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Distributed decentralized control for very large-scale systems with application to LEO satellite mega-constellations

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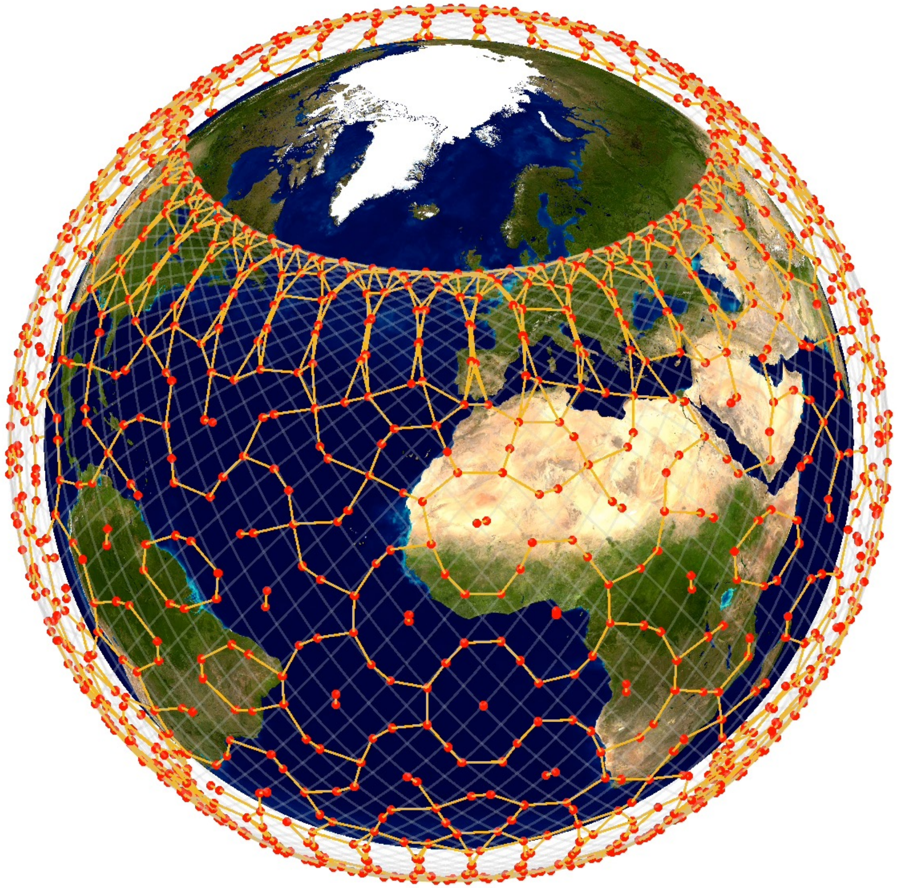

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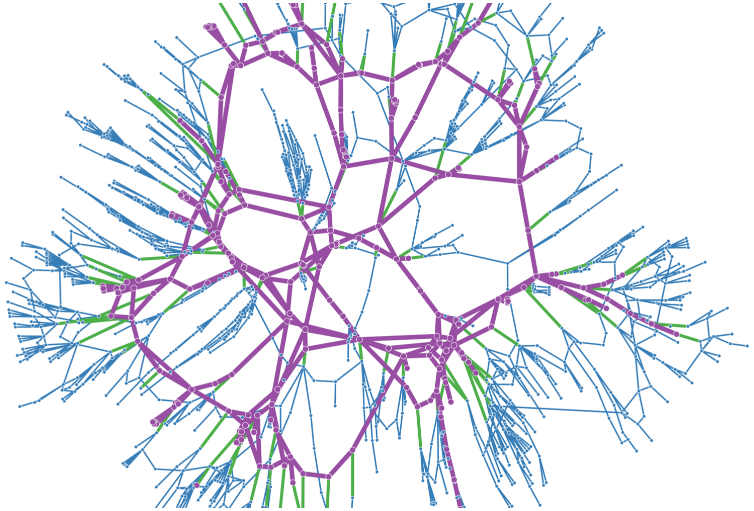
Introduction

Motivation



Introduction

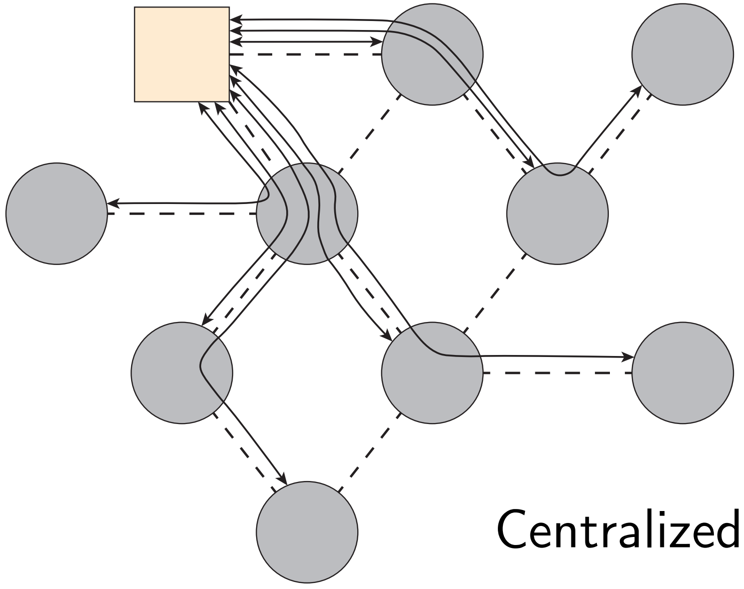
Motivation

⁉️ Yet to transition from concept to **deployment**

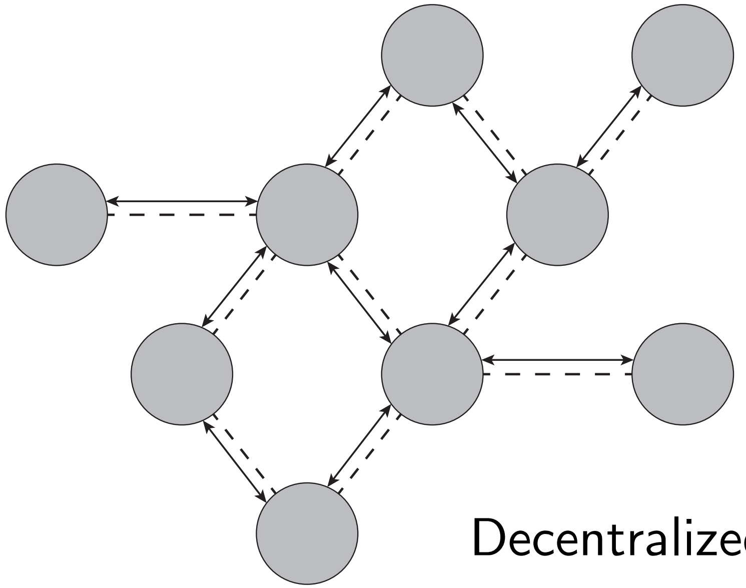
Inhibiting technical **challenges** on a very large-scale

- **Decentralized** framework
- **Distributed** synthesis

Paradigm revolution from a control standpoint



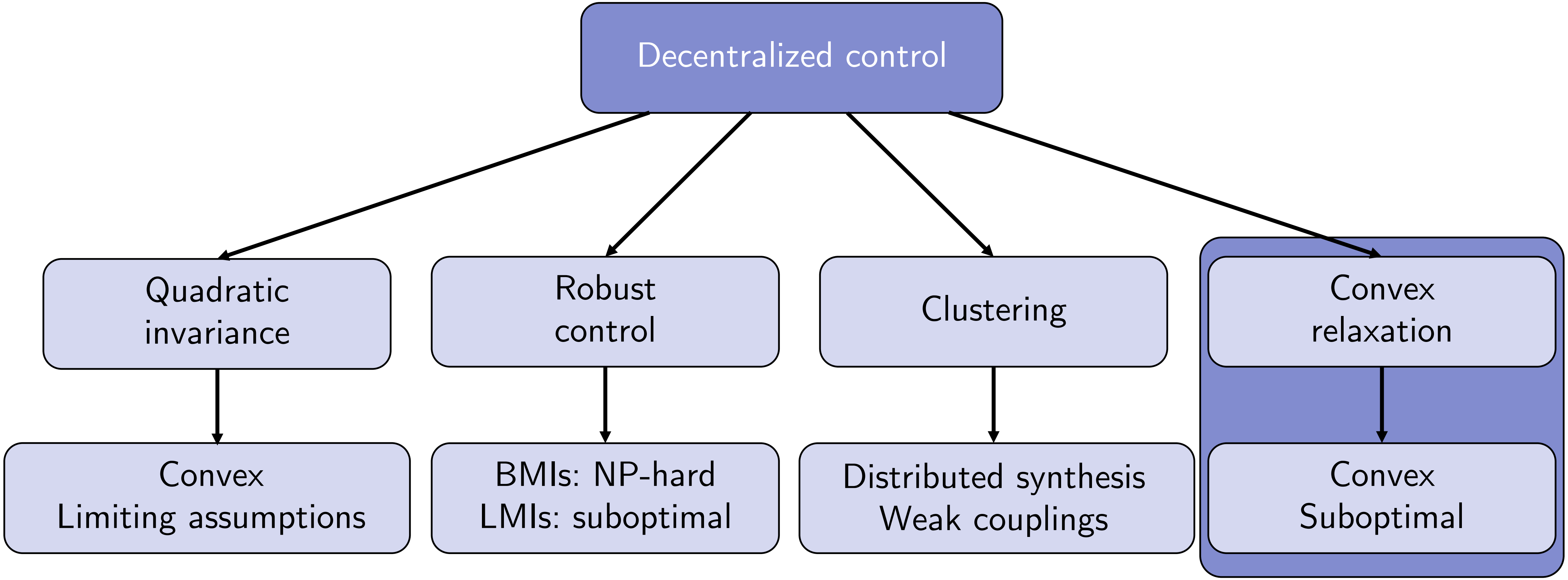
Centralized



Decentralized

Introduction

State-of-the-art overview



Goal: address the void to **enable ground-breaking** very large-scale applications

Introduction

Problem Framework

Control objective

- **Quadratic** cost
- **Regulator**
- **Receding horizon** (RHC)

Local dynamics and couplings

- Network of N systems
- **LTV**
- Approximate **nonlinear** systems
- **Sparse** couplings

Decentralized framework

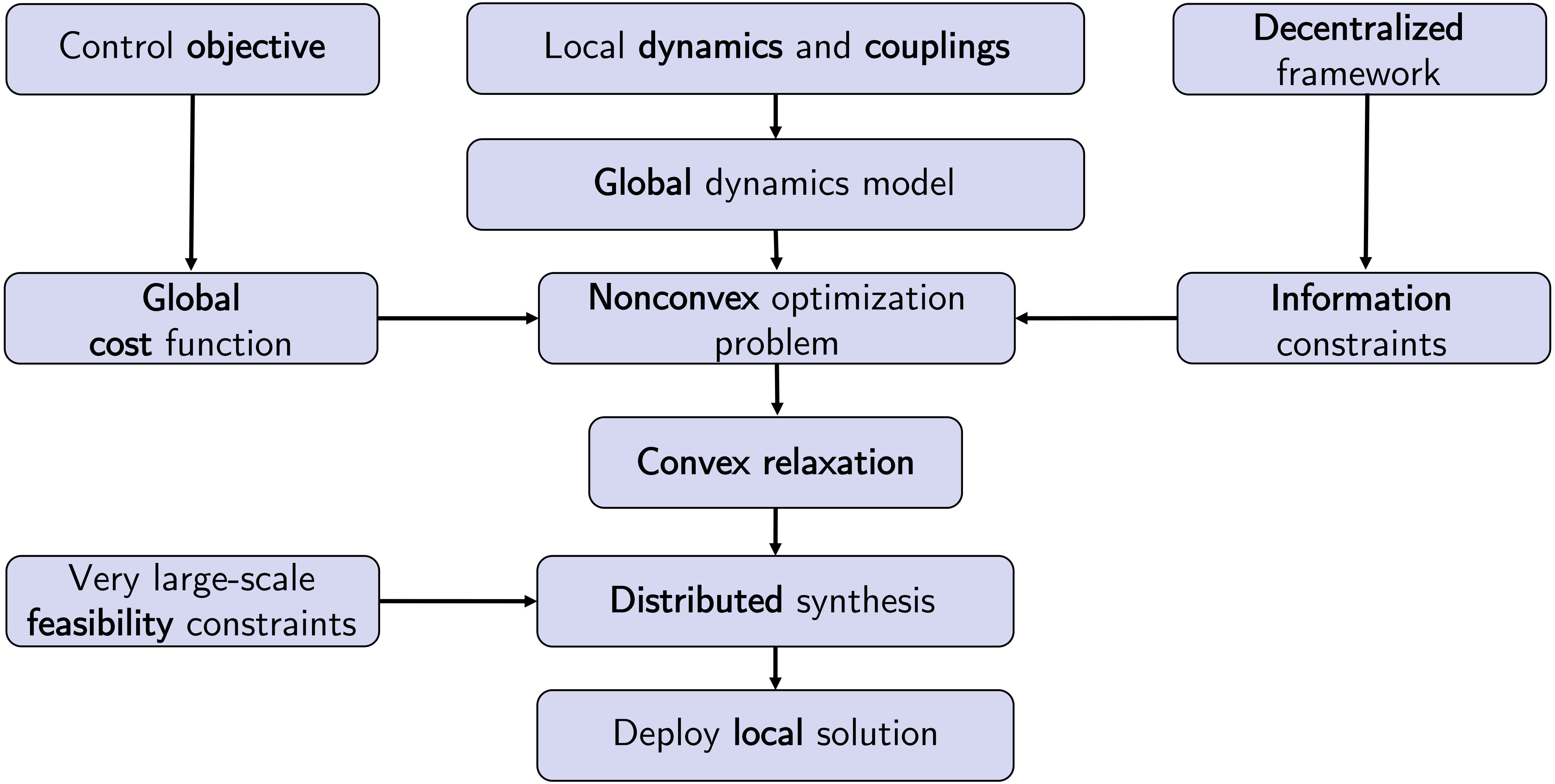
- **Linear** feedback
- **Local** feedback

Very large-scale feasibility constraints

- On controller **synthesis**
- On **communication**, **computational**, and **memory**
- **Feasible** real-time implementation

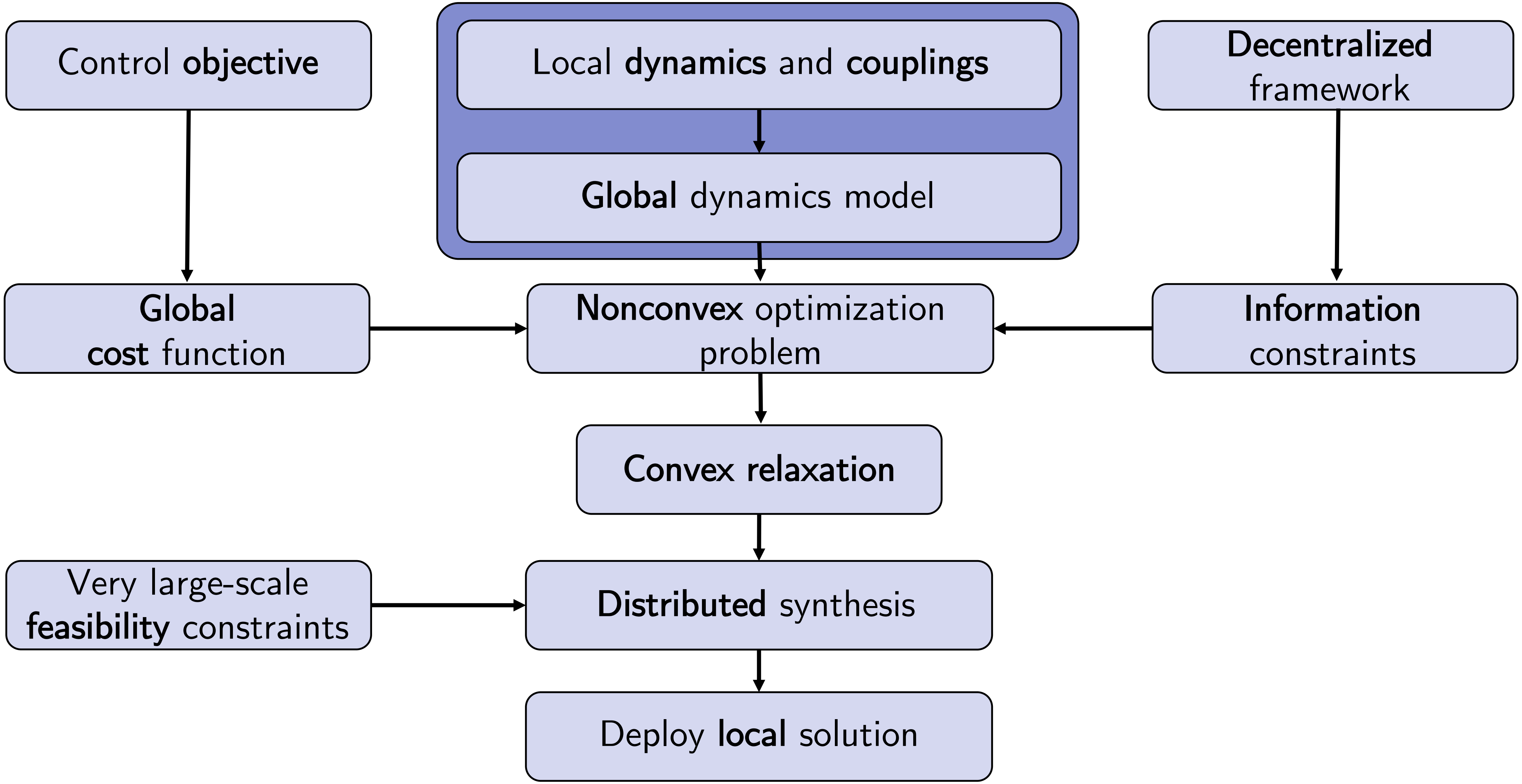
Introduction

Approach overview



Problem Statement

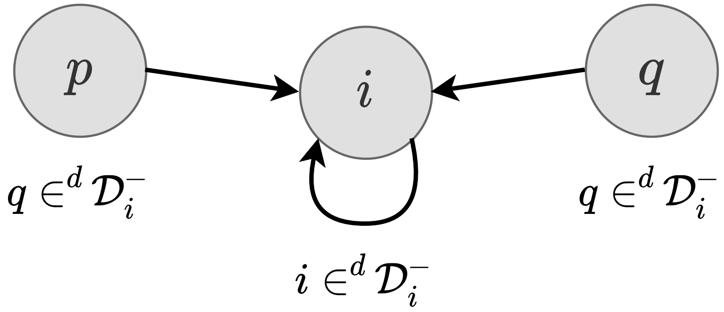
Local dynamics



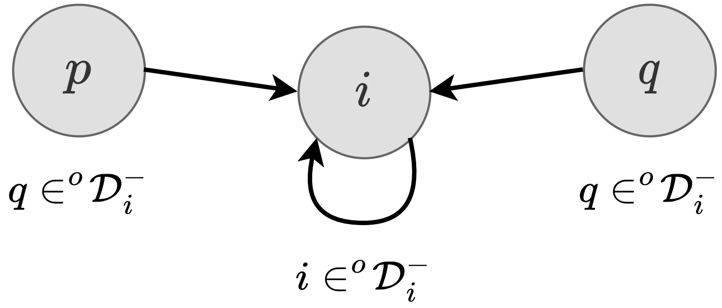
Problem Statement

Network dynamics

Directed **dynamical** coupling graph $\mathcal{G}_d$



Directed **output** coupling graph $\mathcal{G}_o$



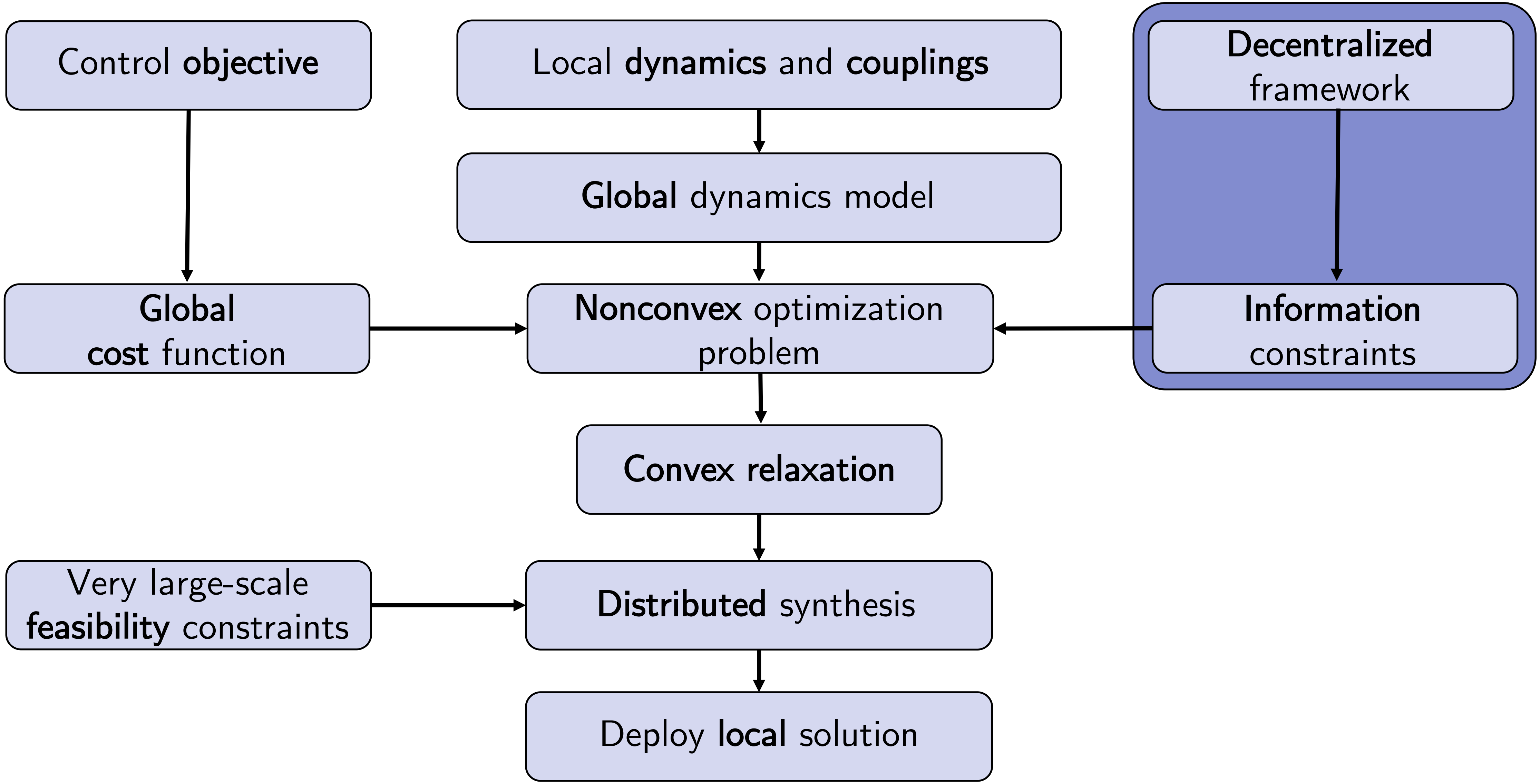
$$\begin{cases} \mathbf{x}_i(k+1) = \sum_{j \in^d \mathcal{D}_i^-} \mathbf{A}_{i,j}(k)\mathbf{x}_j(k) + \sum_{j \in^d \mathcal{D}_i^-} \mathbf{B}_{i,j}(k)\mathbf{u}_j(k) \\ \mathbf{z}_i(k) = \sum_{j \in^o \mathcal{D}_i^-} \mathbf{H}_{i,j}(k)\mathbf{x}_j(k), \end{cases}$$

Grouping local dynamics

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) \\ \mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k), \end{cases}$$

Problem Statement

Decentralized framework

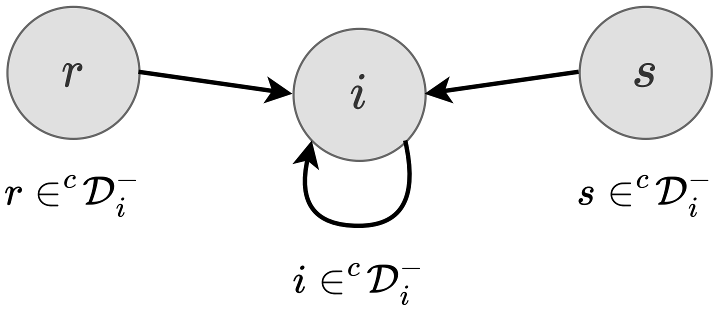


Problem Statement

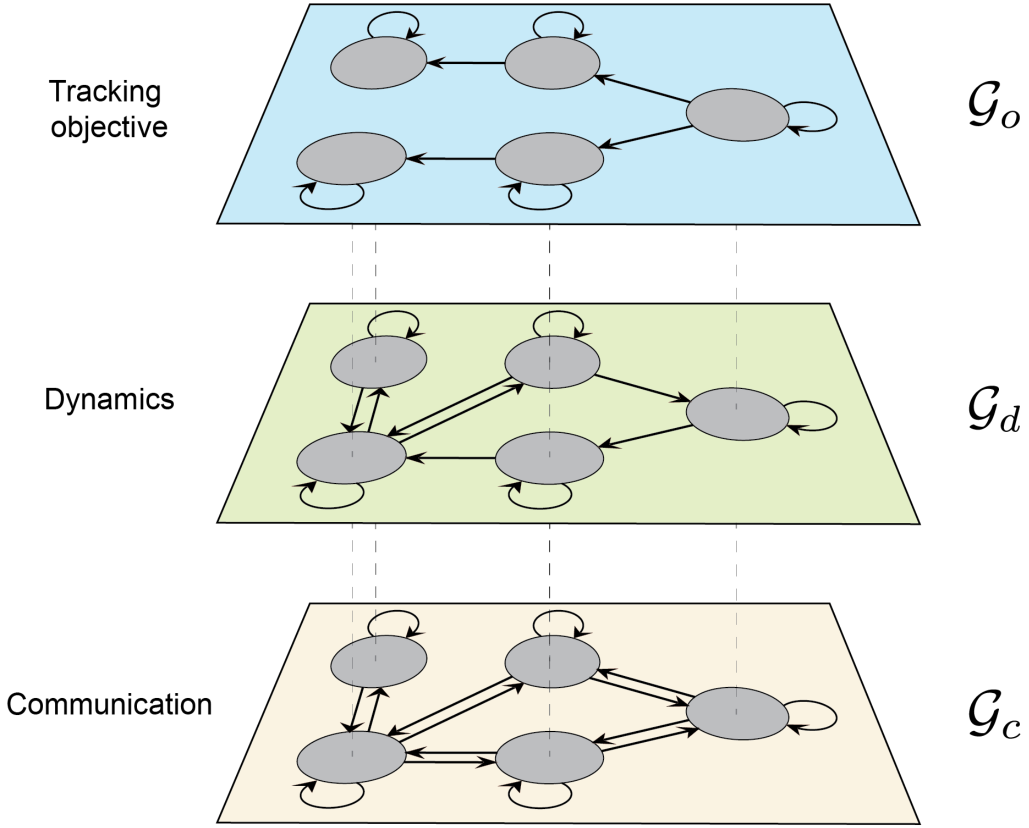
Decentralized framework

Directed state feedback **communication** graph $\mathcal{G}_c$

- Each **system** is a **node**
- Each directed edge represents **access** to the **state** via **communication**



$$\mathbf{u}_i(k) = -\sum_{j\in^c\mathcal{D}_i^-} \mathbf{K}_{i,j}(k)\mathbf{x}_j(k)$$



Problem Statement

Decentralized framework

$$\mathbf{u}_i(k) = -\sum_{j \in {}^c\mathcal{D}_i^-} \mathbf{K}_{i,j}(k)\mathbf{x}_j(k)$$

↓ Grouping local control law

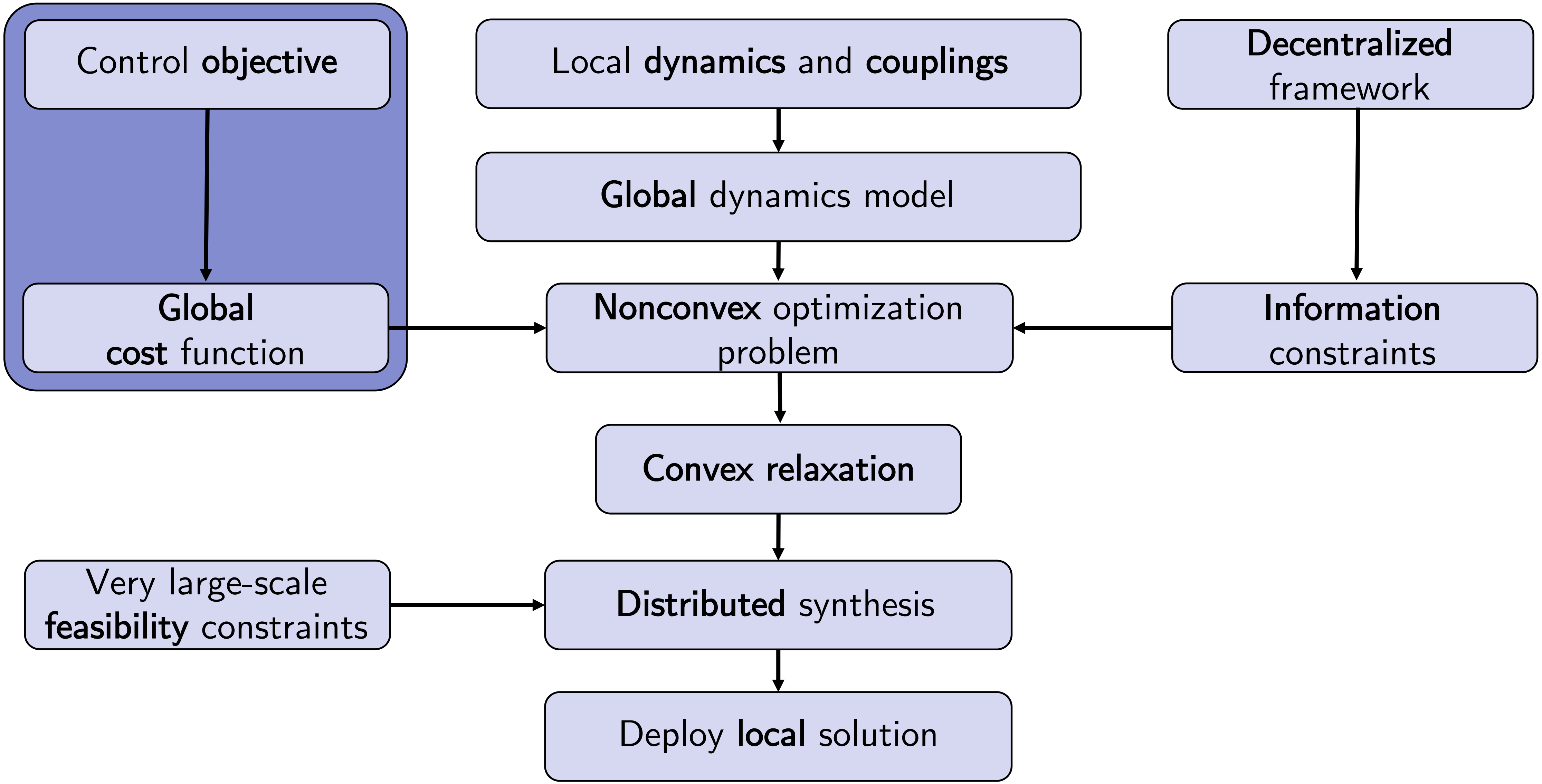
$$\mathbf{u}(k) = -\mathbf{K}(k)\mathbf{x}(k)$$

But $\mathbf{K}(k)$ must be **sparse**: $\mathbf{K}(k) \in \mathrm{Sparse}(\mathbf{E}_{\mathcal{G}_c})$

$$\mathrm{Sparse}(\mathbf{E}) := \left\{[\mathbf{K}]_{ij} \in \mathbb{R}^{m\times n} : [\mathbf{E}]_{ij} = 0 \implies [\mathbf{K}]_{ij} = 0;\ i = 1,...,m,\ j = 1,...,n\right\}$$

Problem Statement

Control objective



Problem Statement

Control objective

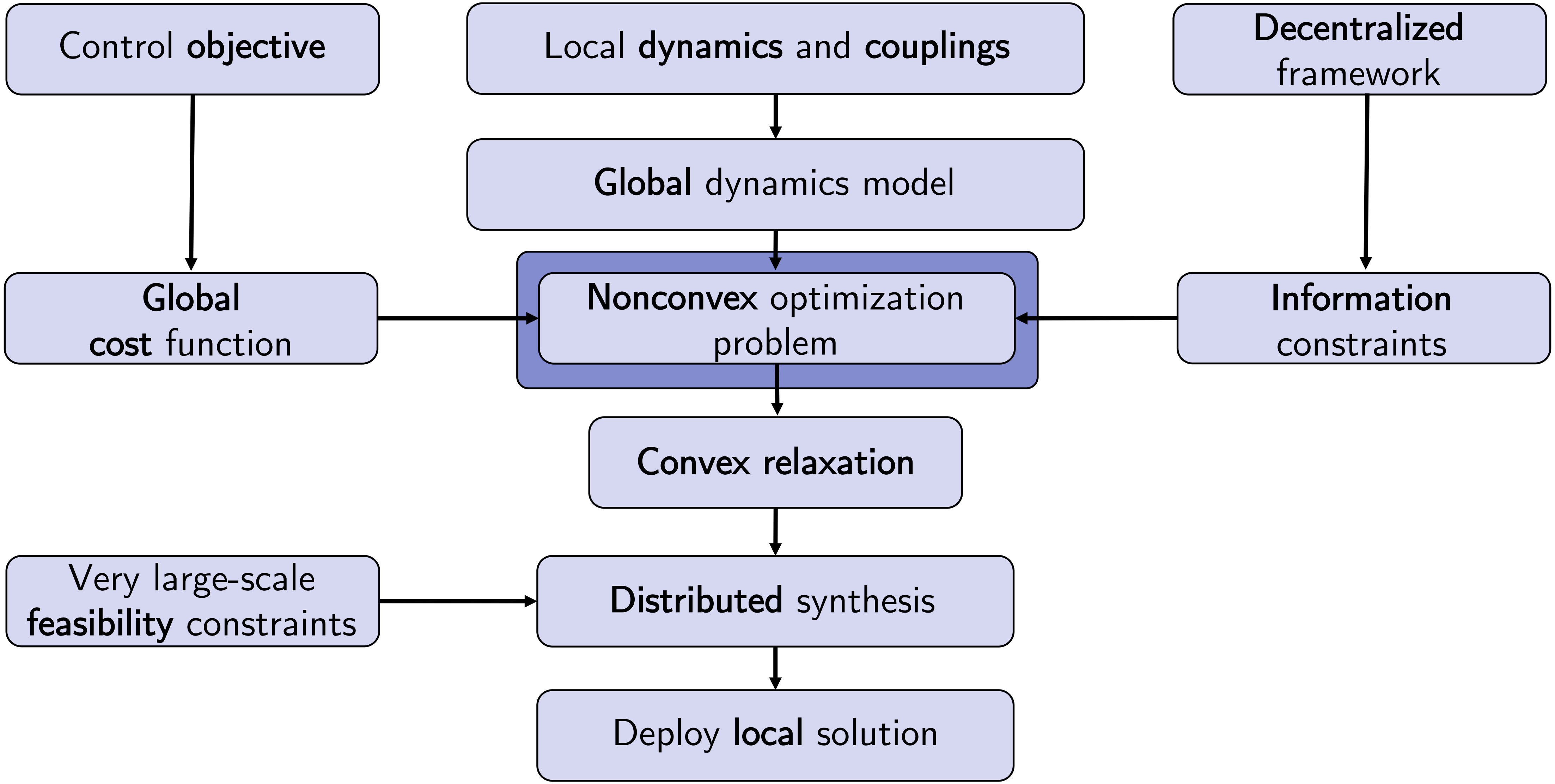
$$J(k) = \mathbf{z}^T(k+H)\mathbf{Q}(k+H)\mathbf{z}(k+H) + \sum_{\tau=k}^{k+H-1} \left(\mathbf{z}^T(\tau)\mathbf{Q}(\tau)\mathbf{z}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}(\tau)\mathbf{u}(\tau)\right),$$

Global **finite-horizon** cost:

- **Finite window** of length H
- Finite **linear-quadratic regulation** problem
- **RHC** framework to approximate the **infinite-horizon** problem

Problem Statement

Nonconvex optimization problem



Problem Statement

Nonconvex optimization problem

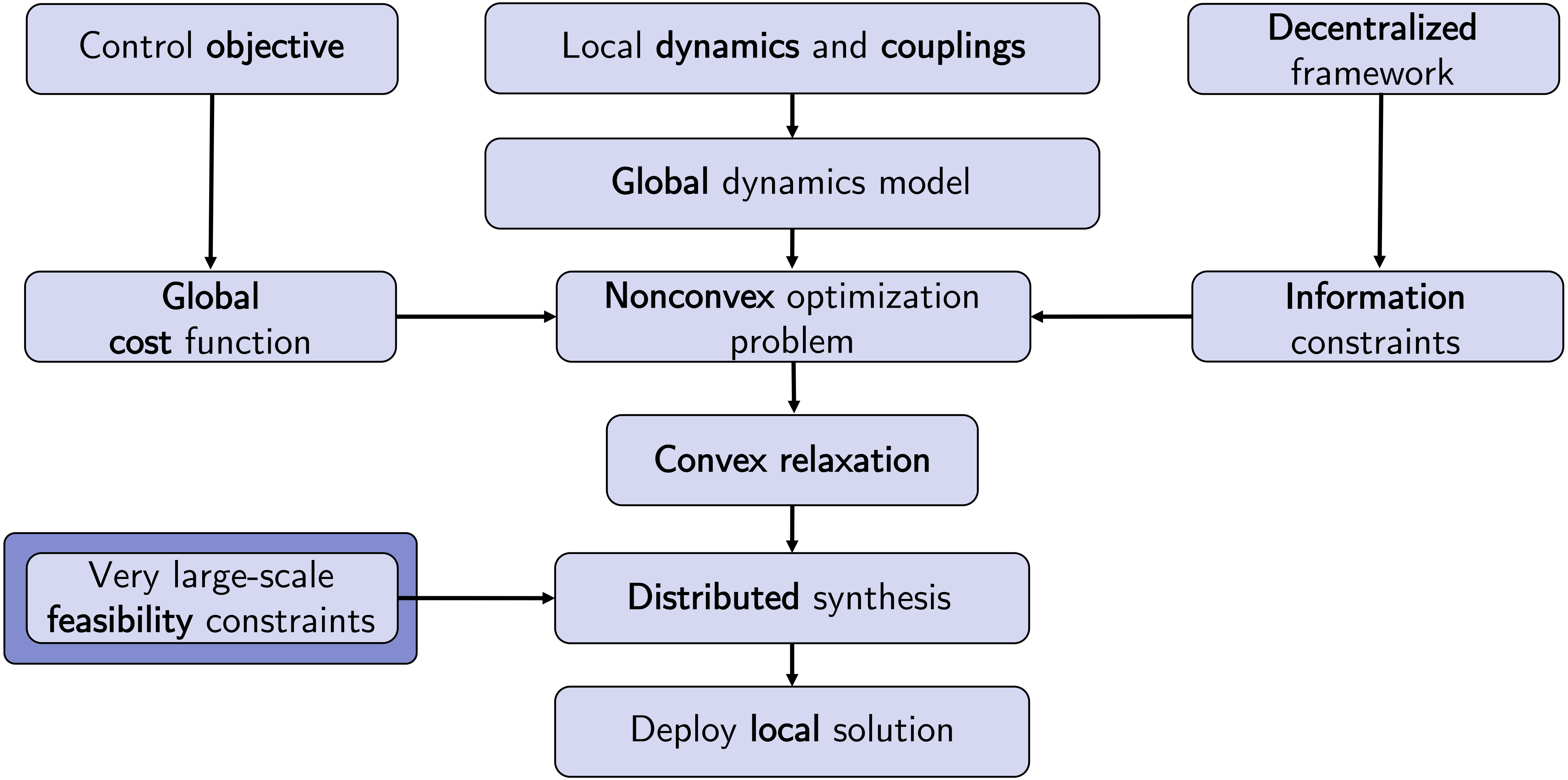
At each discrete time instant k:

$$
\begin{aligned}
\underset{\substack{\mathbf{K}(\tau)\in\mathbb{R}^{m\times n}\\ \tau\in\{k,\dots,k+H-1\}}}{\text{minimize}} \quad & J(k)\\
\text{subject to} \quad & \mathbf{K}(\tau) \in \mathrm{Sparse}(\mathbf{E}_{\mathcal{G}_c}),\ \tau = k,\dots,k+H-1\\
& \mathbf{u}(\tau) = -\mathbf{K}(\tau)\mathbf{x}(\tau),\ \tau = k,\dots,k+H-1\\
& \mathbf{x}(\tau+1) = \mathbf{A}(\tau)\mathbf{x}(\tau) + \mathbf{B}(\tau)\mathbf{u}(\tau),\ \tau = k,\dots,k+H-1
\end{aligned}
$$

Nonconvex!

Problem Statement

Very large-scale feasibility constraints



Problem Statement

Very large-scale feasibility constraints

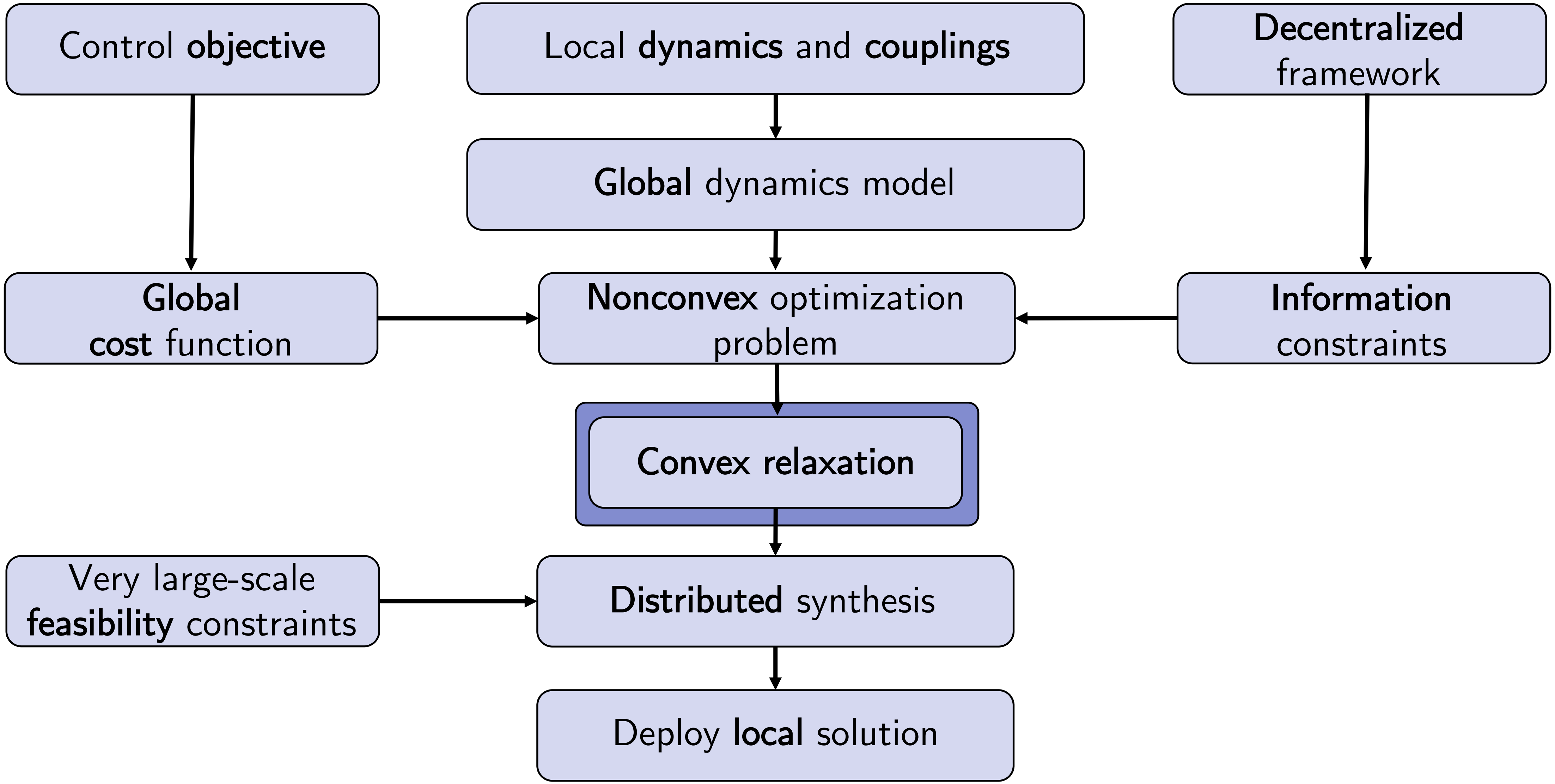
Each system $\mathcal{S}_i$ is associated with a **computational unit** $\mathcal{T}_i$

Synthesis constraints on $\mathcal{T}_i$:

- **Communication**: **instantaneous** communication **not allowed**
- **Communication**: complexity of $\mathcal{O}(1)$ with N
- **Computational**: complexity of $\mathcal{O}(1)$ with N
- **Memory**: complexity of $\mathcal{O}(1)$ with N

Decentralized linear quadratic control

Convex relaxation



Decentralized linear quadratic control

Convex relaxation

Challenges:

- Separation between **optimal** and **relaxed** solutions
- Physically **meaningful** relaxation

Approach:

- Obtain **necessary conditions** for a **constrained minimum**
- **Optimal control** theory
  - **Hamiltonian** function

Decentralized linear quadratic control

Convex relaxation

Augment $J(k)$ to write the **Langrangian**

$$J'(k) = \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T) + \sum_{\tau=k}^{k+T-1} \mathbf{x}^T(\tau)\left(\mathbf{Q}(\tau) + \mathbf{K}^T(\tau)\mathbf{R}(\tau)\mathbf{K}(\tau)\right)\mathbf{x}(\tau)$$

$$+ \sum_{\tau=k}^{k+T-1} \boldsymbol{\lambda}^T(\tau+1)\left[\left(\mathbf{A}(\tau) - \mathbf{B}(\tau)\mathbf{K}(\tau)\right)\mathbf{x}(\tau) - \mathbf{x}(\tau+1)\right]$$

Define the **Hamiltonian**

$$H(k) := \mathbf{x}^T(k)\left(\mathbf{Q}(k) + \mathbf{K}^T(k)\mathbf{R}(k)\mathbf{K}(k)\right)\mathbf{x}(k) + \boldsymbol{\lambda}^T(k+1)\left(\mathbf{A}(k) - \mathbf{B}(k)\mathbf{K}(k)\right)\mathbf{x}(k)$$

Decentralized linear quadratic control

Convex relaxation

Rewrite the **Langrangian**

$$J'(k) = \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T) - \boldsymbol{\lambda}^T(k+T)\mathbf{x}(k+T) + H(k) + \sum_{\tau=k+1}^{k+T-1}\left(H(\tau) - \boldsymbol{\lambda}^T(\tau)\mathbf{x}(\tau)\right)$$

Stationarity:

$$\begin{cases} \dfrac{\partial J'(k)}{\partial \boldsymbol{\lambda}(\tau)} = 0, \quad \tau = k+1,\ldots,k+T \\ \dfrac{\partial J'(k)}{\partial \mathbf{x}(\tau)} = 0, \quad \tau = k+1,\ldots,k+T \\ \mathbf{l}_i^T \dfrac{\partial J'(k)}{\partial \mathbf{K}(\tau)} \mathbf{l}_j = 0, \quad [\mathbf{E}_{\mathcal{G}_c}]_{ij} \neq 0,\ \tau = k,\ldots,k+T-1 \\ \mathbf{l}_i^T \mathbf{K}(\tau) \mathbf{l}_j = 0, \quad [\mathbf{E}_{\mathcal{G}_c}]_{ij} = 0,\ \tau = k,\ldots,k+T-1 \end{cases}$$

$$[\mathbf{l}_i]_k = \begin{cases} 1, & k = i \\ 0, & k \neq i \end{cases}$$

Result: neat identities involving the partial derivatives of the Hamiltonian

Decentralized linear quadratic control

Convex relaxation

Lemma

From the **stationarity** *conditions:* $\boldsymbol{\lambda}(k) = 2\mathbf{P}(k)\mathbf{x}(k)$

$$\begin{cases} \mathbf{P}(k+T) = \mathbf{Q}(k+T) \\ \mathbf{P}(\tau) = \mathbf{Q}(\tau) + \mathbf{K}^T(\tau)\mathbf{R}(\tau)\mathbf{K}(\tau) + (\mathbf{A}(\tau) - \mathbf{B}(\tau)\mathbf{K}(\tau))^T \mathbf{P}(\tau+1)\,(\mathbf{A}(\tau) - \mathbf{B}(\tau)\mathbf{K}(\tau)) \end{cases}$$

and

$$\begin{aligned} \mathbf{x}(i)^T\mathbf{P}(i)\mathbf{x}(i) = &\sum_{\tau=i}^{k+T-1} \mathbf{x}^T(\tau)\left(\mathbf{Q}(\tau) + \mathbf{K}^T(\tau)\mathbf{R}(\tau)\mathbf{K}(\tau)\right)\mathbf{x}(\tau) \\ &+ \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T), \quad i = k, \dots, k+T \end{aligned}$$

- Proof by **induction**[1]
- Similar to **centralized**

[1] Pedroso, L. and Batista, P., 2023. Discrete-time decentralized linear quadratic control for linear time-varying systems. International Journal of Robust and Nonlinear Control, 33(1), pp.67-101.

Decentralized linear quadratic control

Convex relaxation

Lemma

Necessary condition for **optimal gains**:

$$\begin{cases} \mathbf{I}_i^T \left[\left(\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau) \right) \mathbf{x}(\tau)\mathbf{x}^T(\tau) \right] \mathbf{I}_j = 0 & , [\mathbf{E}_{\mathcal{G}_c}]_{ij} \neq 0 \\ \mathbf{I}_i^T \mathbf{K}(\tau) \mathbf{I}_j = 0 & , [\mathbf{E}_{\mathcal{G}_c}]_{ij} \not= 0, \end{cases}$$

for $\tau = k, \ldots, k+T-1$,

$$\mathbf{S}(\tau) := \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{B}(\tau) + \mathbf{R}(\tau)$$

❓ Why is $\mathbf{x}(\tau)\mathbf{x}^T(\tau)$ (of rank 1) here?

Decentralized linear quadratic control

Convex relaxation

Necessary condition for **optimal gains** $(\tau = k, \dots, k+H-1)$

$$\begin{cases} \left[\left(\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right) \mathbf{x}(\tau)\mathbf{x}^T(\tau)\right]_{ji} = 0 & , [\mathbf{E}_{\mathcal{G}_c}]_{ji} \neq 0 \\ [\mathbf{K}(\tau)]_{ji} = 0 & , [\mathbf{E}_{\mathcal{G}_c}]_{ji} = 0 \end{cases}$$

⏪ $\mathbf{P}(\tau+1)$ and $\mathbf{S}(\tau)$ given by a **backward** recursion

💡 **Saddle point** satisfies these conditions

❓ $\mathbf{x}(\tau)$ is **not fully known** by any individual system

Relaxed conditions:

$$\begin{cases} \left[\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} \neq 0 \\ [\mathbf{K}(\tau)]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} = 0 \end{cases}$$

Decentralized linear quadratic control

Convex relaxation

Lemma (One-step relaxed solution)

Let $\mathbf{l}_j$ *denote a column vector whose entries are all set to zero except for the* j*-th one, which is set to* 1*, and* $\boldsymbol{\mathcal{L}}_j := \mathrm{diag}(\mathbf{l}_j)$*. Define* $\mathbf{m}_j \in \mathbb{R}^m$ *as*

$$\begin{cases} [\mathbf{m}_j]_i = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ij} = 0 \\ [\mathbf{m}_j]_i = 1, & [\mathbf{E}_{\mathcal{G}_c}]_{ij} \neq 0 \end{cases}, \; i = 1, ..., m\,,$$

and let $\boldsymbol{\mathcal{M}}_j := \mathrm{diag}(\mathbf{m}_j)$*. Then, the gains of the one-step relaxation are given by*

$$\mathbf{K}(\tau) = \sum_{j=1}^{n} (\mathbf{I} - \boldsymbol{\mathcal{M}}_j + \boldsymbol{\mathcal{M}}_j \mathbf{S}(\tau) \boldsymbol{\mathcal{M}}_j)^{-1} \boldsymbol{\mathcal{M}}_j \mathbf{B}^T(\tau) \mathbf{P}(\tau + 1) \mathbf{A}(\tau) \boldsymbol{\mathcal{L}}_j\,,$$

$\tau = k, \ldots, k + H - 1$.

Decentralized linear quadratic control

Convex relaxation

Overview:

- 💥 Does **not depend** on the initial condition $\mathbf{x}(\tau)$
  - **is not fully known** by any individual system
- 💡 **Closed-form** solution
- ⚡ **Computational complexity**[1] of $\mathcal{O}(n^3)$
  - **same as centralized**
- ❓ Can we find any **physical interpretation**?

[1]Pedroso, L. and Batista, P., 2021. Efficient algorithm for the computation of the solution to a sparse matrix equation in distributed control theory. Mathematics, 9(13), p.1497.

Decentralized linear quadratic control

Convex relaxation

❓ Can we find any **physical interpretation**?

💡 Yes!

One-step relaxation is **equivalent** to

$$\begin{aligned} \underset{\mathbf{K}(\tau)\in\mathbb{R}^{m\times n}}{\text{minimize}} \quad & \operatorname{tr}(\mathbf{P}(\tau)) \\ \text{subject to} \quad & \mathbf{K}(\tau) \in \operatorname{Sparse}(\mathbf{E}_{\mathcal{G}_c}) \end{aligned}$$

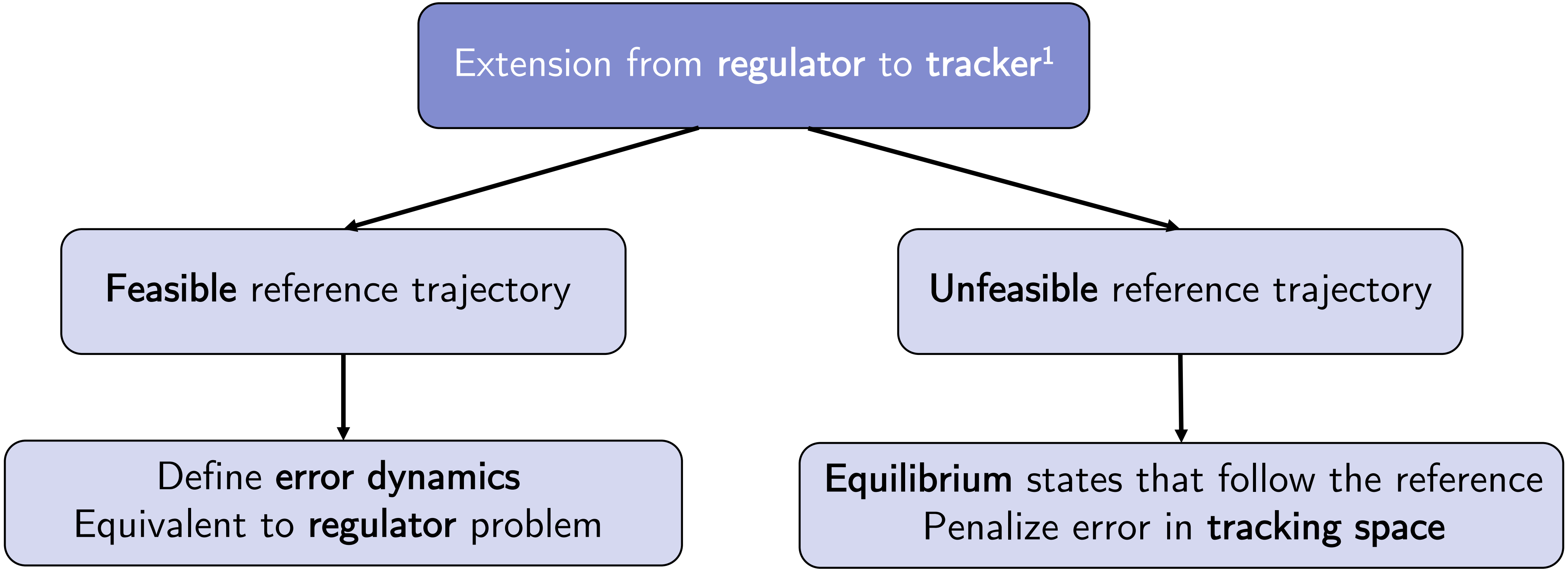
for $\tau = k, ..., k+H-1$

- **Decoupled in time** (greedy)
- **Ignores cross-correlation** between states
- Proof [1]

[1] Pedroso, L. and Batista, P., 2023. Discrete-time decentralized linear quadratic control for linear time-varying systems. International Journal of Robust and Nonlinear Control, 33(1), pp.67-101.

Decentralized linear quadratic control

Linear quadratic tracker



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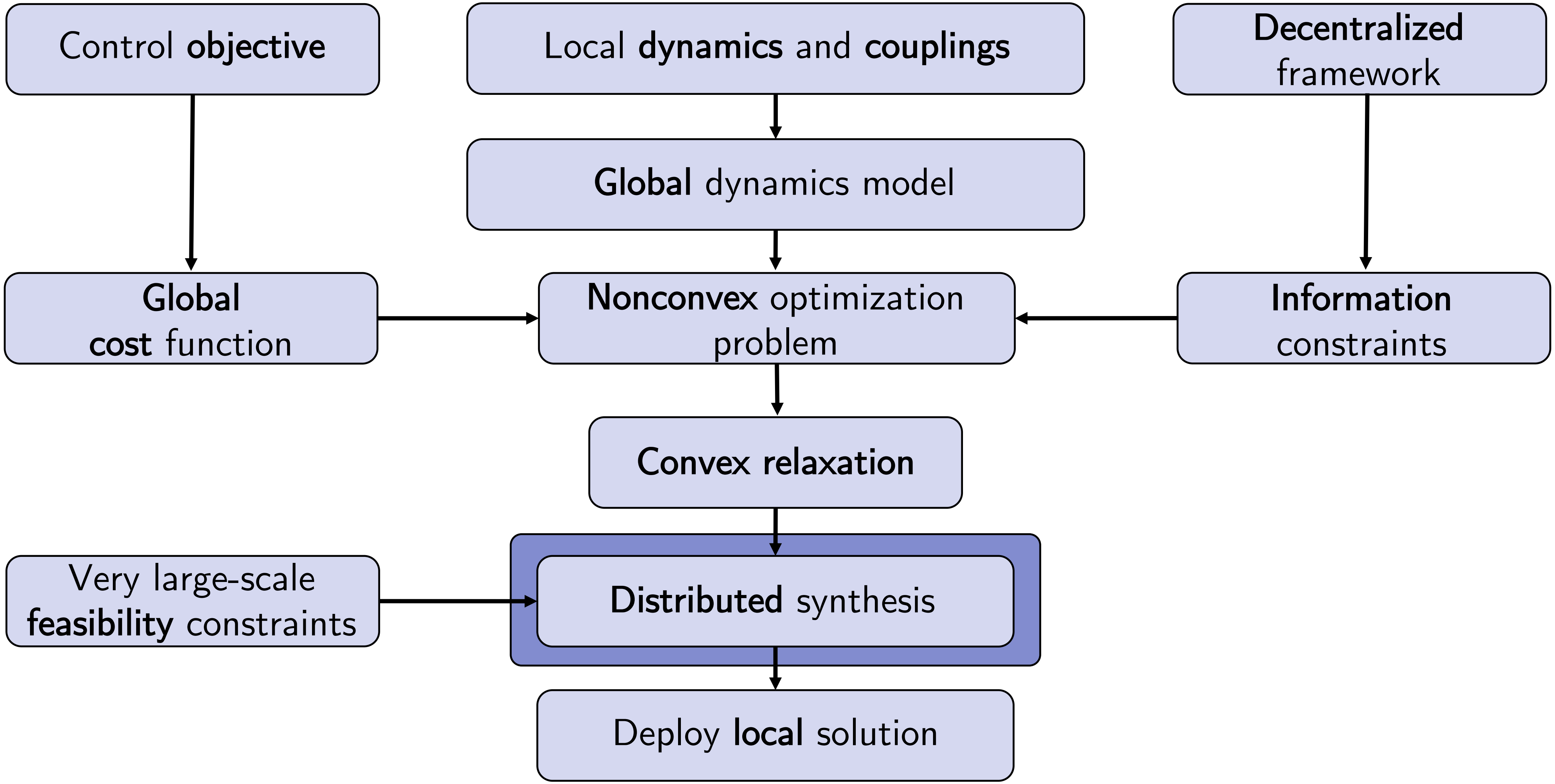
Distributed and decentralized RHC



Distributed and decentralized RHC

Challenges and approach

Challenges:

- **One-step** synthesis is **not distributed**
- **One-step** synthesis **does not follow** the very large-scale feasibility constraints

Approach:

- Particular case of **dynamically decoupled** systems
- An **approximation** to **decouple** the **one-step** synthesis
- Local computations **scheduling**

Distributed and decentralized RHC

Gain synthesis decoupling

- **Decoupled** dynamic coupling graph and $\mathcal{G}_c = \mathcal{G}_o = \mathcal{G}$

Relaxed original one-step conditions:

$$\begin{cases} \left[\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} \neq 0 \\ [\mathbf{K}(\tau)]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} = 0 \end{cases}$$

↓

Block matrix
decomposition

$$\begin{cases} \sum_{p \in \mathcal{D}_i^+} \mathbf{S}_{j,p}(\tau)\mathbf{K}_{p,i}(\tau) - \mathbf{B}_j^T(\tau)\mathbf{P}_{j,i}(\tau+1)\mathbf{A}_i(\tau) = \mathbf{0}, & j \in \mathcal{D}_i^+ \\ \mathbf{K}_{j,i}(\tau) = \mathbf{0}, & j \notin \mathcal{D}_i^+ \end{cases}$$

Distributed and decentralized RHC

Gain synthesis decoupling

- **Decoupled** dynamic coupling graph and $\mathcal{G}_c = \mathcal{G}_o = \mathcal{G}$

Relaxed original one-step conditions:

$$\begin{cases} \left[\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} \neq 0 \\ [\mathbf{K}(\tau)]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} = 0 \end{cases}$$

↓

Block matrix decomposition

$$\mathbf{P}_{p,q}(\tau) = \sum_{r\in\mathcal{D}_p^+\cap\mathcal{D}_q^+} \mathbf{H}_{r,i}^T(\tau)\mathbf{Q}_r(\tau)\mathbf{H}_{r,j}(\tau) + \sum_{r\in\mathcal{D}_p^+\cap\mathcal{D}_q^+} \mathbf{K}_{r,i}^T(\tau)\mathbf{R}_r(\tau)\mathbf{K}_{r,j}(\tau)$$
$$+ \sum_{r\in\mathcal{D}_p^+}\sum_{s\in\mathcal{D}_q^+} \left(\mathbf{A}_p(\tau)\boldsymbol{\delta}_{pr} - \mathbf{B}_r(\tau)\mathbf{K}_{r,p}(\tau)\right)^T \mathbf{P}_{r,s}(\tau+1)\left(\mathbf{A}_q(\tau)\boldsymbol{\delta}_{qs} - \mathbf{B}_s(\tau)\mathbf{K}_{s,q}(\tau)\right)$$

Distributed and decentralized RHC

Gain synthesis decoupling

- **Decoupled** dynamic coupling graph and $\mathcal{G}_c = \mathcal{G}_o = \mathcal{G}$

Relaxed original one-step conditions:

$$\begin{cases} \left[\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} \neq 0 \\ [\mathbf{K}(\tau)]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} = 0 \end{cases}$$

↓

Block matrix decomposition

🔍 The local gains $\mathbf{K}_{j,i}(\tau),\ j \in \mathcal{D}_i^+$ can be **computed locally** in $\mathcal{T}_i$

🧐 But the propagation of $\mathbf{P}_{j,i}(\tau+1)$ **cannot!**

$$\mathbf{P}_{p,q}(\tau) = \sum_{r\in\mathcal{D}_p^+\cap\mathcal{D}_q^+} \mathbf{H}_{r,i}^T(\tau)\mathbf{Q}_r(\tau)\mathbf{H}_{r,j}(\tau) + \sum_{r\in\mathcal{D}_p^+\cap\mathcal{D}_q^+} \mathbf{K}_{r,i}^T(\tau)\mathbf{R}_r(\tau)\mathbf{K}_{r,j}(\tau)$$
$$+ \sum_{r\in\mathcal{D}_p^+}\sum_{s\in\mathcal{D}_q^+} \left(\mathbf{A}_p(\tau)\boldsymbol{\delta}_{pr} - \mathbf{B}_r(\tau)\mathbf{K}_{r,p}(\tau)\right)^T \mathbf{P}_{r,s}(\tau+1)\left(\mathbf{A}_q(\tau)\boldsymbol{\delta}_{qs} - \mathbf{B}_s(\tau)\mathbf{K}_{s,q}(\tau)\right)$$

Distributed and decentralized RHC

Gain synthesis decoupling

- **Decoupled** dynamic coupling graph and $\mathcal{G}_c = \mathcal{G}_o = \mathcal{G}$

Relaxed original one-step conditions:

$$\begin{cases} \left[\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} \neq 0 \\ [\mathbf{K}(\tau)]_{ji} = 0, & [\mathbf{E}_{\mathcal{G}_c}]_{ji} = 0 \end{cases}$$

↓

Block matrix decomposition

↓

Thoughtful **approximation**

The local gains $\mathbf{K}_{j,i}(\tau),\ j \in \mathcal{D}_i^+$ can be **computed locally** in $\mathcal{T}_i$

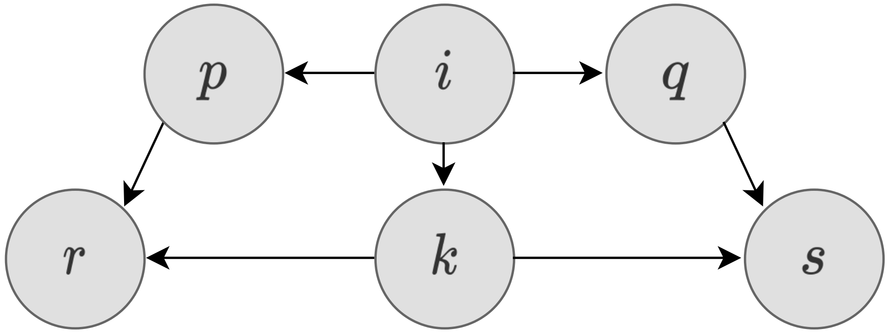
But the propagation of $\mathbf{P}_{j,i}(\tau+1)$ **cannot!**

Distributed and decentralized RHC

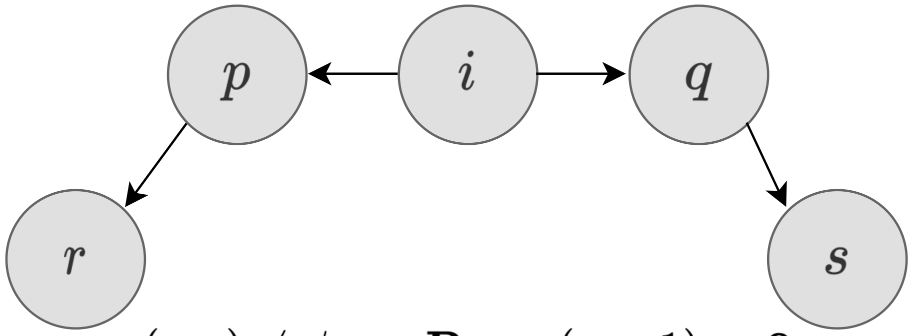
Gain synthesis decoupling

💡 **Thoughtful approximation** to decouple it

- Computation unit $\mathcal{T}_i$
- $\mathbf{P}_{r,s}(\tau+1)$ computed in $\mathcal{T}_i$ denoted as $\mathbf{P}_{i,(r,s)}(\tau+1)$
- **Approximation**:



$(r,s) \in \psi_i$



$(r,s) \notin \psi_i \rightarrow \mathbf{P}_{i,(r,s)}(\tau+1) \approx \mathbf{0}$

💥 **Exact** for some topologies

Distributed and decentralized RHC

Gain synthesis decoupling

But the propagation of $\mathbf{P}_{j,i}(\tau+1)$ **cannot!**

Block matrix decomposition + Thoughtful **approximation**

$\mathcal{T}_i$ **stores** and **updates** $\mathbf{P}_{i,(p,q)}(\tau), \forall p,q \in \mathcal{D}_i^+$

Can be **computed locally** in $\mathcal{T}_i$

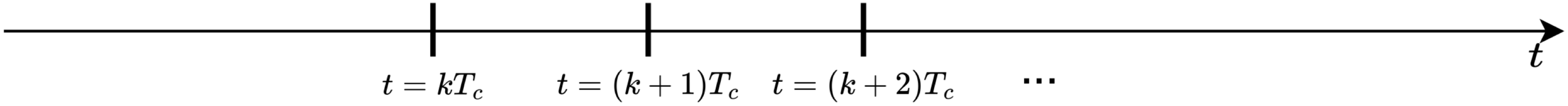
Distributed and decentralized RHC

Scheduling

Obstacle:

- **Only the first gain** in each finite-window used
- The propagation of $\mathbf{P}_{i,(p,q)}(\tau), \forall p, q \in \mathcal{D}_i^+$ is **backward in time!**
- Computation of a **whole finite-window every time instant**
  - **Infeasible** due to **transmission delays**

Local RHC computations **scheduling**



Distributed and decentralized RHC

Scheduling

Obstacle:

- **Only the first gain** in each finite-window used
- The propagation of $\mathbf{P}_{i,(p,q)}(\tau), \forall p, q \in \mathcal{D}_i^+$ is **backward in time!**
- Computation of a **whole finite-window every time instant**
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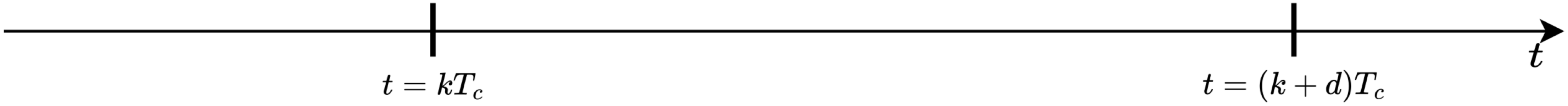
Local RHC computations **scheduling**



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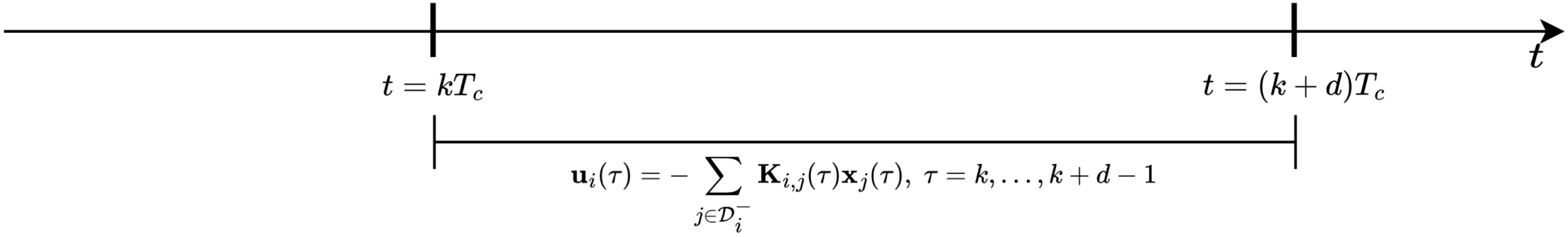

Distributed and decentralized RHC

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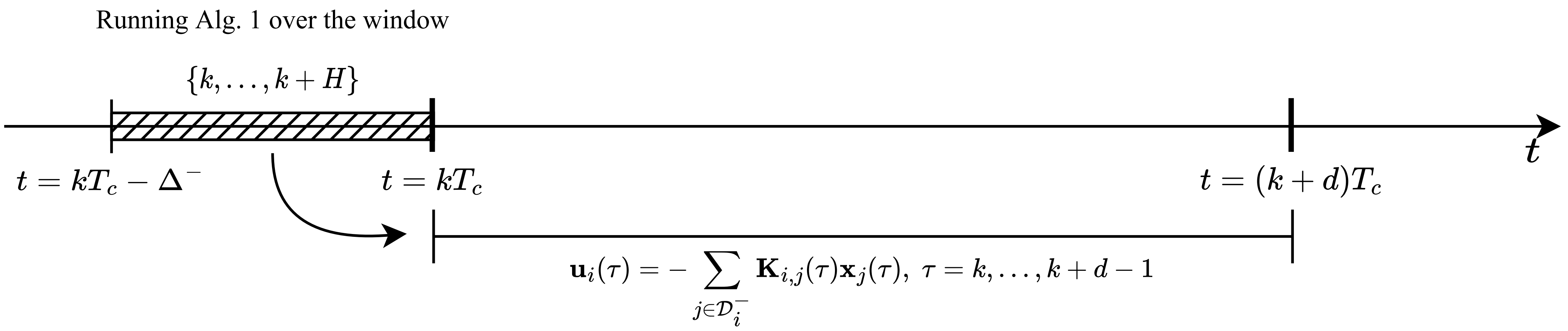

Distributed and decentralized RHC

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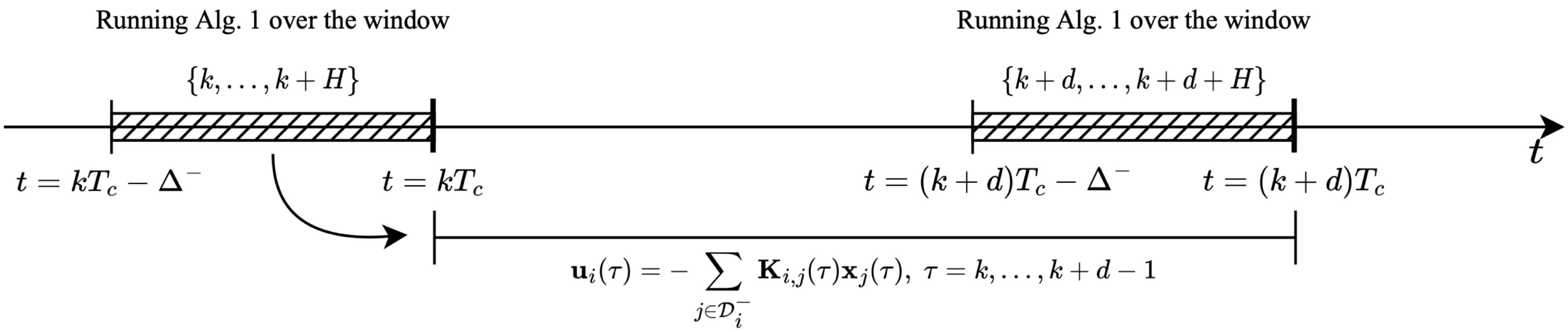

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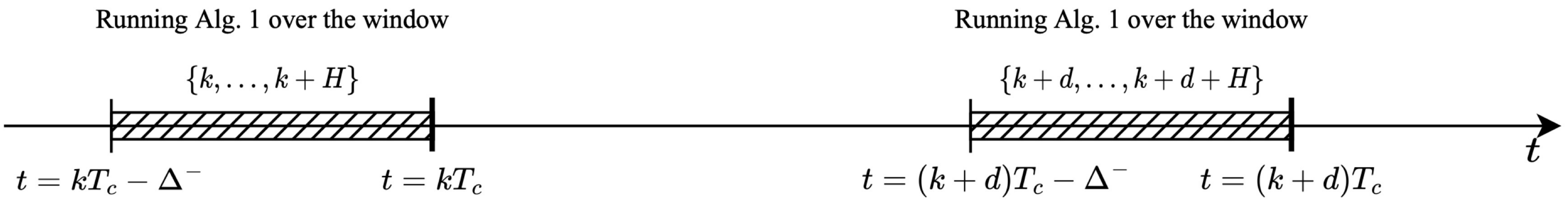
Local RHC computations **scheduling**



Satisfies the **very large-scale feasibility** constraints!

Distributed and decentralized RHC

Overview and extension

Overview:

- 🎯 **Distributed decentralized** real-time synthesis
- 📆 **Distributed** RHC computations **scheduling**
- ⚡ **Satisfies** the **very large-scale feasibility** constraints!

Extension:

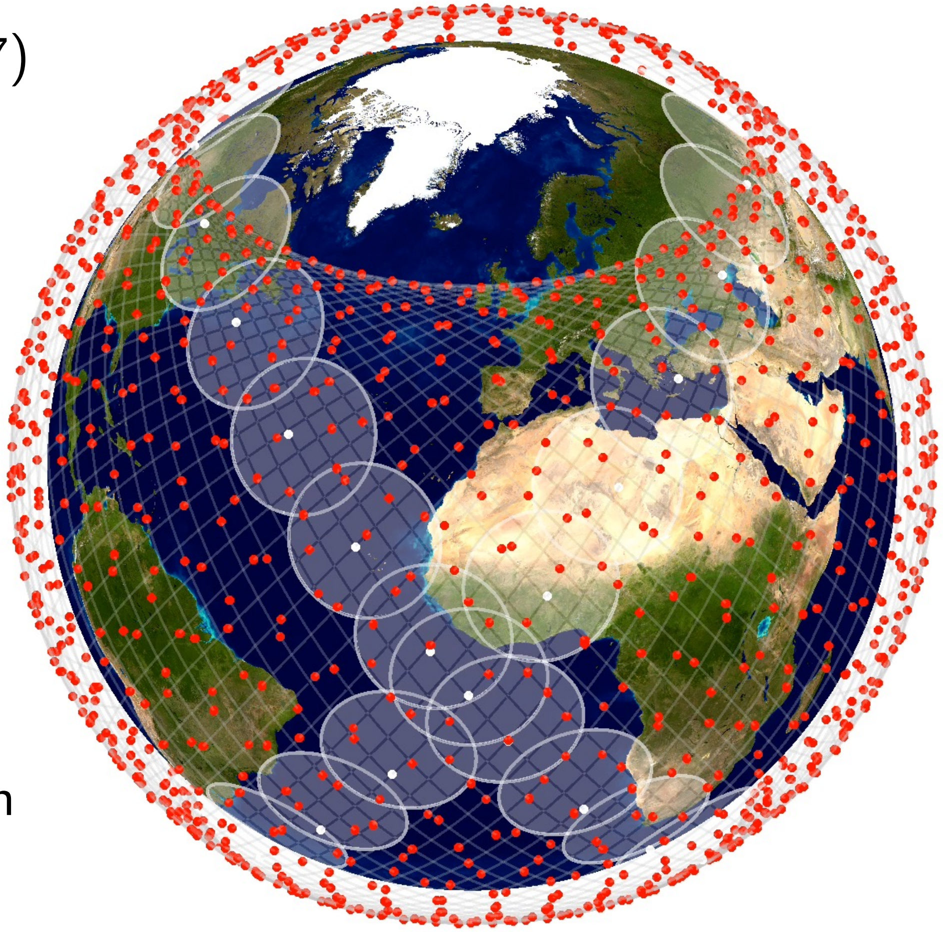
- 📈 **Time-varying** topology[1]

[1]Pedroso, L. and Batista, P., 2022. Distributed decentralized receding horizon control for very large-scale networks with application to LEO satellite mega-constellations. arXiv preprint arXiv:2209.14951.

On-board orbit control of LEO mega-constellations

Motivation

- **Very large-scale** (Starlink: 11 900 satellites planned by 2027)
- **Low-latency high capacity** global broadband connectivity
- Soaring **demand**
- Greatest concern is **economic viability**
- **Centralized TT&C** architecture
  - **Expensive** and **challenging** to maintain
- Inevitably evolving towards an **on-board distributed** solution
- **State-of-the-art** decentralized solutions **fail in this scale**



Single shell of the Starlink mega-constellation

On-board orbit control of LEO mega-constellations

Constellation model

- **Walker** constellation $\bar{i}:T/P/F$
- **Electric** Hall effect thrusters
- **State** of a satellite $\mathcal{S}_i$

$$\mathbf{x}_i(t) = \begin{bmatrix} a_i(t) \\ u_i(t) \\ e_{xi}(t) \\ e_{yi}(t) \\ i_i(t) \\ \Omega_i(t) \end{bmatrix}$$

(semi-major axis)

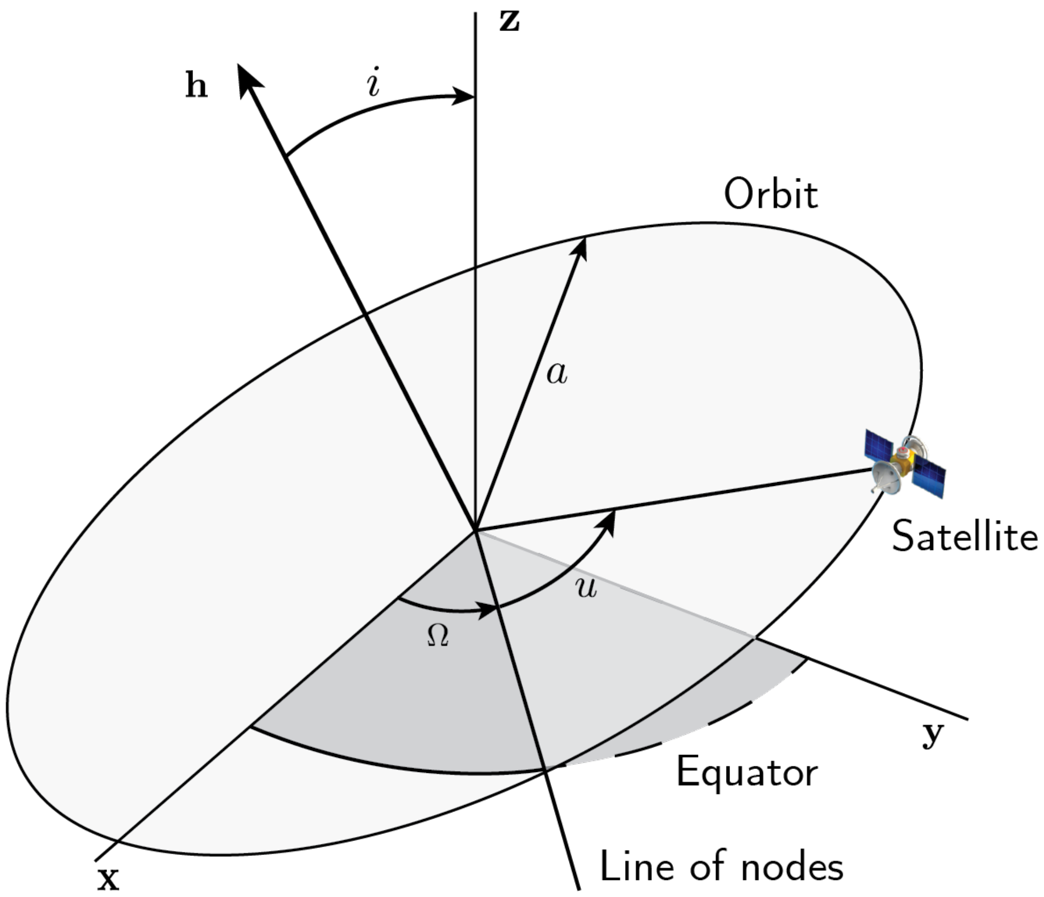
(mean argument of latitude)

(x component of eccentricity vector)

(y component of eccentricity vector)

(inclination)

(longitude of ascending node)



Nonsingular orbital elements of circular orbit

On-board orbit control of LEO mega-constellations

Nominal constellation

- **Nonlinear** orbital dynamics

- **Linearization** about **nominal** orbit $\bar{\mathbf{x}}_i(t) = \left[\bar{a}_i(t)\ \bar{u}_i(t)\ \bar{e}_{xi}(t)\ \bar{e}_{yi}(t)\ \bar{i}_i(t)\ \bar{\Omega}_i(t)\right]^T$

$$
\begin{cases}
\bar{a}_i(t) = \bar{a} \\
\bar{u}_i(t) = \bar{u}_{t_0} + ((i-1) \bmod T/P)\, 2\pi P/T + \lfloor (i-1)P/T \rfloor 2\pi F/T + (\dot{M} + \dot{\omega})(t - t_0) \\
\bar{e}_{x,i}(t) = 0 \\
\bar{e}_{y,i}(t) = 0 \\
\bar{i}_i(t) = \bar{i} \\
\bar{\Omega}_i(t) = \bar{\Omega}_{t_0} + \lfloor (i-1)P/T \rfloor 2\pi/P + \dot{\Omega}(t - t_0)
\end{cases}
$$

On-board orbit control of LEO mega-constellations

Nominal constellation

Nonlinear orbital dynamics

Linearization about **nominal** orbit $\bar{\mathbf{x}}_i(t) = \left[\bar{a}_i(t)\ \bar{u}_i(t)\ \bar{e}_{xi}(t)\ \bar{e}_{yi}(t)\ \bar{i}_i(t)\ \bar{\Omega}_i(t)\right]^T$

$$\begin{cases} \bar{a}_i(t) = \bar{a} \\ \bar{u}_i(t) = \bar{u}_{t_0} + ((i-1) \bmod T/P)\, 2\pi P/T + \lfloor (i-1)P/T \rfloor 2\pi F/T + (\dot{M} + \dot{\omega})(t - t_0) \\ \bar{e}_{x,i}(t) = 0 \\ \bar{e}_{y,i}(t) = 0 \\ \bar{i}_i(t) = \bar{i} \\ \bar{\Omega}_i(t) = \bar{\Omega}_{t_0} + \lfloor (i-1)P/T \rfloor 2\pi/P + \dot{\Omega}(t - t_0) \end{cases}$$

Secular perturbations due to J_2

On-board orbit control of LEO mega-constellations

Nominal constellation

Nonlinear orbital dynamics

Linearization about **nominal** orbit $\bar{\mathbf{x}}_i(t) = \left[\bar{a}_i(t)\ \bar{u}_i(t)\ \bar{e}_{xi}(t)\ \bar{e}_{yi}(t)\ \bar{i}_i(t)\ \bar{\Omega}_i(t)\right]^T$

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\end{cases}
$$

Secular perturbations due to J_2

$(t_0, \bar{u}_{t_0}, \bar{\Omega}_{t_0})$ is the **anchor** of the nominal constellation

On-board orbit control of LEO mega-constellations

Linearization

Relative orbital elements (in relation to **nominal state**)

$$\delta \mathbf{x}_i(t) := \begin{bmatrix} a_i(t) \\ \delta u_i(t) \\ \delta e_{x,i}(t) \\ \delta e_{y,i}(t) \\ \delta i_i(t) \\ \delta \Omega_i(t) \end{bmatrix} = \begin{bmatrix} a_i(t)/\bar{a}_i(t) - 1 \\ u_i(t) - \bar{u}_i(t) + \left(\Omega_i(t) - \bar{\Omega}_i(t)\right) \cos \bar{i}_i(t) \\ e_{x,i}(t) - \bar{e}_{x,i}(t) \\ e_{y,i}(t) - \bar{e}_{y,i}(t) \\ i_i(t) - \bar{i}_i(t) \\ \left(\Omega_i(t) - \bar{\Omega}_i(t)\right) \sin \bar{i}_i(t) \end{bmatrix}$$

D'Amico, S. (2010).
Autonomous formation flying in low earth orbit.
PhD thesis, TU Delft.

LTV model

$$\delta \mathbf{x}_i((k+1)T_c) = \mathbf{A}_i(k) \delta \mathbf{x}_i(kT_c) + \mathbf{B}_i(k) \mathbf{u}_i(kT_c)/m_i(kT_c)$$

On-board orbit control of LEO mega-constellations

Tracking output: Inertial

Decoupled tracking of

- Semi-major axis
- Eccentricity
- Inclination

Inertial tracking output of satellite $\mathcal{S}_i$

$$\mathbf{z}_{i,in}(k) = \begin{bmatrix} a_i(k) - \bar{a}_i(k) \\ e_{x,i}(k) - \bar{e}_{x,i}(k) \\ e_{y,i}(k) - \bar{e}_{y,i}(k) \\ i_i(k) - \bar{i}_i(k) \end{bmatrix} = \begin{bmatrix} \bar{a}_i(k)\delta a_i(k) \\ \delta e_{x,i}(k) \\ \delta e_{y,i}(k) \\ \delta i_i(k) \end{bmatrix}$$

On-board orbit control of LEO mega-constellations

Tracking output: Relative

🕹️ **Relative** tracking of

- Mean argument of latitude
- Longitude of ascending node

⛳ **Goal**: maintain the constellation **shape**

🔗 **Sparse couplings** with satellites in **close proximity**

- Within a **range** R up to a **maximum** of $|\mathcal{D}^-|_{\max}$ satellites in each in-neighborhood

🛰️ **Relative tracking output** of satellite $\mathcal{S}_i$ with respect to $\mathcal{S}_j,\ j \in \mathcal{D}_i^-$

$$\mathbf{z}_{i,j}^{ref}(k) := \begin{bmatrix} u_i(k) - u_j(k) - (\bar{u}_i(k) - \bar{u}_j(k)) \\ \Omega_i(k) - \Omega_j(k) - (\bar{\Omega}_i(k) - \bar{\Omega}_j(k)) \end{bmatrix} = \begin{bmatrix} \delta u_i(k) - \delta u_j(k) - (\delta\Omega_i(k) - \delta\Omega_j(k)) \,/\, \tan \bar{i} \\ (\delta\Omega_i(k) - \delta\Omega_j(k)) \,/\, \sin \bar{i} \end{bmatrix}$$

On-board orbit control of LEO mega-constellations

Tracking output: Relative

Relative tracking output of satellite $\mathcal{S}_i$

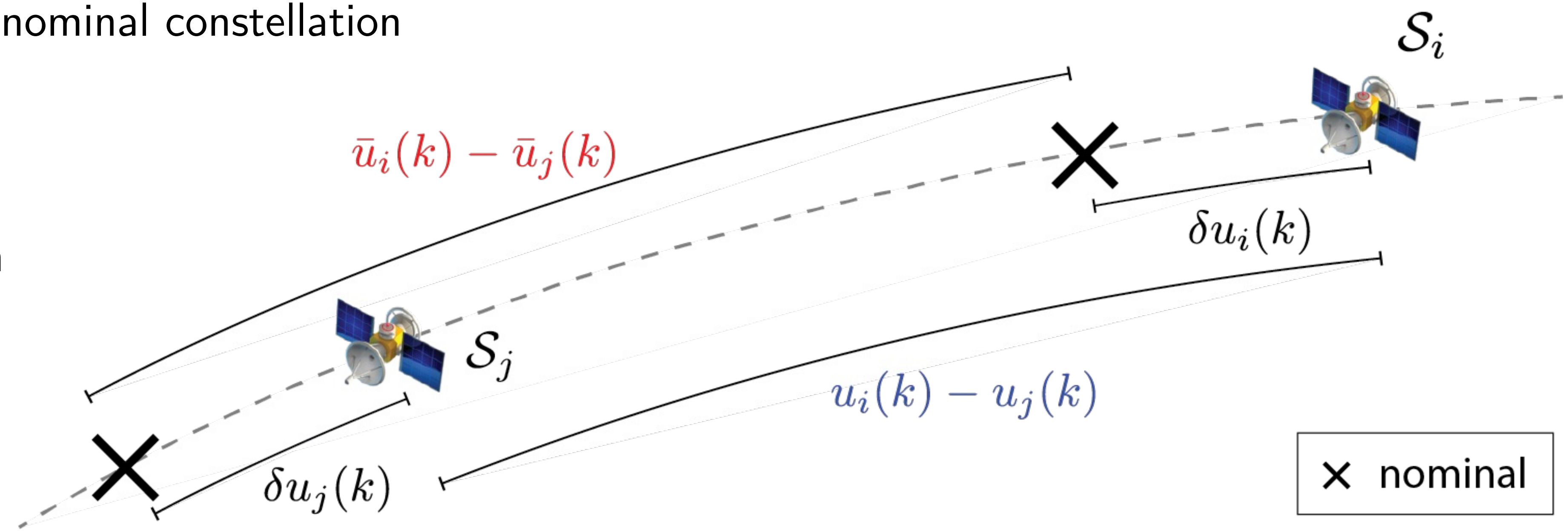
$$\mathbf{z}_{i,j}^{ref}(k) := \begin{bmatrix} u_i(k) - u_j(k) - (\bar{u}_i(k) - \bar{u}_j(k)) \\ \Omega_i(k) - \Omega_j(k) - (\bar{\Omega}_i(k) - \bar{\Omega}_j(k)) \end{bmatrix} = \begin{bmatrix} \delta u_i(k) - \delta u_j(k) - (\delta\Omega_i(k) - \delta\Omega_j(k)) \,/\, \tan \bar{i} \\ (\delta\Omega_i(k) - \delta\Omega_j(k)) \,/\, \sin \bar{i} \end{bmatrix}$$

Satellites are **not driven** towards the **nominal** constellation!

Drifts away from nominal constellation

Fuel saving!

Novel formulation



On-board orbit control of LEO mega-constellations

Illustrative Constellation, Tuning and Simulation

Single shell of **Starlink** mega-constellation

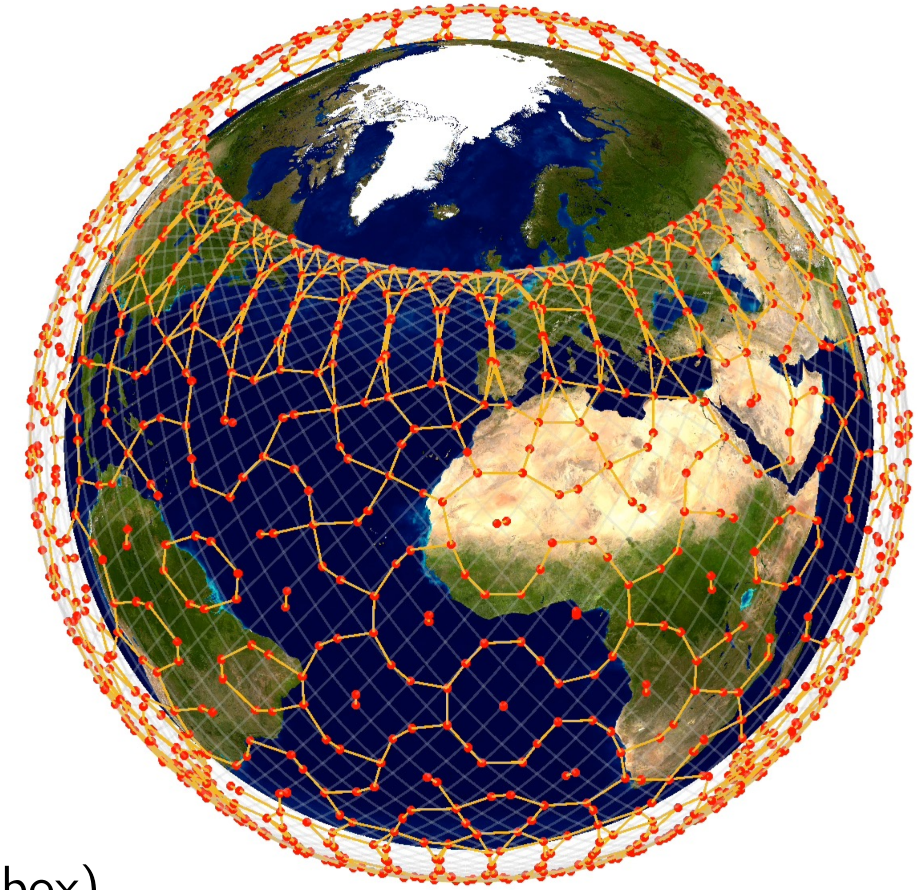
- **1584** satellites
- $h = 540\,\mathrm{km}$

Tuning

- $R = 750\,\mathrm{km}$
- $|\mathcal{D}^-|_{\max} = 6$

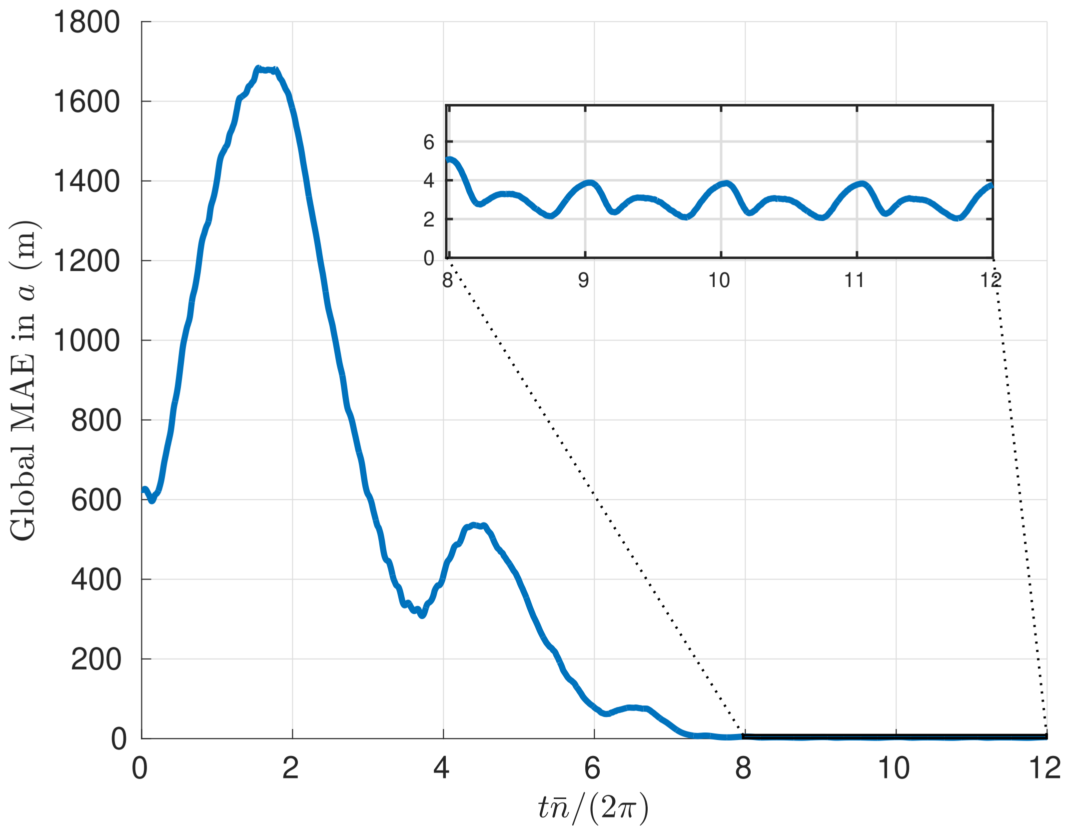
Simulation

- **High-fidelity** orbit propagation (TUDAT Toolbox)
- **Perturbations:** EGM96 **gravity** potential, NRLMSISE-00 atmospheric **drag**, solar **radiation** pressure, **third body**

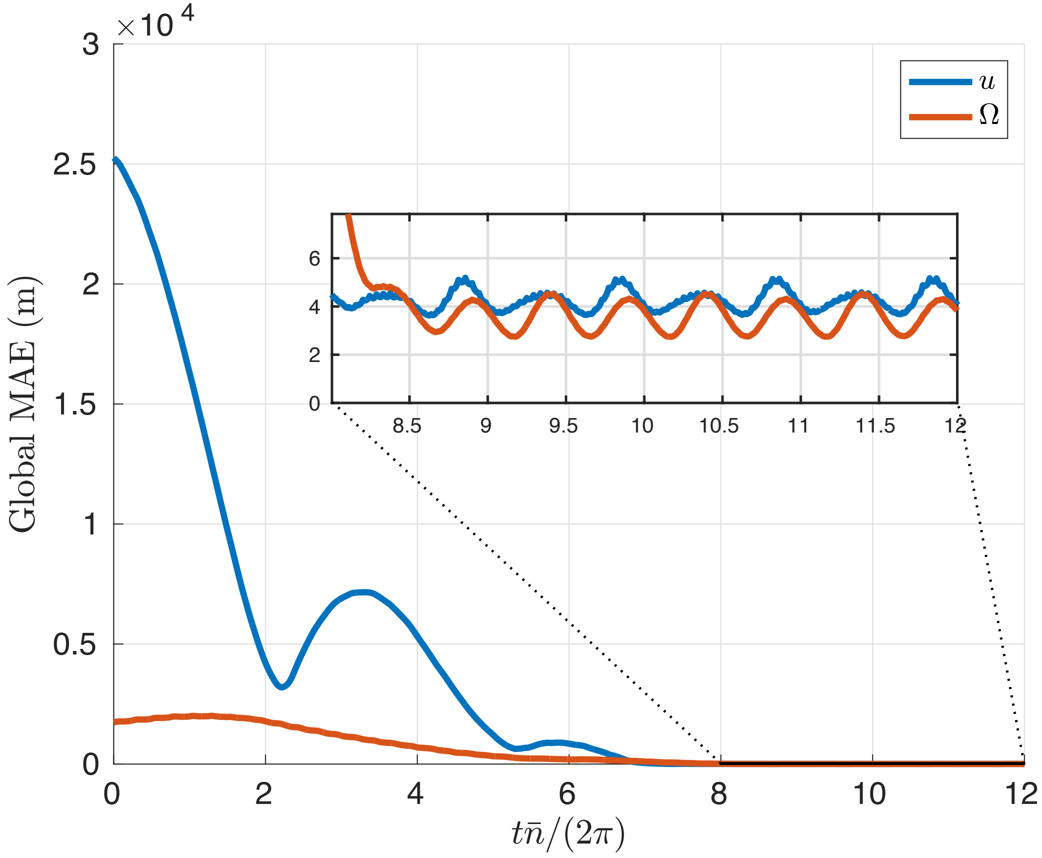


On-board orbit control of LEO mega-constellations

Simulation results



Global mean absolute error (MAE) of semi-major axis

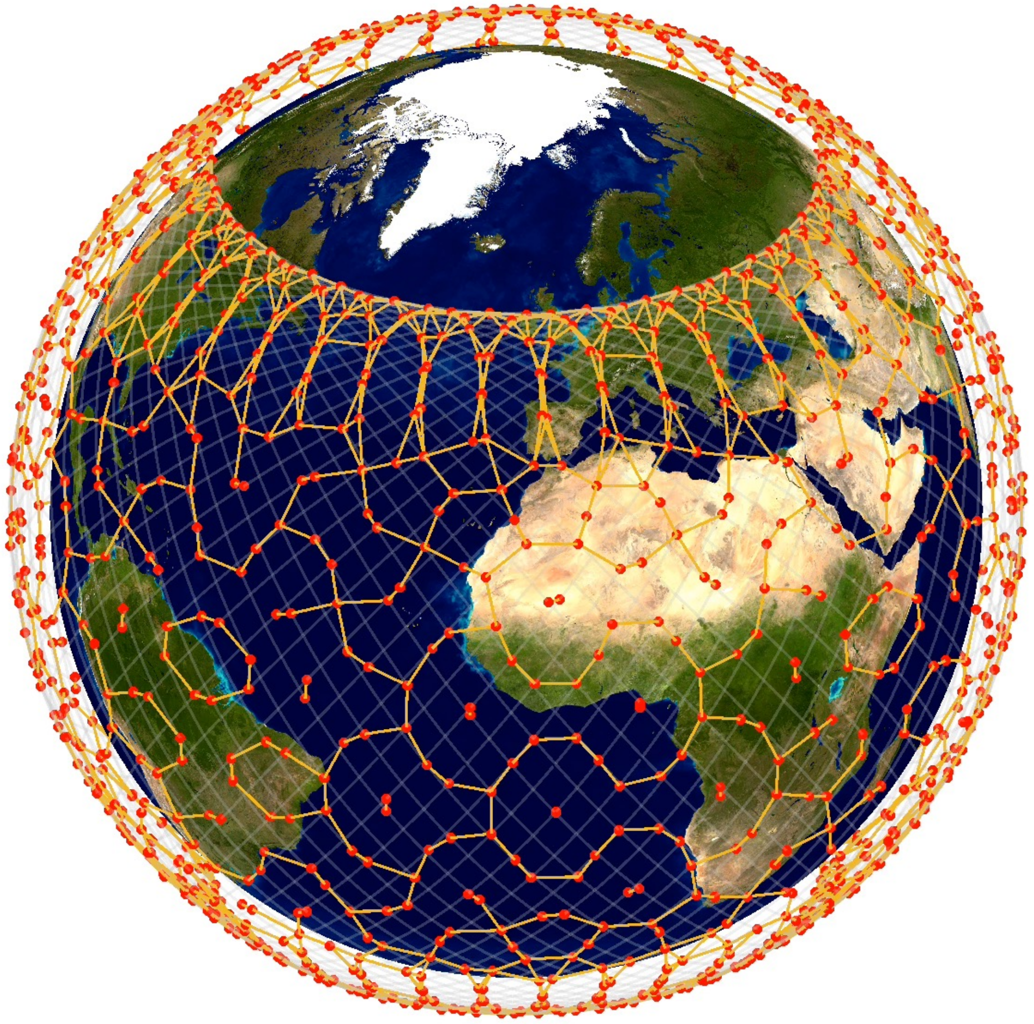


Global mean absolute error (MAE) of mean argument of latitude and longitude of ascending node

Meter-level accuracy!

Conclusion

- **Inhibiting** technical **challenges** on a **very large-scale networked control systems**
- **Convex relaxation** of the decentralized problem
- **Distributed** real-time synthesis
- **Promising** performance on the **pressing shape-keeping** task of LEO mega-constellations
- **No state-of-the-art solutions** can address this problem **in such scale**



May 2023

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