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Urgency-aware Routing in Single Origin-destination Itineraries through Artificial Currencies

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Introduction

Motivation

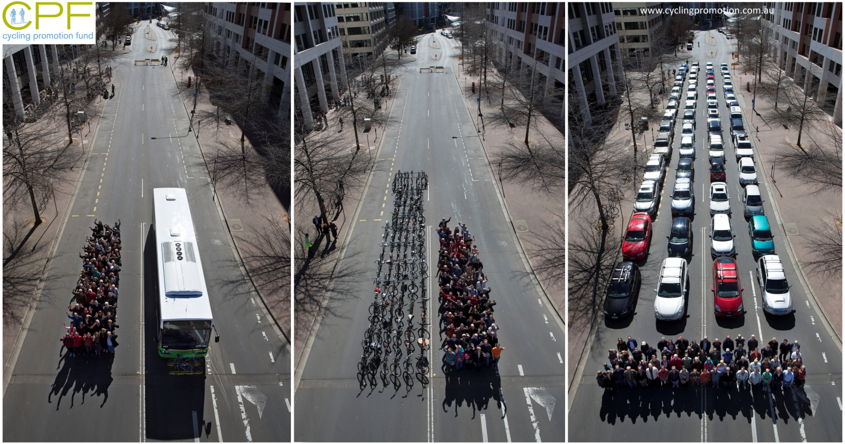

Figure 1: 69 people in bus, bikes, and cars. (Cycling Promotion Fund, 9th September 2012 [C.P.F., 2012])

Introduction

Opportunity

- Vehicle autonomy
- Car sharing
- Public transport
- Connectivity

Figure 2: New opportunities. ([Raysonho, 2015, Grendelkhan, 2017])

Centralized controlled **intermodal mobility** $\rightarrow$ **system's optimum** performance![1,2]

[1] Salazar, Rossi, Schiffer, Onder, Pavone. "On the interaction between autonomous mobility-on-demand and public transportation systems." ITSC, 2018. [Salazar et al., 2018]

[2] Wollenstein-Betech, Salazar, *et al.*. "Routing and rebalancing intermodal autonomous mobility-on-demand systems in mixed traffic." IEEE T-ITS, 2021. [Wollenstein-Betech et al., 2021]

Literature Review

Self-interested behavior $\longleftrightarrow$ **Societal Welfare**

Monetary Tolls [1]

- ✓ Easy to implement
- ✓ Easy to use
- ✗ Unfair

Artificial Currencies [2]

- ✓ Fair
- ✗ Bidding
- ✗ Uncertainty

Idea: Bridge the gap[3]

Payment-transaction of **artificial currency** $\rightarrow$ **urgency-aware** system's **optimum**

[1] [Pigou, 1920, Morrison, 1986, Bergendorff et al., 1997, Fleischer et al., 2004, Paccagnan et al., 2019]

[2] [Prendergast, 2016, Gorokh et al., 2019, Censi et al., 2019, Elokda et al., 2022]

[3] Salazar, Paccagnan, Agazzi, Heemels. "Urgency-aware optimal routing in repeated games through artificial currencies." European Journal of Control 62 (2021). [Salazar et al., 2021]

Problem Statement

Repeated game-framework

- User choice: $\mathbf{y}^i(t) \in \{0,1\}^n$
- Traveling probability: P_{go}
- Each arc has a **price**:

$$k^i(t+1) = k^i(t) - \mathbf{p}^\top \mathbf{y}^i(t)$$

- **Aggregate** flows of M users:

$$\mathbf{x}(t) = \frac{1}{M}\sum_i \mathbf{y}^i(t)$$

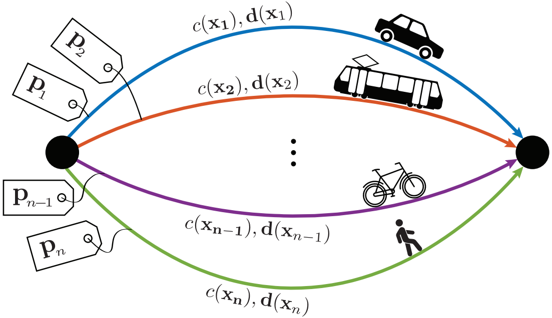

Figure 3: Parallel-arc network.

Self-interested at a cost ⚔ **Altruistic** for a reward

Three-level Analysis

Macroscopic
(Central Operator)

Microscopic
(User)

Mesoscopic
(Incentive Mechanism)

Problem Statement

Three-level Analysis: Macroscopic

Social cost of arc j: $\mathbf{c}_j(\mathbf{x}_j)$

Minimize overall social cost: $\mathbf{c}^\top \mathbf{x}$

Problem (Central Operator's Problem)

The **central operator** aims at routing customers so that the **aggregate flows** are

$$\mathbf{x}^\star \in \arg \min_{\mathbf{x}\in[0,1]^n} \mathbf{c}(\mathbf{x})^\top \mathbf{x}$$
$$\text{s.t.} \qquad \mathbf{1}^\top \mathbf{x} = P_{\mathrm{go}}.$$

Problem Statement

Three-level Analysis: Microscopic

- **Discomfort** of arc j: $\mathbf{d}_j(\mathbf{x}_j)$
- Daily **sensitivity** to discomfort: s^i
- Min. **daily perceived** discomfort + average **future** discomfort over T days

Problem (Individual User's Problem)

A traveling user with Karma level $k \geq 0$, reference k_{ref}, and sensitivity s will choose his/her route as $\mathbf{y}^\star$ resulting from

$$(\mathbf{y}^\star, \bar{\mathbf{y}}^\star) \in \underset{\mathbf{y}\in\mathcal{Y},\, \bar{\mathbf{y}}\in\bar{\mathcal{Y}}}{\mathrm{argmin}} \; s\,\mathbf{d}(\mathbf{x})^\top \mathbf{y} + T\,\bar{s}\,\mathbf{d}(\mathbf{x})^\top \bar{\mathbf{y}}$$

$$\text{s.t.} \;\; k - \mathbf{p}^\top \mathbf{y} - T\mathbf{p}^\top \bar{\mathbf{y}} \geq 0$$

$$\mathbf{p}^\top \mathbf{y} \leq k,$$

Problem Statement

Three-level Analysis: Mesoscopic

- **Infinite-user** population: $M \to \infty$
- Users achieve daily **Wardrop Equilibrium** (WE): $\mathbf{x}^{\mathrm{WE}}(t)$
- Design **prices p**

Problem (Pricing Problem)

Given a desired system optimum $\mathbf{x}^\star$, select $\mathbf{p} \in \mathbb{R}^n$ so that

$$\lim_{t \to \infty} \mathbf{x}^{\mathrm{WE}}(t) = \mathbf{x}^\star.$$

Best-response strategy

Closed-form Solution

Theorem (User's Best Response Strategy)[1]

An **optimal response strategy** of a with Karma k, sensitivity s, and Karma reference k_{ref} is $\mathbf{y}^\star = \mathbf{e}_{\mathbf{j}^\star}$ iff

$$\bar{\gamma}_{j^\star} \geq \underline{\gamma}_{j^\star} \quad \text{and} \quad \gamma_{j^\star} \leq s/\bar{s} \leq \gamma_{j^\star-1}$$

[1] Pedroso, Salazar, Heemels. "Urgency-aware Routing in Single Origin-destination Itineraries through Artificial Currencies" (*preprint*)

Best-response strategy

Closed-form Solution

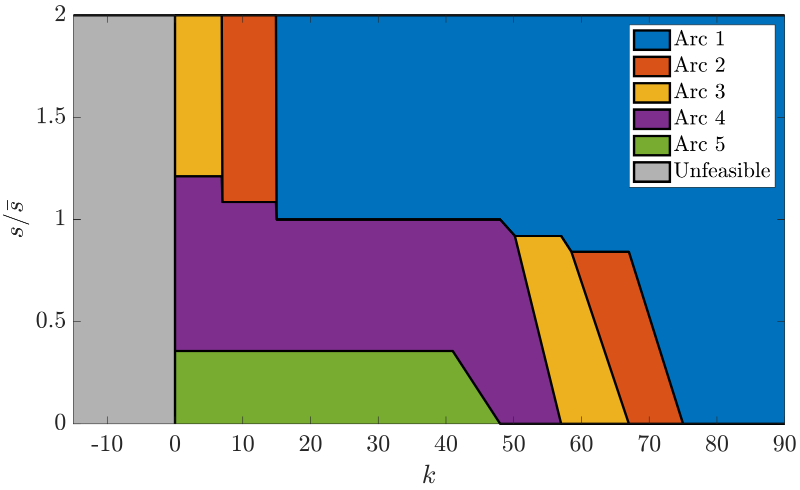

Figure 4: Decision landscape of individual user's problem.

Pricing Design Problem

At **steady-state** in $\mathbf{x}^\star$

- **Total Karma** remains constant: $\mathbf{p}^\top \mathbf{x}^\star = 0$

For $n = 2$ arcs[1]

- $\mathbf{p}^\top \mathbf{x}^\star = 0$ **alone** defines the **optimal prices**

For n arcs[2]

- Much more intricate

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[2] Pedroso, Salazar, Heemels. "Urgency-aware Routing in Single Origin-destination Itineraries through Artificial Currencies" (*preprint*)

Pricing Design Problem: n arcs

Markov chain

- $\mathrm{P}(j^\star | k^i, \mathbf{p}, \mathbf{x}^\star)$ from the **best response strategy**
- Stationary **Karma distribution** $\pi_\infty(\mathbf{p}, \mathbf{x}^\star)$

Aggregate of Markov chains

$$\mathbf{x}_j^\star = \sum_{k=k_{\min}}^{k_{\max}} \mathrm{P}(j^\star = j | k, \mathbf{p}, \mathbf{x}^\star)[\pi_\infty(\mathbf{p}, \mathbf{x}^\star)]_k, \quad j = 1, \ldots, n$$

Challenge for $n > 2$

- The support of the chain depends on $\mathbf{p}$
- **Gradient-free** optimization

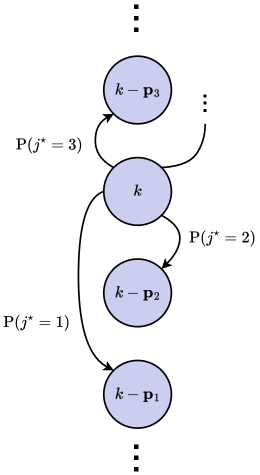


Figure 5: Markov chain.

Numerical Results

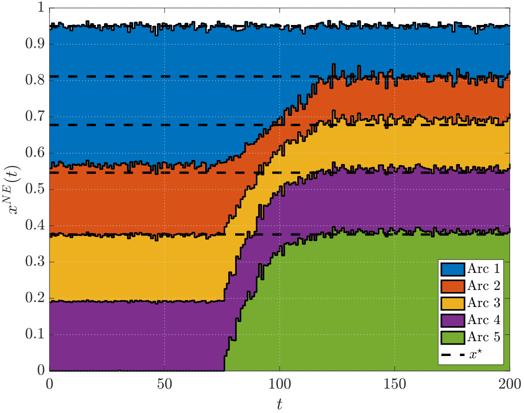


Figure 6: Aggregate flows.

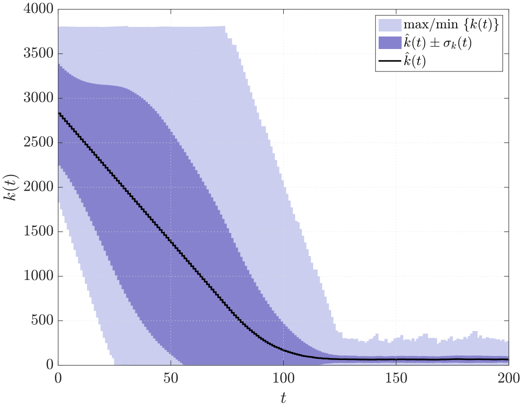


Figure 7: Karma level.

Numerical Results

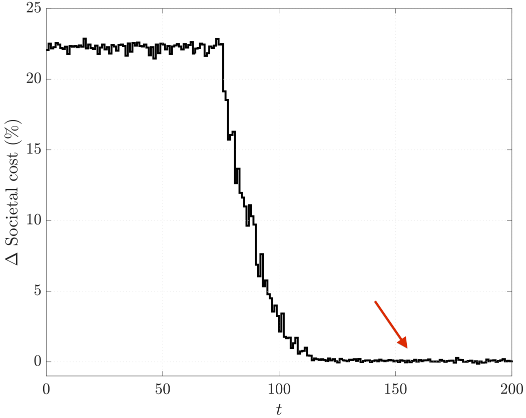


Figure 8: Societal cost.

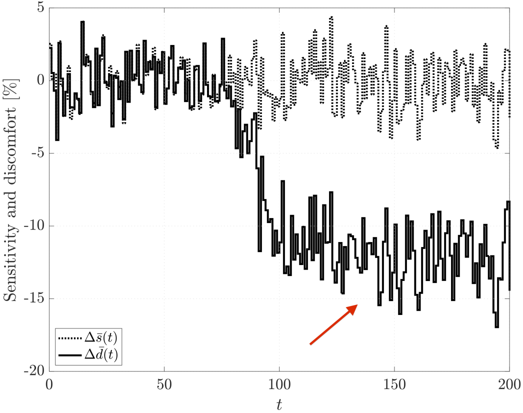
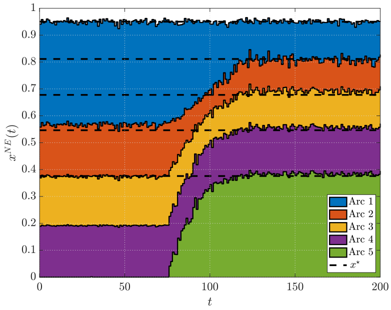


Figure 9: Sensitivity w.r.t. urgency-unaware.

Conclusion

- Incentive scheme: **fair** and **urgency-aware**
- Solution for the **user's best response strategy**
- **Pricing design** procedure for n arcs
- Aggregate decision achieves **system's optimum**
- 12% **improvement** w.r.t. urgency-unaware policy



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