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Application of spline interpolation to speckle shearography measurements for damage identification

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Abstract

This paper aims to explore a new technique for structural damage identification using cubic spline interpolation. The method is based on the interpolation of modal rotations measured with speckle shearography. In order to locate the damaged areas, we make use of the analytical derivative of the cubic spline function to compute the modal curvature, which is known to be very sensitive to damage. A comprehensive parametric study of the spatial sampling interval is carried out to find its influence on noise filtering. Furthermore, the identification quality dependency on the mode shape and respective noise is also examined. The results obtained with the proposed method show the consistency of the localizations. Additionally, the challenging tasks of identifying small and multiple damage are tackled, yielding a good performance.

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Keywords: damage identification; beam; shearography; cubic spline; modal curvature;

1. Introduction

Structural damage identification is a field of great concern, since it provides a solution to the prevention of structural failure, whose occurrence leads to catastrophic harm at various levels. Given this fact, it is essential to have a range of effective methods so that it is possible to monitor structural integrity and identify potential damaged areas, which

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proves to be very useful across several engineering areas. Concerned with its implications, researchers have been giving much attention to this field and a wide spectrum of methods arose. Among them, vibration-based methods, being nondestructive and not requiring that the vicinity of the damage is known *a priori*, became very popular, whose underlying principle is the fact that a localized damage alters the dynamic characteristics of the structure. In particular, as it is put forward in Pandey *et al.* (1991), by using numerical examples, the analysis of the curvature mode shape (second spatial derivative of the vibrational displacement mode shape of a beam) and comparison with its undamaged counterpart, yields a localized anomaly. Although several techniques based on this fact have been proposed, the vast majority still fail to identify small and multiple damage, when applied to experimental data. As a matter of fact, plenty of work has been carried out using finite differences and wavelet transform. However, very little research has been undergone on using cubic spline interpolation as a means of obtaining an approximation to the modal curvature fields.

Sazonov and Klinkhachorn (2005) compute the modal curvature shape by applying finite differences as an approximation to the second derivative of the modal displacement. They also provide an optimal spatial sampling interval, in order to minimize the effects of measurement uncertainty and its propagation, validated by numerical examples. Mininni *et al.* (2016) present a method based on the computation of the curvature mode shape via finite differences of modal rotation fields (first spatial derivative of the vibrational displacement mode shape of a beam) obtained experimentally using speckle shearography. Not only is this technique not as prone to measurement error as the one proposed by Sazonov *et al.* (2005), since it requires only an approximation of a first derivative, but also an optimal spatial sampling is deduced, yielding an acceptable performance overall. Rucka (2011) explores the application of the wavelet transform to damage identification, as well as the benefits and limitations of considering higher vibrational modes. In her research, and in order to reduce the boundary effects, a cubic spline interpolation was used, but only to extrapolate additional points from the numeric simulation data. These comprise three of the various methods proposed which take advantage of an optimal spatial sampling for the use of finite differences or the well-known noise rejection properties of the continuous wavelet transform.

This paper aims to present a new method for damage identification, taking advantage of the smoothing properties of cubic spline interpolation. On top of that, this approach is based on the differentiation of modal rotation fields, obtained using speckle shearography, as suggested in Mininni *et al.* (2016), leading to lower uncertainty propagation and amplification. Speckle shearography is an optical technique to measure the gradient of the displacement fields based on the interferometric comparison between light rays illuminating the vibrating beam and a reference, making it less susceptible to external noise. This technique is thoroughly defoned in Francis *et al.* (2010). In addition, a cubic spline is a function defined in a piecewise fashion by third order polynomials, but such that continuity in the function, its first and second derivative is assured. Not only does it provide smoothness in the interpolation, but also the analytical derivative of each piece of the spline consists of a smoothened approximation of the derivative of the interpolated data. Therefore, computing an approximation to the modal curvature shape using the analytical derivative of the interpolating cubic spline of the rotation fields, provided experimentally by shearography, should yield a good immunity to both noise and measurement uncertainty, and, thus allow a clearer identification of damage. A comprehensive study of the dependence of the spatial sampling interval on the quality of the identification, as well as on the noise and measurement uncertainty rejection properties is also carried out in this paper. In addition, we seek to point out the differences in the quality of the identification between different vibrational modes. Finally, the performance of proposed method is also assessed when applied to small and multiple damage.

2. Theoretical background

The method for damage identification presented in this paper relies on cubic spline interpolation. In this section, a thorough theoretical analysis is conducted, addressing basic definitions, the properties of this tool, as well as its application the problem we aim at solving.

2.1. Cubic spline interpolation

The formal definition of a cubic spline is as follows: given the one-dimensional mesh, $\Gamma = \{x_0, ..., x_n\}$, with $x_0 < x_1 < ... < x_n$, a function $s : [x_0, x_1] \to \mathbb{R}$ is said to be a cubic spline which interpolates the points $(x_0, y_0), ..., (x_n, y_n)$ if the following conditions are met:

- $s \in C^2[x_0, x_n]$;
- for $x \in [x_{i-1}, x_i]$, $i = 1, 2, ..., n$, $s(x)$ is a third-degree polynomial;
- $s(x_i) = y_i$, $i = 0, 1, ..., n$.

To build an interpolating spline of the points $(x_0, y_0),...,(x_n, y_n)$, one starts by constraining the continuity of its second derivative, given in Lagrange´s form by

$$s_i''(x) = m_{i-1}\frac{x_i - x}{h_i} + m_i\frac{x - x_{i-1}}{h_i},\ h_i = x_i - x_{i-1} \tag{1}$$

where m_i is the value of the second derivative of the spline in node x_i. Integrating (1) twice and constraining the continuity of the spline by setting

$$s_i(x_{i-1}) = y_{i-1}$$
$$s_i(x_i) = y_i$$

for each piece, yields

$$s_i = m_{i-1}\frac{(x_i - x)^3}{6h_i} + m_i\frac{(x - x_{i-1})^2}{2h_i} + (y_{i-1} - m_{i-1}\frac{h_i^2}{6})\frac{x_i - x}{h_i} + (y_i - m_i\frac{h_i^2}{6})\frac{x - x_{i-1}}{h_i}. \tag{2}$$

Differentiating (2), one obtains

$$s'_i = -m_{i-1}\frac{(x_i - x)^2}{2h_i} + m_i\frac{(x - x_{i-1})^2}{2h_i} + \frac{y_i - y_{i-1}}{h_i} - (m_i - m_{i-1})\frac{h_i}{6} \tag{3}$$

and containing the continuity of (3) in the transition between pieces, one obtains the following identity

$$m_{i-1}\frac{h_i}{6} + m_i\frac{h_i + h_{i+1}}{3} + m_{i+1}\frac{h_{i+1}}{6} = \frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i-1}}{h_i}$$

which, for the particular case of uniform spacing, *h*, is given by

$$m_{i-1} + 4m_i + m_{i+1} = \frac{6}{h^2}(y_{i+1} - 2y_i + y_{i-1}),\ i = 1,...,n-1 \tag{4}$$

It is clear, that once the constants $m_0, m_1, \dots, m_n$ are computed via equation (4) it is possible to approximate the derivative of the interpolated data. However, we have $n-1$ equations, given by (4), and $n+1$ unknowns, thus one needs two impose two additional constraints, the so-called boundary conditions. In the present work, we will be studying a beam held freely at both extremities and using the experimental modal rotation field as the interpolation data, therefore following Euler-Bernoulli beam theory the third derivative of the modal vibrational displacement, *i.e.*, the second derivative of the modal rotation field, is null. Given this fact, the couple of additional constrains are $m_0 = m_n = 0$, resulting on what is known as a natural spline, from which it is possible to write the following linear system of equations to find the remaining $n-1$ unknowns:

$$[A]\{m\} = \{b\}, \tag{5}$$

where $[A]$ is a diagonally dominant tridiagonal matrix, meaning one may solve (5) using the Thomas algorithm (Hoffman *et al.* (2001)) allowing, therefore, for an efficient method of determining $\{m\}$.

2.2. The proposed method

As discussed previously, Pandey *et al.* (1991) suggest that the second derivative of a beam vibrational curvature can be used as a means of identifying any variation to the dynamical properties of the beam, by differentiating the modal displacement shape. Nevertheless, we propose to compute the modal curvature shape of a vibrating beam, from its modal rotation field, obtained experimentally using speckle shearography, as in Mininni *et al.* (2016), and additionally, instead of applying finite differences, we advocate the use of cubic spline interpolation, explored in detail in the previous subsection. As a matter of fact, using the rotation field of the beam as interpolationed data, one can easily compute the second derivative of the spline with natural boundary conditions in each of the nodes, *i.e.* $m_0, m_1, \dots, m_n$, via the linear system of equations in (5). Furthermore, having obtained the value of such constants it is possible to approximate the modal curvature shape of the beam making use of equation (3). It is important to notice that equation (3) is smooth, in the sense that its derivative is continuous, therefore it seems that this technique has greater resistance to measurement uncertainty and noise when compared to the finite differences method used in Sazonov *et al.* (2005) and Mininni *et al.* (2016), allowing, seemingly, for a better damage identification.

It is important to remark that the spatial sampling interval, h, is a parameter of paramount importance. As a matter of fact, decreasing it increases the susceptibility to noise when computing the derivative of the rotation field.On the other hand, if this parameter is set too high the abnormalities in the profile due to the damage are treated as noise and smoothed, making it difficult to identify the damage. Therefore, one must balance this two effects in order to allow for an effective damage identification.

3. Results

3.1. Experimental setup and damage scenarios

As introduced in the previous sections, the presented method will be subjected to a validation using experimental data, obtained from shearography, for a broad variety of damage scenarios. The experimental apparatus consists of an aluminum beam held by rubber bands at both extremities, corresponding to a free-free condition. The natural frequency for each mode depends on the damage scenario being considered, as the stiffness of the beam decreases with the presence of damage, which was verified by Minnini *et al.* (2016). Therefore, to excite the beam at its natural frequency, one has to measure it experimentally, for each damage scenario, as well as for each mode. As a matter of

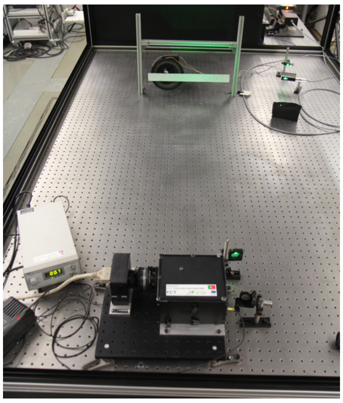

Fig. 1. Experimental apparatus used.

fact, such values were determined with the help of a microphone after having excited the beam with an impact hammer. A loudspeaker was placed behind the beam to correctly excite the beam to the natural frequency corresponding to each mode, while the modal rotation fields were being measured. The shearography system was placed at 1.2 m from the suspended beam, allowing for the recording of the phase map of the entirety of the beam, which was posteriorly processed by applying filtering and unwrapping techniques, as described in Minnini *et al.* (2016). The apparatus used is shown in Fig. 1.

The aluminum beam of dimensions 400 mm × 40 mm × 3 mm was used under different damage scenarios. The damages inflicted to the beam were made using an electronically controlled milling machine, which carved slots on the beam of a certain width and depth, with the precision of 5 microns. Furthermore, the slots were created in two distinct locations and for several depths, as a means of testing the method for a wide range of situations. Measurements have also been taken of the undamaged beam, followed by increasingly deeper slots at one location and, finally, multiple damage scenarios, in which increasingly deeper slots are carved in the second location. The depth of each slot is measured in different points and averaged. Slot 1 had a width of 5 mm and slot 2 a width of 3 mm, and the depths of the slots corresponding to each damage scenario are presented in Table 1.

Table 1. Damage Scenarios.

Damage Scenarios	Slot 1 Depth [mm]	Slot 2 Depth [mm]
1	0.10	
2	0.22	
3	0.30	
4	0.41	
5	0.41	0.03
6	0.41	0.10
7	0.41	0.19
8	0.41	0.30

After the unwrapping and filtering of the phase maps we obtain the rotation fields for each damage scenario and for the first four modes. It is important to remark that, throughout this paper, the modal rotation field data is normalized, such that its amplitude is unitary, allowing for a better comparison between damage scenarios. The modal rotation profiles for the undamaged case and fourth damage scenario, taken along the mid width of the beam, are represented, respectively, in Fig. 2 and Fig.3.. Comparing the profiles of the rotation fields of the beam in the undamaged case and fourth damage scenario, corresponding respectively to Fig. 2 and Fig. 3, it is very difficult, to notice any difference between them, validating the need to analyze the profile of the curvature instead.

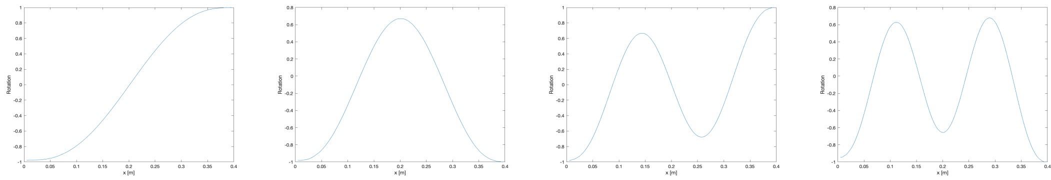

Fig. 2. Rotation profiles of the beam in the undamaged case for the first four modes, respectively, from left to right.

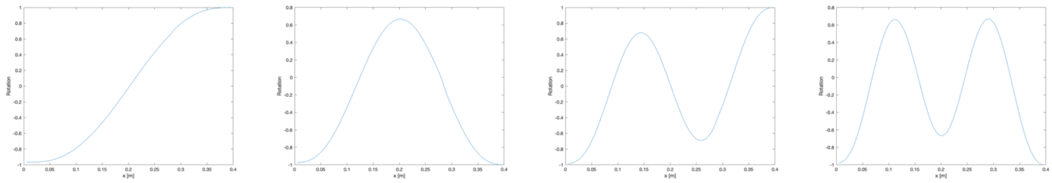

Fig. 3. Rotation profiles of the beam in the fourth damage case for the first four modes, respectively, from left to right.

3.2. Application to damage identification

Applying the method detailed in section 2.2 to the experimental data for several sampling intervals, h, one obtains results in which a damage signature is visible. Some examples are presented in Figs. 4 – 6 and will be subjected to a

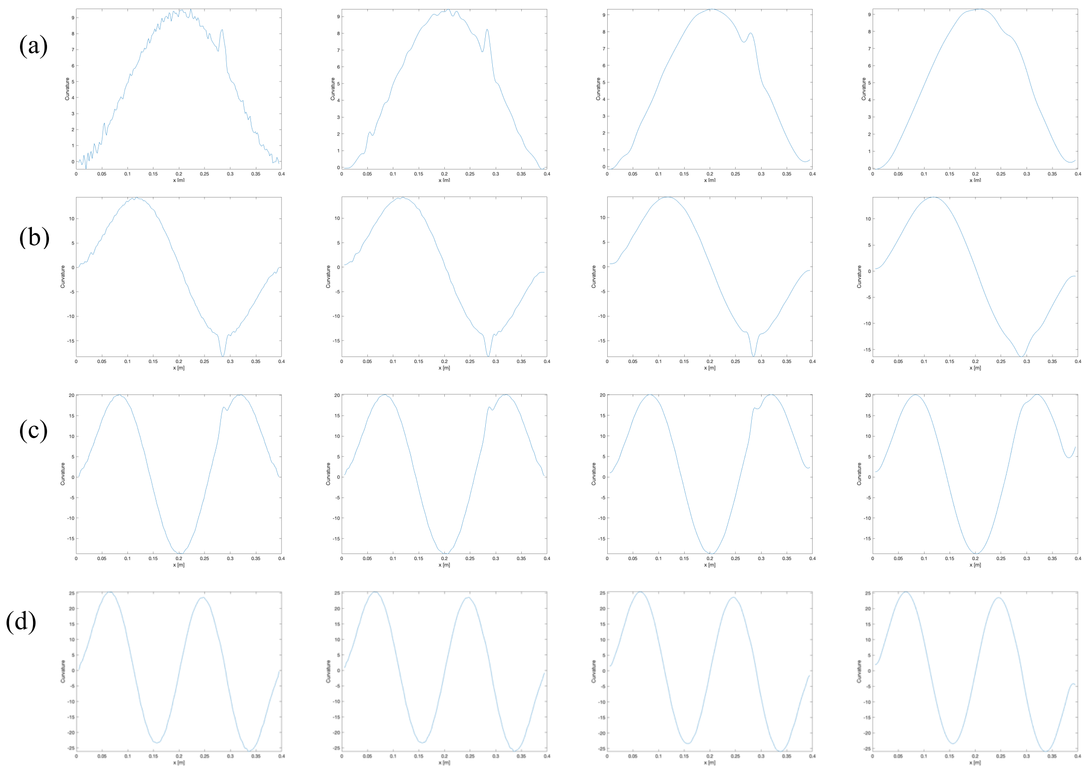

Fig. 4. Parametric study of h for the curvature of the fourth damage scenario of mode: (a) one; (b) two; (c) three; (d) four.

thorough analysis. The sampling intervals shown in such examples were chosen so that the results of the parametric study are visible, displaying, at the same time, a sufficient number of plots.

Table 2. Spatial sampling interval used in Figs. 4 – 6.

Mode	Spatial sampling interval from left to right [mm]
1	0.17; 7.4; 15.9; 20.0
2	0.17; 4.7; 9.5; 18.4
3	0.17; 3.4; 6.9; 14.0
4	0.17; 2.6; 5.3; 11.0

As a matter of fact, Fig. 4 represents the curvature mode shapes for the most dramatic single damage scenario, *i.e.* damage scenario four, in which a parametric study of *h* is carried out, for each of the first four modes. The spatial sampling interval, *h*, for each graph of Fig. 4 is presented in Table 2. Analyzing, these plots one can clearly localize a consistent anomaly for the first three modes corresponding to the damage. It very interesting to point out that as predicted in section 2.2, for small values of the spatial sampling interval the curve is very prone to perturbations caused by noise, and as *h* increases so does its resistance to those perturbations, having the side effect, however, of smoothing the spike caused by the damage. One must, therefore, seek a balance of these two effects. Looking at Fig. 2, corresponding to the first mode, it is clear that for *h*=0.17mm all the modes develop perturbations due to noise. Increasing *h* these perturbations are no longer noticeable, yielding a distinct spike in an otherwise smooth curve, as visible in the third plot of the first three modes for a spatial sampling interval of *h*=15.9 mm, *h*=9.5 mm and *h*=6.9 mm, respectively for the first, second and third modes. This spike, whose location is consistent across the modes, corresponds to the damage inflicted to the beam, validating the use of the modal curvature shape as a mean of identifying damage. Furthermore, if one increases the parameter h, the abnormality in the curve due to the damage is significantly reduced, decreasing the quality of the identification, as visible in the fourth plot of the first three modes for a spatial sampling interval of *h*=20.0 mm, *h*=18.4 mm and *h*=14.0 mm, respectively for the first, second and third modes. It is also very interesting to remark that the damage signature is not visible for the fourth mode. In fact, looking at the location of the damage signatures for the first three modes, one concludes that it falls in the vicinity of the zero-crossing for the fourth mode, leading to an inconclusive identification. This fact is one of the drawbacks of using

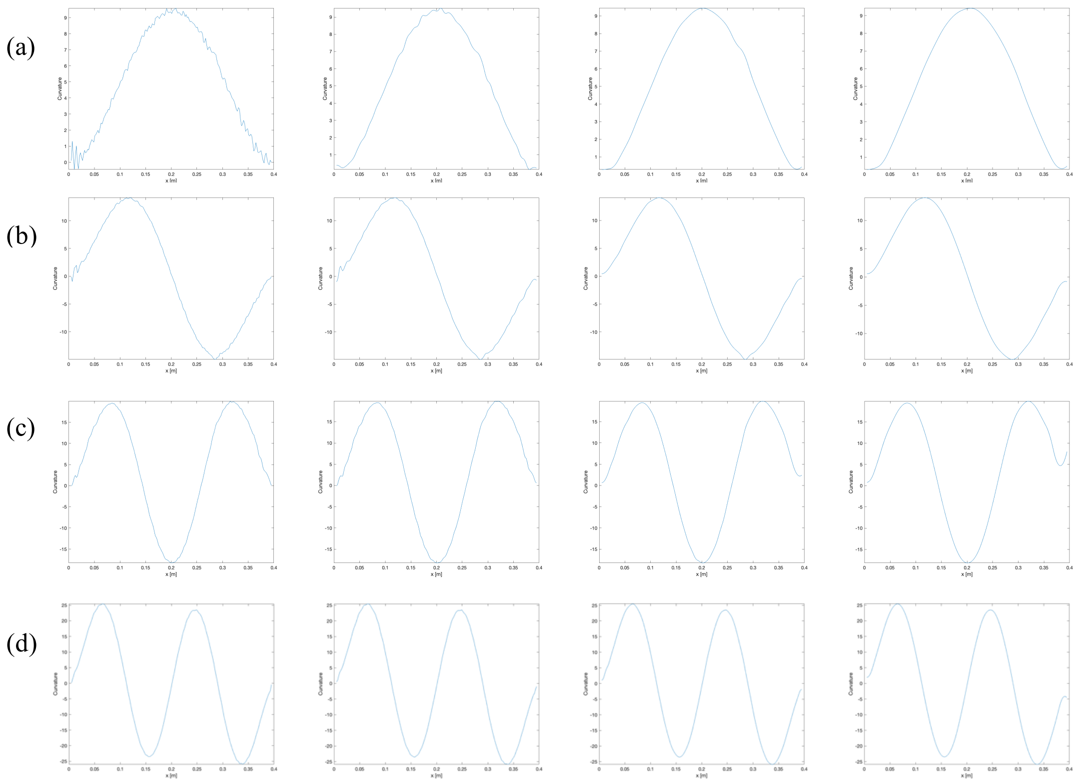

Fig. 5. Parametric study of *h* for the curvature of the first damage scenario of mode: (a) one; (b) two; (c) three; (d) four.

higher vibrational modes, since the higher the mode considered the greater is the number of zero-crossing points, augmenting the likelihood of a damage falling in such regions. Furthermore, it is of great importance to compare the proneness to noise of the different modes. It is noticeable from the analysis of the plots of Fig. 2 that the higher the mode the lowest the noise. For instance, for h=0.17 mm the first mode the perturbations due to noise are tremendous, but considering higher modes this effect starts to fade, resulting in barely visible oscillations in the curve of the fourth mode. This trait is an advantage of using higher vibration modes, allowing for the use of a smaller sampling interval and therefore considering more data points, which leads to a more marked peak. In other words, for higher vibrational modes noise perturbations are eliminated for a smaller and therefore, the damage signature is not as smoothed as in lower modes. Finally, one notices that the width of the peak, corresponding to the damage, is of a higher value in comparison to the width of the carved slot. Fig. 5 consists of the curvature mode shapes for the smallest single damage scenario, *i.e.* damage scenario one, in which a parametric study of h is presented, for each of the first four modes. The spatial sampling interval, h, for each graph of Fig. 5 is presented in Table 2. This scenario exhibits a considerable challenge to any damage identification method. In fact, the vast majority of the available methods fail to tackle successfully small damage, mainly due to the difficulty of the distinction between noise perturbations and damage signatures, given the similarity in their orders of magnitude. In fact, a technique allowing for such identification should provide for a means of computing the derivative of the modal rotation field in a manner so that it takes into account the smoothness of the resulting curve, reducing to a minimum the irregularities resulting of noise errors. The method presented in this paper was designed to be endowed with such properties. In fact, its use to this damage scenario, yielded in Fig. 5, alongside with an adequate choice of the spatial sampling interval successfully identifies the small damage. As a matter of fact, considering the third plot of the first three modes, for a spatial sampling interval of h=15.9 mm, h=9.5 mm and h=6.9 mm, respectively for the first, second and third modes one consistently identifies a clear damage signature, which stands out against the smoothness of the curve. Again, as seen in the analysis of Fig. 4, given the location of the damage it is not possible to make any conclusive identification from the plots of the fourth mode. Mininni *et al.* (2016) also analyze this damage scenario using finite differences as a means of computing the modal curvature from the rotation field of the beam using shearography. Despite the good results obtained overall by the method presented by these researchers, using an optimal spatial sampling interval, it is not able to identify the first damage scenario, as seen in Fig. 10 (a) and Fig. 11 (a) of Mininni *et al.* (2016). Comparing Fig. 4 and Fig. 5 one

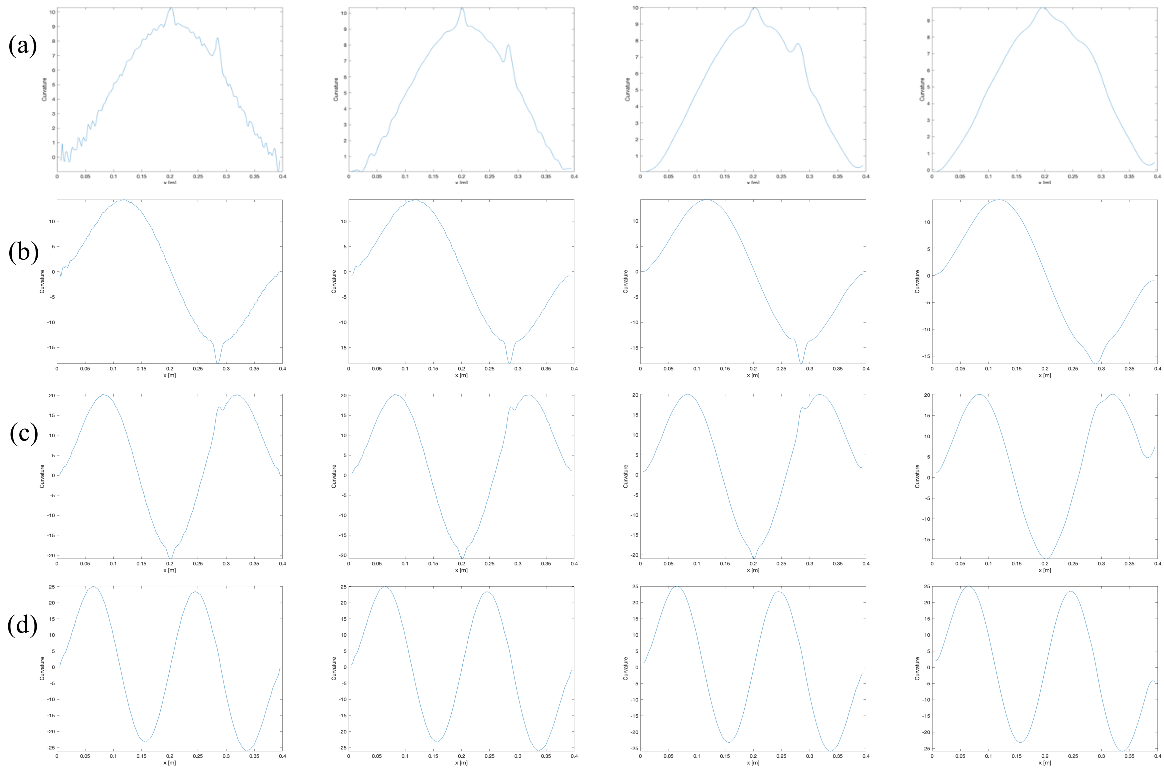

Fig. 6. Parametric study of h for the curvature of the eighth damage scenario of mode: (a) one; (b) two; (c) three; (d) four.

readily notices the consistency of the damage localization of the presented method. Furthermore, it is also visible that the amplitude of the damage signature provides a means of a relative damage quantification, given that the abnormality in the curvature increases in significance with the severity of the damage. This result is expected since a more significant damage reduces, locally, the stiffness of the beam more dramatically and, thus, its dynamical response.

Fig. 6 represents the curvature mode shapes for the most dramatic multiple damage scenario, *i.e.* damage scenario eight, in which a parametric study of h is carried out, for each of the first four modes. The spatial sampling interval, h, for each graph of Fig. 6 is presented in Table 2. This scenario also poses a challenge to damage identification techniques. However, there are some methods, which can achieve this type of identification, like, for instance, the one proposed by Mininni *et al.* (2016). Using an adequate value of spatial sampling interval, one can clearly notice, by analyzing Fig. 6, that the method presented in this paper also identifies successfully multiple damage. In fact, the damage corresponding to the first slot is marked with a clear peak in the third plot of the first three modes for a spatial sampling interval of h=15.9 mm, h=9.5 mm and h=6.9 mm, respectively for the first, second and third modes. One can also easily spot a distinct damage signature of the second slot in the third plot of the modes one and three, for a spatial sampling interval of h=15.9 mm and h=6.9 mm, respectively. Given that the location of the slots coincides with the zero-crossing vicinity of some modal shapes, it is not possible to identify the first slot in the fourth mode, neither the second slot in the second and fourth modes. Comparing Fig. 6 with Figs. 4 and 5, one can readily conclude that this method is very reliable since the damage of the first slot is localized consistently. Furthermore, comparing Fig. 4 and Fig. 6 the amplitude of the damage signature is unvarying for the modes where it is localized. For these reasons, the method put forward in this paper shows clear signs of robustness.

4. Conclusions

The proposed technique consists of a new method for damage identification based on the analysis of the modal curvature shape. This modal curvature is determined by using cubic spline interpolation as a means of differentiating the modal rotation fields, obtained experimentally with speckle shearography. Several conclusions were reached: (i) the spatial sampling interval was found to play a role of great importance when it comes to balancing the noise errors and the contrast of the damage signature; (ii) using the smoothing properties of the analytical derivative of the cubic spline function applied to the modal rotation field data, one is able to obtain, with an adequate sampling interval, very smooth curvature profiles with a distinct damage signature; (iii) the technique put forward in this paper identifies, successfully, the scenarios of small and multiple damage, for experimental data, yielding very good results, a hurdle the large majority of the available methods fail to overcome; (iv) it important to remark that the amplitude of the damage signature, in a given mode, increases with the severity of the damage it refers to, but also that its width does not correspond to the width of the slot carved on the beam; (v) we analyzed the benefits and drawbacks of using distinct order modes, being the lower ones more susceptible to noise and the higher more likely to fail to localize the damage when this is located in the zero-crossing vicinity of the modes; (vi) throughout the analysis of the results it was possible to conclude that the presented method is very robust, consistently not only localizing the damage but also yielding a damage signature amplitude identical for the same slot, when comparing single and multiple damage.

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