference associated with the locus of ρ_1, one can obtain a first estimate for the unknown permittivity ϵ' by solving (14). Generally this equation has several solutions. The physical solution is extracted by comparing $\rho_1(\omega_{\min}, \epsilon')$ with the arc initial point. Once ϵ' is known one can compute ξ from (6), and then, from the radius R, we can find $|\rho_2|$

$$|\rho_2| = \frac{1}{2}\left[-\left(\frac{\xi^2 - 1}{R}\right) + \sqrt{\left(\frac{\xi^2 - 1}{R}\right)^2 + 4\xi^2}\right]. \quad (15)$$

This result is easily obtained from (10). Finally a first approximation for ϵ'' is found using

$$\varepsilon'' \approx -\frac{\ln(|\rho_2|)}{l\,\omega_c\sqrt{\mu_0\varepsilon_0}}\sqrt{\varepsilon' - \left(\frac{\pi}{a\omega_c\sqrt{\mu_0\varepsilon_0}}\right)^2}. \quad (16)$$

III. Example of Application

This section describes the application of the method to the determination of the complex permittivity of Polyethylene in the interval $[55, 65]$ GHz.

A V-band rectangular waveguide receptacle with nominal dimensions $a = 3.759$ mm, $b = 1.88$ mm and length $l = 5$ mm was fabricated and terminated with a short-circuit. A calibration procedure is used to accurately determine the actual values of a and l. This calibration involves the measurement of the reflection coefficient of the empty receptacle at the $\mathbf{AA}'$ interface for the frequency span of interest. An optimization procedure is used to fit the measured data (circumference in the Smith Chart) with (3). In the present example, this yields $a = 3.744$ mm and $l = 4.880$ mm.

The Polyethylene sample is then placed inside the sample holder, and ρ_1 is measured for this setup. The locus of the experimental data in the complex plane is shown in Fig. 3. Using a standard least square minimization one can find the circumference that best fits the measured data. It is found that it is centered at $(x_0, y_0) = (-0.002, -0.005)$, that the radius is $R = 0.995$ and that the swept arc is $\Delta\phi = 3.98$ radians. Desirably, the length l of the sample and the frequency span $\Delta\omega$ must be such that the measured arc amplitude is larger than 180° in order to reduce the estimation error in the fitting process.

Using(14) and (16), it is found that $\epsilon_r = 2.341 - j0.0009$. This first guess value is then fed to an optimizing routine that finds the best fit between the experimental data and the theoretical model from (5). The optimization goal is to minimize the squared distance between measured data and the theoretical model (5). This refinement gives $\epsilon_r = 2.337 - j0.0006$. The use

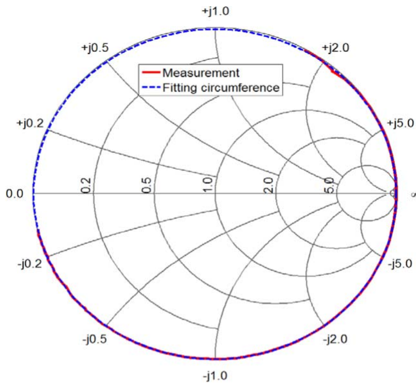

Fig. 3. Locus of the measured reflection coefficient of a Polyethylene sample with length $l = 4.88$ mm from $f = 55$ to 65 GHz on the Smith Chart.

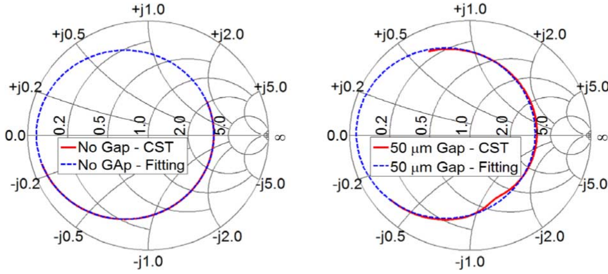

Fig. 4. Locus of the simulated reflection coefficient of a MACOR™ sample with length $l = 5$ mm from $f = 60$ to 65 GHz on the Smith Chart, with and without a $\Delta b = 50\mu\mathrm{m}$ air gap adjacent to the top wall of the waveguide.

of curve fitting methods to determine dielectric properties with waveguide measurements has been considered in other works [4], but the novelty of our approach is that it allows obtaining a quite accurate first estimate for the permittivity from the swept arc length and curvature radius in the Smith chart. In this example the error in the initial estimate of the real part of the permittivity is less than 0.2%.

In order to validate the proposed method, a disk sample from the same Polyethylene batch was cut, with a diameter $d = 30$ mm and thickness $t = 1.56$ mm and its complex permittivity was measured using the open resonator method Fabry-Perot [5]. The obtained complex permittivity value is $\epsilon_\mathrm{r} = 2.349 - j0.0009$ [6]. The mismatch in the imaginary parts of the measured permittivities is larger than for the real parts, because it is difficult to measure very precisely ϵ'' (with any method) in case of very low loss materials.

IV. Influence of Air Gaps

It is well known that air-gaps, mainly in the E-plane where the E-field is the stronger, can introduce considerable error in the complex permittivity estimation [2]. This error decreases with the permittivity value of the material [7]. A solution for this problem is to apply a conducting paste to the edges of the sample [7]. Alternatively, if the height of the gap Δb can be measured and is known, it is possible to relate the "correct" complex permittivity ϵ_ra with the measured value ϵ_rm as [2]

$$\tan\left[\omega b\sqrt{\varepsilon_0\mu_0\left(\varepsilon_{r\,a}-\varepsilon_{r\,m}\right)}\right] + \chi\tan\left[\omega\,\Delta b\sqrt{\varepsilon_0\mu_0\left(1-\varepsilon_{r\,m}\right)}\right] = 0 \quad (17)$$

where $\chi = \varepsilon_{r\,a}\sqrt{1-\varepsilon_{r\,m}}/\sqrt{\varepsilon_{r\,a}-\varepsilon_{r\,m}}$. Equation (17) can be numerically solved with respect to ϵ_ra.

A particularly interesting feature of our graphical method is that the presence of air gaps or other perturbations (e.g., a misplaced sample or an imperfect short-circuit termination) can be detected during the measurements. In fact, air gaps originate hybrid or high order modes in the sample holder. Theses modes introduce perturbations in the measured reflection coefficient ρ_1 that are manifested as ripples, curls or deviations from the expected circumference representation. In order to exemplify this behavior two simulations were performed in CST Microwave Studio [8] (Fig. 4) for a MACOR sample ($\epsilon_\mathrm{r} = 5.4 - j0.07; l = 5$ mm) with and without an air gap in the E-plane of $\Delta b = 50\mu\mathrm{m}$.

For the case without air gap the proposed graphical method gives the permittivity $\epsilon_\mathrm{r} = 5.409 - j0.070$, while in the presence of the $\Delta b = 50\mu\mathrm{m}$ gap the method gives $\epsilon_\mathrm{r} = 5.138 - j0.070$. Using (17) one can eliminate the influence of the air gap, and this yields $\epsilon_\mathrm{r} = 5.415 - j0.074$.

V. Discussion

The proposed method is a simple graphic fitting procedure and yet it has several advantages. Since it is a waveguide based method it only requires small samples. The samples can be cut from different locations of the material batch and used to evaluate its homogeneity and anisotropy. The method is not plagued with multiple solutions that require the measurement of multiple samples. Most importantly, during the measurement procedure it is simple to identify errors and perturbations such as the misplacement of the sample or air gaps. Therefore, a simple visual inspection of the data representation in the Smith Chart allows for a first validation and, if needed, an adequate correction of the measurement procedure, even before the post-processing of the measured data and the calculation of the complex permittivity.

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