Decentralized linear quadratic control for networks with time-varying dynamics: design and applications

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Who's Leonardo?

- Bachelor (3 years) in Aerospace Engineering in **2020**
- Master (2 years) expected in October **2022**
- **DECENTER** project since **2019**
  - 5 papers in peer-reviewed journals
  - 2 papers in peer-reviewed conferences
- Looking for a **high-impact challenging PhD** position

Outline

Introduction

Motivation

commons.wikimedia.org/w/index.php?curid=36426738

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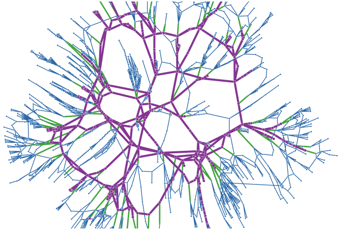

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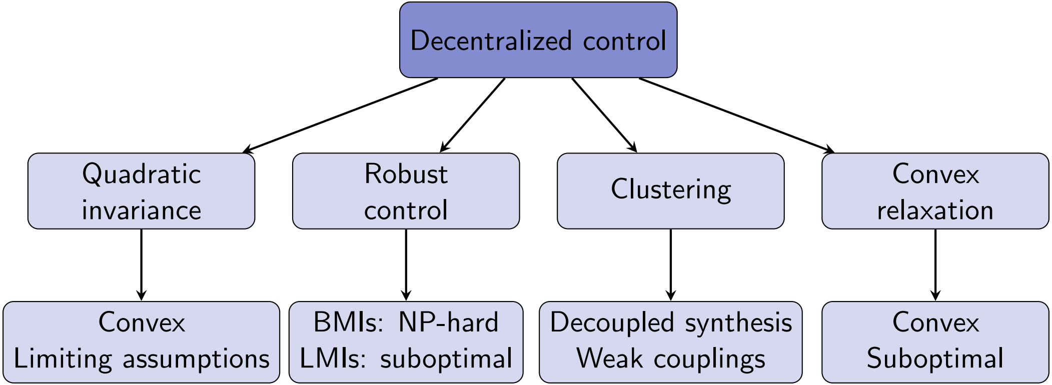
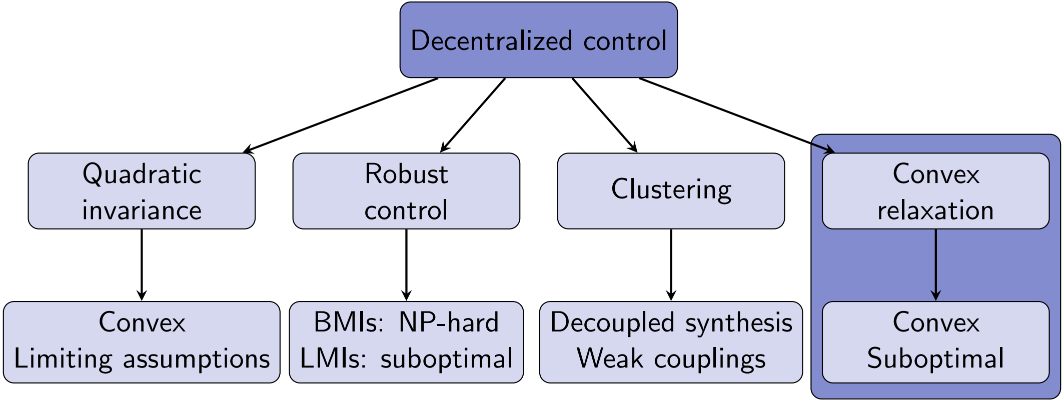
Introduction

State-of-the-art overview



Introduction

State-of-the-art overview



Outline

Decentralized linear quadratic control

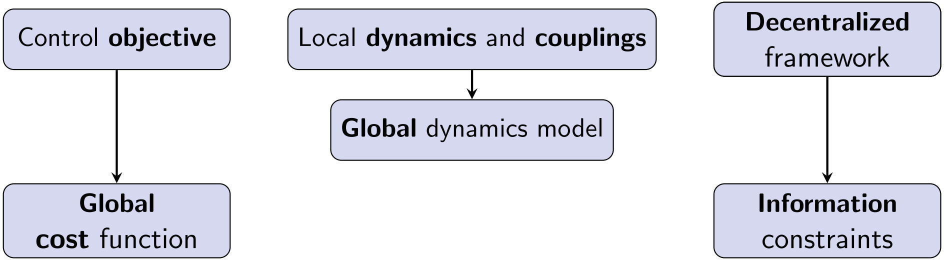
Approach overview

Control **objective**	Local **dynamics** and **couplings**	**Decentralized** framework
▶ **Common/decoupled** ▶ **Quadratic** cost ▶ **Regulator**	▶ **LTV** ▶ Approx. **nonlinear** sys. ▶ **Sparse** couplings	▶ **Linear** feedback ▶ **Local** feedback

Pedroso, L. and Batista, P. (2021b).
Discrete-time decentralized linear quadratic control for linear time-varying systems.
International Journal of Robust and Nonlinear Control.

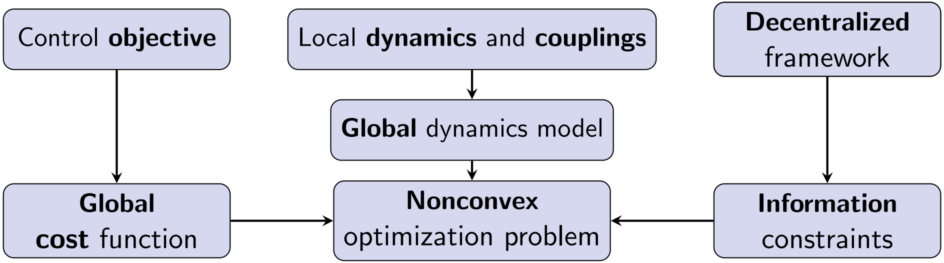
Decentralized linear quadratic control

Approach overview



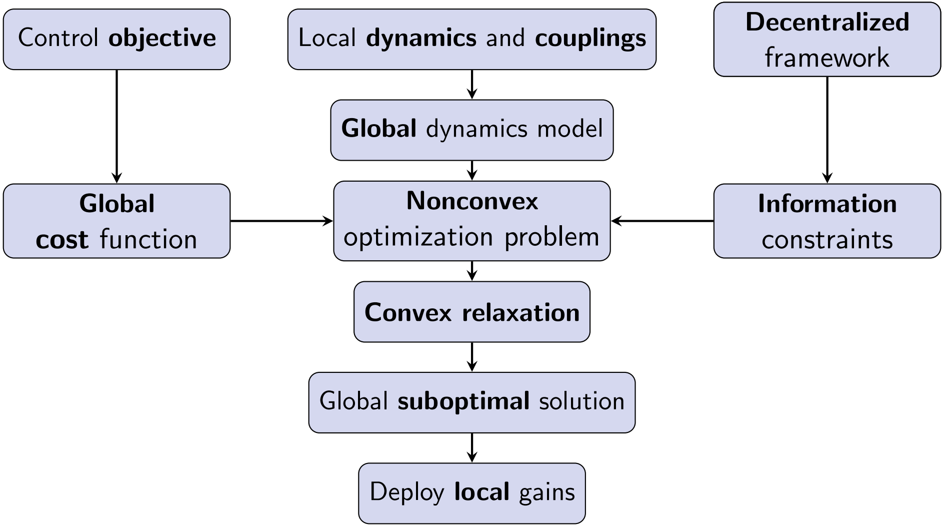
Decentralized linear quadratic control

Approach overview



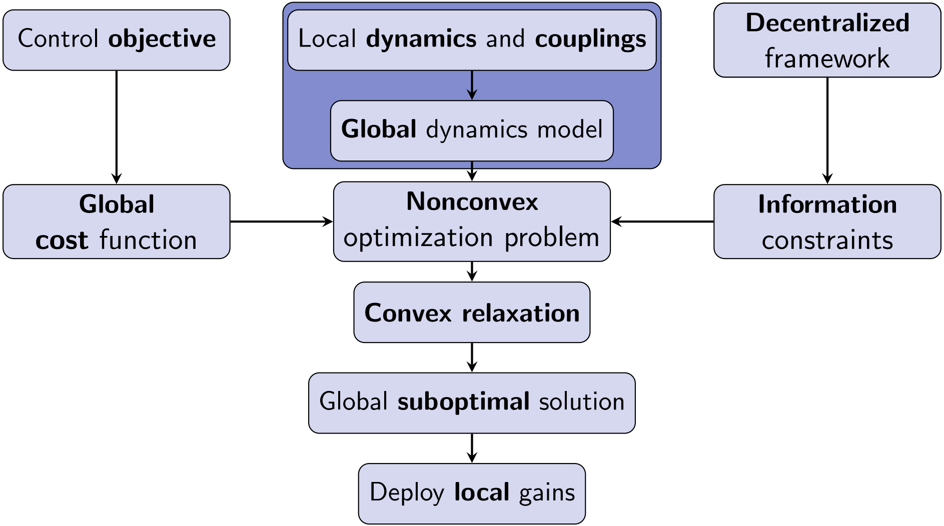
Decentralized linear quadratic control

Approach overview



Decentralized linear quadratic control

Problem formulation: Local dynamics

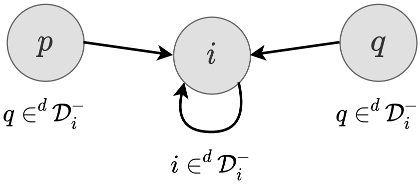


Decentralized linear quadratic control

Problem formulation: Local dynamics

Directed dynamic coupling graph ${}^{d}\mathcal{G}$:

- Each **system** is a **node**
- Each directed **edge** is a dynamical **coupling**



$$\mathbf{x}_i(k+1) = \sum_{j \in {}^{d}\mathcal{D}_i^-} \left(\mathbf{A}_{i,j}(k)\mathbf{x}_j(k) + \mathbf{B}_{i,j}(k)\mathbf{u}_j(k)\right)$$

Decentralized linear quadratic control

Problem formulation: Global dynamics

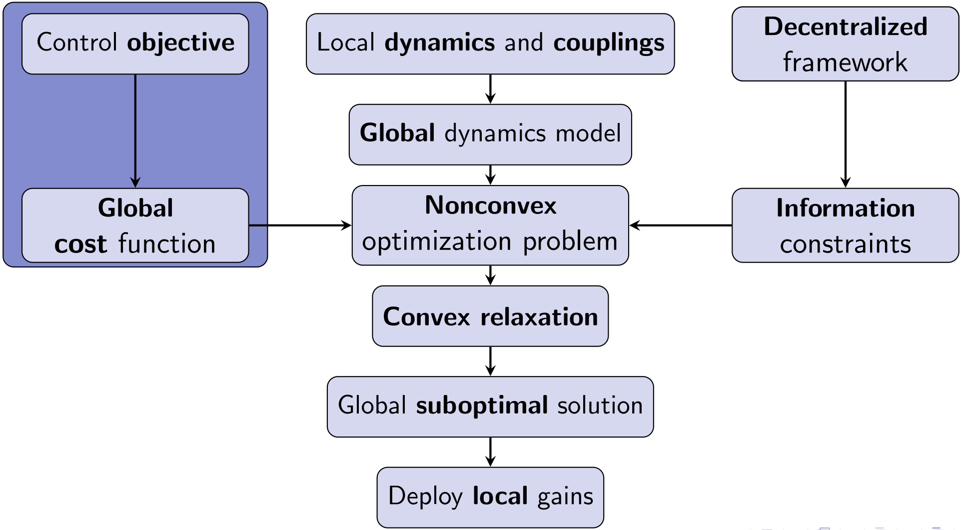
$$\mathbf{x}_i(k+1) = \sum_{j \in {}^d\mathcal{D}_i^-} \left(\mathbf{A}_{i,j}(k)\mathbf{x}_j(k) + \mathbf{B}_{i,j}(k)\mathbf{u}_j(k)\right)$$

$$\downarrow$$

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k)$$

Decentralized linear quadratic control

Problem formulation: Cost function



Decentralized linear quadratic control

Problem formulation: Cost function

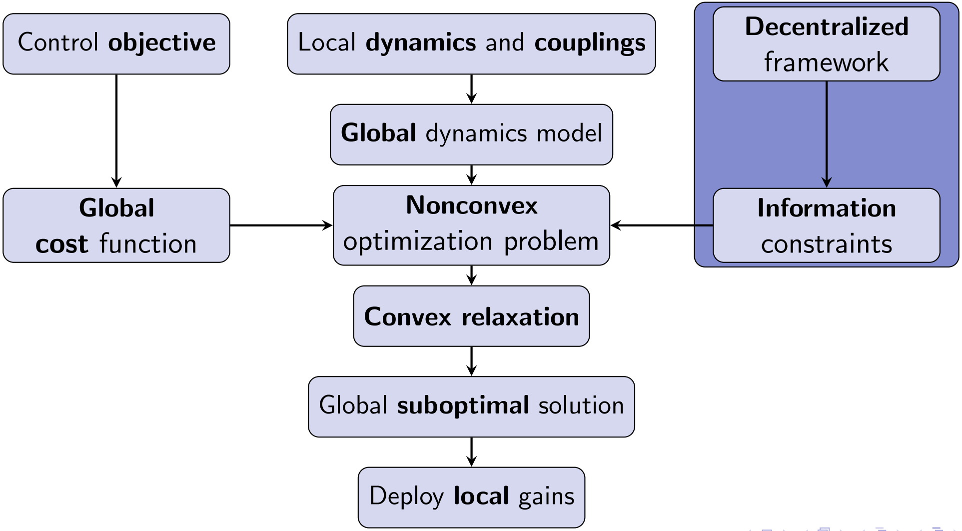
$$J(k) := \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T) + \sum_{\tau=k}^{k+T-1}\Big(\mathbf{x}^T(\tau)\mathbf{Q}(\tau)\mathbf{x}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}(\tau)\mathbf{u}(\tau)\Big)$$

Global **finite-horizon** cost:

- **MPC**-like scheme to solve **infinite-horizon** problem
- **Network-wise** or **decoupled** control objectives

Decentralized linear quadratic control

Problem formulation: Decentralized framework

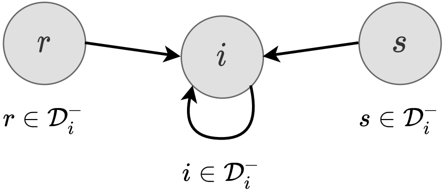


Decentralized linear quadratic control

Problem formulation: Decentralized framework

Directed communication graph $\mathcal{G}$:

- Each **system** is a **node**
- If system i has access to $\mathbf{x}_j$ is represented by edge $j \to i$



$$\mathbf{u}_i(k) = -\sum_{j \in \mathcal{D}_i^-} \mathbf{K}_{i,j}(k)\mathbf{x}_j(k)$$

Decentralized linear quadratic control

Problem formulation: Information constraints

$$\mathbf{u}_i(k) = -\sum_{j \in \mathcal{D}_i^-} \mathbf{K}_{i,j}(k)\mathbf{x}_j(k)$$

$$\downarrow$$

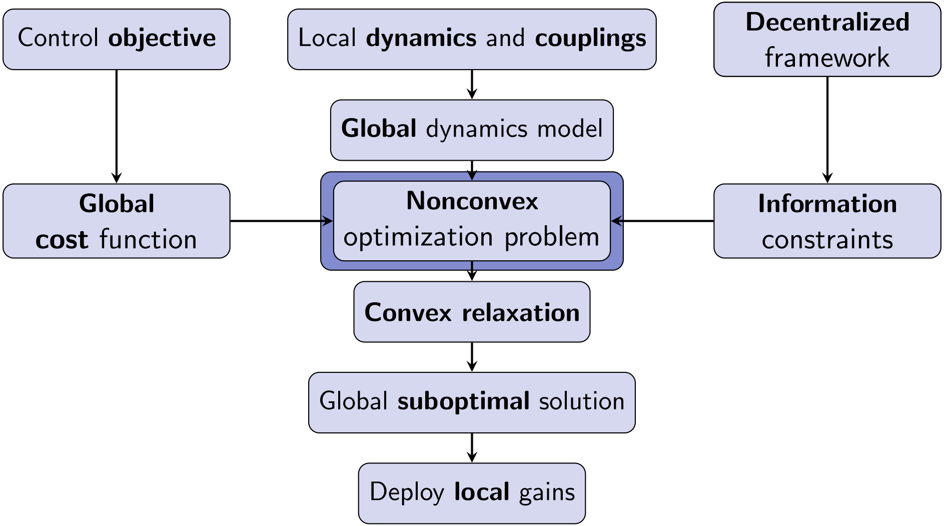
$$\mathbf{u}(k) = -\mathbf{K}(k)\mathbf{x}(k)$$

But $\mathbf{K}(k)$ is sparse: $\mathbf{K}(k) \in \mathrm{Sparse}(\mathbf{E}_{\mathcal{D}})$

$$\mathrm{Sparse}(\mathbf{E}) := \left\{\mathbf{K} \in \mathbb{R}^{m \times n} : [\mathbf{E}]_{ij} = 0 \implies [\mathbf{K}]_{ij} = 0;\ i = 1, \ldots, m,\ j = 1, \ldots, n\right\}$$

Decentralized linear quadratic control

Problem formulation: Nonconvex optimization problem



Decentralized linear quadratic control

Problem formulation: Nonconvex optimization problem

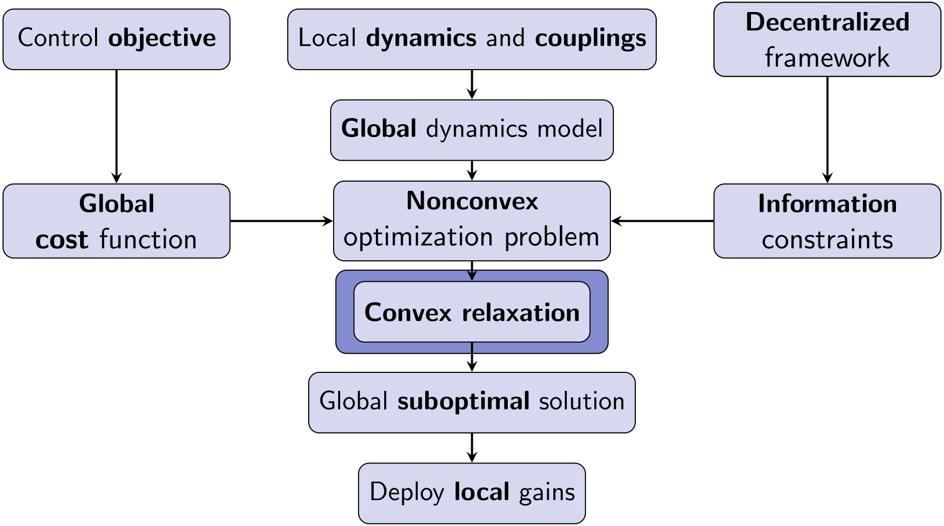
At each discrete-time instant k:

$$\begin{aligned}
\underset{\substack{\mathbf{K}(\tau)\in\mathbb{R}^{m\times n}\\ \tau=k,\ldots,k+T-1}}{\text{minimize}} \quad & J(k)\\
\text{subject to} \quad & \mathbf{x}(\tau+1)=\mathbf{A}(\tau)\mathbf{x}(\tau)+\mathbf{B}(\tau)\mathbf{u}(\tau),\ \tau=k,\ldots,k+T-1\,,\\
& \mathbf{K}(\tau)\in \mathrm{Sparse}(\mathbf{E}),\ \tau=k,\ldots,k+T-1\,,
\end{aligned}$$

Nonconvex!

Decentralized linear quadratic control

One-step convex relaxation



Decentralized linear quadratic control

One-step convex relaxation

Challenges:

- Physically **meaningful** relaxation
- Separation between **optimal** and **relaxed** solutions

Approach:

- Obtain **necessary conditions** for a **constrained minimum**
- Analyze a convenient potential **saddle point**

Decentralized linear quadratic control

One-step convex relaxation

Augment $J(k)$ to write the **Lagrangian**

$$J'(k) = \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T) + \sum_{\tau=k}^{k+T-1} \mathbf{x}^T(\tau)\left(\mathbf{Q}(\tau) + \mathbf{K}^T(\tau)\mathbf{R}(\tau)\mathbf{K}(\tau)\right)\mathbf{x}(\tau)$$

$$+ \sum_{\tau=k}^{k+T-1} \boldsymbol{\lambda}^T(\tau+1)\left[(\mathbf{A}(\tau) - \mathbf{B}(\tau)\mathbf{K}(\tau))\,\mathbf{x}(\tau) - \mathbf{x}(\tau+1)\right]$$

Define the **Hamiltonian**

$$H(k) := \mathbf{x}^T(k)\left(\mathbf{Q}(k) + \mathbf{K}^T(k)\mathbf{R}(k)\mathbf{K}(k)\right)\mathbf{x}(k) + \boldsymbol{\lambda}^T(k+1)\left(\mathbf{A}(k) - \mathbf{B}(k)\mathbf{K}(k)\right)\mathbf{x}(k)$$

Rewrite the **Lagrangian**

$$J'(k) = \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T) - \boldsymbol{\lambda}^T(k+T)\mathbf{x}(k+T) + H(k) + \sum_{\tau=k+1}^{k+T-1}\Big(H(\tau) - \boldsymbol{\lambda}^T(\tau)\mathbf{x}(\tau)\Big)$$

Stationarity:

$$\begin{cases} \dfrac{\partial J'(k)}{\partial \boldsymbol{\lambda}(\tau)} = 0, & \tau = k+1,\dots,k+T \\ \dfrac{\partial J'(k)}{\partial \mathbf{x}(\tau)} = 0, & \tau = k+1,\dots,k+T \\ \mathbf{I}_i^T \dfrac{\partial J'(k)}{\partial \mathbf{K}(\tau)} \mathbf{I}_j = 0, & [\mathbf{E}_{\mathcal{D}}]_{ij} \neq 0,\ \tau = k,\dots,k+T-1 \\ \mathbf{I}_i^T \mathbf{K}(\tau) \mathbf{I}_j = 0, & [\mathbf{E}_{\mathcal{D}}]_{ij} = 0,\ \tau = k,\dots,k+T-1 \end{cases}$$

$$[\mathbf{I}_i]_k = \begin{cases} 1, & k = i \\ 0, & k \neq i \end{cases}$$

Result: Neat identities involving the partial derivatives of the Hamiltonian

Decentralized linear quadratic control

One-step convex relaxation

Lemma

From the **stationarity** *conditions:* $\boldsymbol{\lambda}(k) = 2\mathbf{P}(k)\mathbf{x}(k)$

$$\begin{cases} \mathbf{P}(k+T) = \mathbf{Q}(k+T) \\ \mathbf{P}(\tau) = \mathbf{Q}(\tau) + \mathbf{K}^T(\tau)\mathbf{R}(\tau)\mathbf{K}(\tau) + (\mathbf{A}(\tau) - \mathbf{B}(\tau)\mathbf{K}(\tau))^T \mathbf{P}(\tau+1)(\mathbf{A}(\tau) - \mathbf{B}(\tau)\mathbf{K}(\tau)) \end{cases}$$

and

$$\mathbf{x}(i)^T\mathbf{P}(i)\mathbf{x}(i) = \sum_{\tau=i}^{k+T-1} \mathbf{x}^T(\tau)\left(\mathbf{Q}(\tau) + \mathbf{K}^T(\tau)\mathbf{R}(\tau)\mathbf{K}(\tau)\right)\mathbf{x}(\tau) + \mathbf{x}^T(k+T)\mathbf{Q}(k+T)\mathbf{x}(k+T), \quad i = k, \ldots, k+T$$

- Proof by **mathematical induction** in [Pedroso and Batista, 2021a]
- Similar to **centralized**

Decentralized linear quadratic control

One-step convex relaxation

Lemma

Necessary condition for **optimal gains***:*

$$\begin{cases} \mathbf{I}_i^T \left[\left(\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right) \mathbf{x}(\tau)\mathbf{x}^T(\tau)\right] \mathbf{I}_j = 0 & , [\mathbf{E}_{\mathcal{D}}]_{ij} \neq 0 \\ \mathbf{I}_i^T \mathbf{K}(\tau)\mathbf{I}_j = 0 & , [\mathbf{E}_{\mathcal{D}}]_{ij} \not\neq 0, \end{cases}$$

for $\tau = k, \ldots, k+T-1,$

$$\mathbf{S}(\tau) := \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{B}(\tau) + \mathbf{R}(\tau)$$

❓ **Why** is $\mathbf{x}(\tau)\mathbf{x}^T(\tau)$ (of rank 1) here?

Decentralized linear quadratic control

One-step convex relaxation

Necessary condition for **optimal gains**:

$$\begin{cases} \mathbf{I}_i^T \left[\left(\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right) \mathbf{x}^T(\tau)\mathbf{x}(\tau)\right] \mathbf{I}_j = 0 & , [\mathbf{E}_\mathcal{D}]_{ij} \neq 0 \\ \mathbf{I}_i^T \mathbf{K}(\tau)\mathbf{I}_j = 0 & , [\mathbf{E}_\mathcal{D}]_{ij} \neq 0, \end{cases}$$

- **Saddle point** satisfies these conditions
- $\mathbf{x}(k)$ is **not fully known** by any system
- Robust feedback

Relaxed one-step conditions:

$$\begin{cases} \mathbf{I}_i^T \left[\left(\mathbf{S}(\tau)\mathbf{K}(\tau) - \mathbf{B}^T(\tau)\mathbf{P}(\tau+1)\mathbf{A}(\tau)\right)\right] \mathbf{I}_j = 0 & , [\mathbf{E}_\mathcal{D}]_{ij} \neq 0 \\ \mathbf{I}_i^T \mathbf{K}(\tau)\mathbf{I}_j = 0 & , [\mathbf{E}_\mathcal{D}]_{ij} \neq 0, \end{cases}$$

Decentralized linear quadratic control

One-step convex relaxation

Theorem (One-step relaxed solution)

Let $\mathbf{l}_j$ *denote a column vector whose entries are all set to zero except for the* j*-th one, which is set to* 1*, and* $\mathcal{L}_j := \mathrm{diag}(\mathbf{l}_j)$*. Define* $\mathbf{m}_j \in \mathbb{R}^m$ *as*

$$\begin{cases} \mathbf{m}_j(i) = 0, & [\mathbf{E}]_{ij} = 0 \\ \mathbf{m}_j(i) = 1, & [\mathbf{E}]_{ij} \neq 0 \end{cases}, \; i = 1, \dots, m \,,$$

and let $\mathcal{M}_j := \mathrm{diag}(\mathbf{m}_j)$*. Then, the gains of the one-step relaxation are given by*

$$\mathbf{K}(\tau) = \sum_{j=1}^{n} \left(\mathbf{I} - \mathcal{M}_j + \mathcal{M}_j \mathbf{S}(\tau) \mathcal{M}_j\right)^{-1} \mathcal{M}_j \mathbf{B}^T(\tau) \mathbf{P}(\tau+1) \mathbf{A}(\tau) \mathcal{L}_j \,,$$

$\tau = k, \dots, k+T-1$

Decentralized linear quadratic control

One-step convex relaxation

Overview:

🎯 Satisfies **necessary** conditions of a **saddle point**

💥 Does **not depend** on the initial condition $\mathbf{x}(k)$

- is **not fully known** by any system

💡 **Closed-form** solution

⚡ **Computational complexity** of $\mathcal{O}(n^3)$ [Pedroso and Batista, 2021b]

- **same as centralized**

❓ Can we find any **physical interpretation**?

Decentralized linear quadratic control

One-step convex relaxation

❓ Can we find any **physical interpretation**?

💡 Yes!

One-step relaxation is **equivalent** to

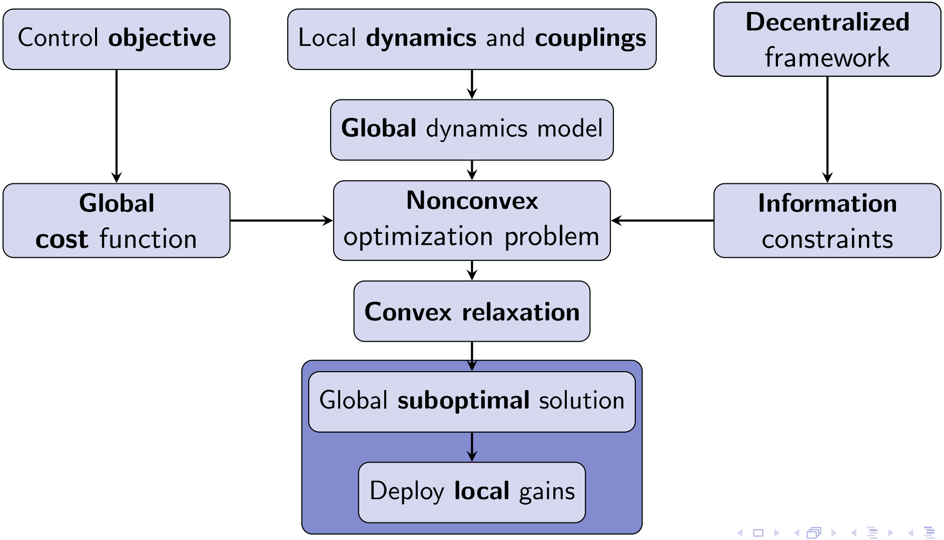
$$\begin{aligned} \underset{\mathbf{K}(\tau)\in\mathbb{R}^{m\times n}}{\text{minimize}} \quad & \operatorname{tr}(\mathbf{P}(\tau)) \\ \text{subject to} \quad & \mathbf{K}(\tau) \in \operatorname{Sparse}(\mathbf{E}) \end{aligned}$$

for $\tau = k + T - 1, \ldots, k$

- **Decoupled in time** (greedy)
- **Ignores cross-correlation** between states
- Proof in [Pedroso and Batista, 2021a]

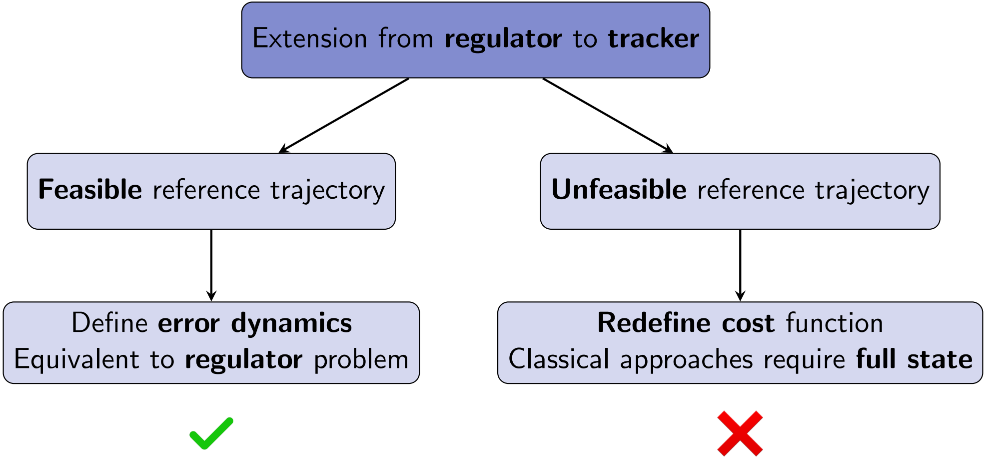
Decentralized linear quadratic control

One-step convex relaxation



Decentralized linear quadratic control

Linear quadratic tracker



Decentralized linear quadratic control

Linear quadratic tracker

Unfeasible reference trajectory $\mathbf{r}(k) \in \mathbb{R}^o$

Goal:

- Track $\mathbf{r}(k)$ with $\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k)$

Assumptions:

- $o = m$
- $\mathbf{H}(\tau)$ is **full-rank**
- **Slowly** time-varying dynamics

Decentralized linear quadratic control

Linear quadratic tracker

Define **equilibrium** $\bar{\mathbf{x}}(k)$ and $\bar{\mathbf{u}}(k)$

$$\begin{cases} \bar{\mathbf{x}}(k) = \mathbf{A}(k)\bar{\mathbf{x}}(k) + \mathbf{B}(k)\bar{\mathbf{u}}(k) \\ \mathbf{H}(k)\bar{\mathbf{x}}(k) = \mathbf{r}(k) \end{cases}$$

Define the **error** $\mathbf{e}(k) := \mathbf{x}(k) - \bar{\mathbf{x}}(k)$

$$\mathbf{e}(k+1) = \mathbf{A}(k)\mathbf{e}(k) + \mathbf{B}(k)(\mathbf{u}(k) - \bar{\mathbf{u}}(k)) - (\bar{\mathbf{x}}(k+1) - \bar{\mathbf{x}}(k))$$

Define $\mathbf{u_a}(k)$

$$\bar{\mathbf{x}}(k+1) - \bar{\mathbf{x}}(k) = \mathbf{B}(k)\mathbf{u_a}(k) + \mathbf{d}(k)$$

Decentralized linear quadratic control

Linear quadratic tracker

Error dynamics:

$$\mathbf{e}(k+1) = \mathbf{A}(k)\mathbf{e}(k) + \mathbf{B}(k)(\mathbf{u}(k) - \bar{\mathbf{u}}(k) - \mathbf{u_a}(k)) - \mathbf{d}(k)$$

Minimize the component of the error in the **tracking space**

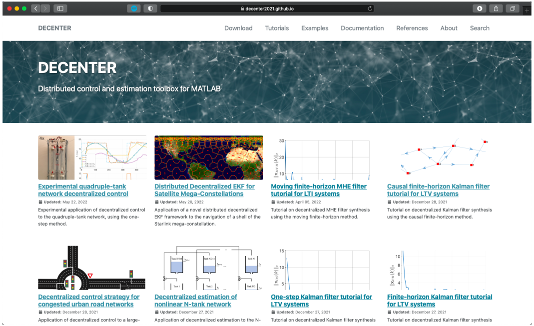
$$\begin{aligned} \underset{\substack{\bar{\mathbf{x}}(\tau),\bar{\mathbf{u}}(\tau),\tau=k,\dots,k+T \\ \mathbf{u_a}(\tau),\tau=k,\dots,k+T-1}}{\text{minimize}} \quad & \sum_{\tau=k}^{k+T-1} ||\mathbf{H}(\tau+1)\mathbf{d}(\tau)||^2 \\ \text{subject to} \quad & \begin{cases} \bar{\mathbf{x}}(\tau) = \mathbf{A}(\tau)\bar{\mathbf{x}}(\tau) + \mathbf{B}(\tau)\bar{\mathbf{u}}(\tau) \\ \mathbf{H}(\tau)\bar{\mathbf{x}}(\tau) = \mathbf{r}(\tau) \end{cases} \quad , \tau = k, \dots, k+T\,. \end{aligned}$$

⚡ Closed-form solution in [Pedroso and Batista, 2021a]

Outline

DECENTER Toolbox

- Implementations in **MATLAB**
- **Documentation**
- **Simulations** source code



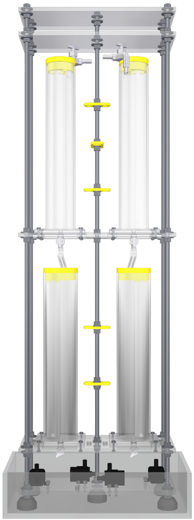
`http://decenter2021.github.io`

Outline

Example: Experimental quadruple-tank process

Experimental setup

- **MATLAB/Simulink** interface
- Shift between **numeric**/**experimental**
- **Inexpensive** and **fast** to assemble
- **Open-source** and **reproducible**
- Suitable **education**/**research**
- *github.com/decenter2021/quadruple-tank-setup*



Example: Experimental quadruple-tank process

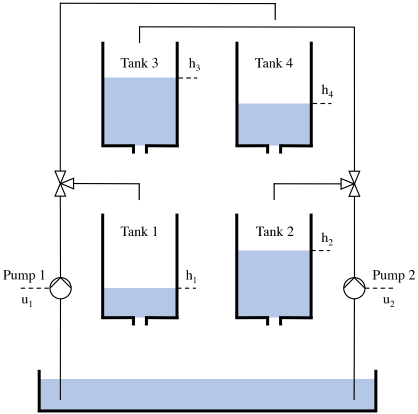
Process nonlinear model

$$\dot{h}_1(t) = -\frac{a_1}{A_1}\sqrt{2gh_1(t)} + \frac{a_3}{A_1}\sqrt{2gh_3(t)} + \frac{\gamma_1 k_1}{A_1}u_1(t)$$

$$\dot{h}_2(t) = -\frac{a_2}{A_2}\sqrt{2gh_2(t)} + \frac{a_4}{A_2}\sqrt{2gh_4(t)} + \frac{\gamma_2 k_2}{A_2}u_2(t)$$

$$\dot{h}_3(t) = -\frac{a_3}{A_3}\sqrt{2gh_3(t)} + \frac{(1-\gamma_2)k_2}{A_3}u_2(t)$$

$$\dot{h}_4(t) = -\frac{a_4}{A_4}\sqrt{2gh_4(t)} + \frac{(1-\gamma_1)k_1}{A_4}u_1(t)$$

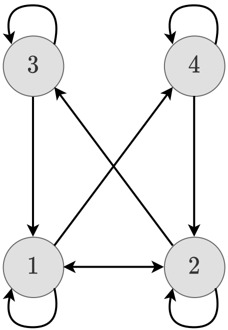


Goal: Track $\mathbf{r}(t)$ with $[h_1(t)\ h_2(t)]^T$

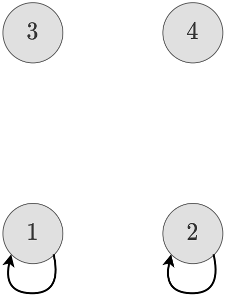
Example: Experimental quadruple-tank process

LTV model and decentralized framework

Dynamic coupling graph ${}^d\mathcal{G}$



Communication graph $\mathcal{G}$

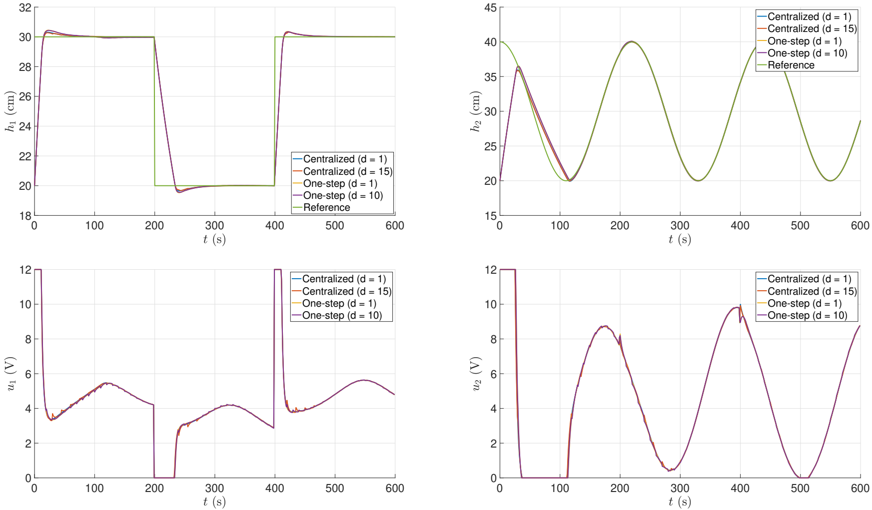


$$\mathbf{x}(k) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k)$$

$$u_i(k) = -\mathbf{K}_{i,i}(k)\mathbf{x}_i(k) + \bar{u}_i(k) + u_i^a(k)$$

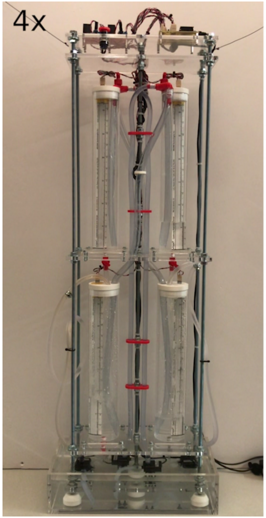
Example: Experimental quadruple-tank process

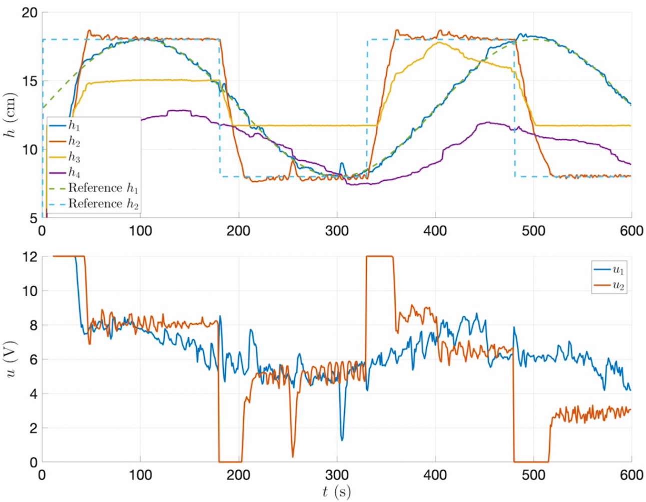
Numeric comparison with centralized



Example: Experimental quadruple-tank process

Experimental results





Outline

Example: Signal control for urban traffic networks

Two **decentralized** signal control methods

- DTUC
- D2TUC

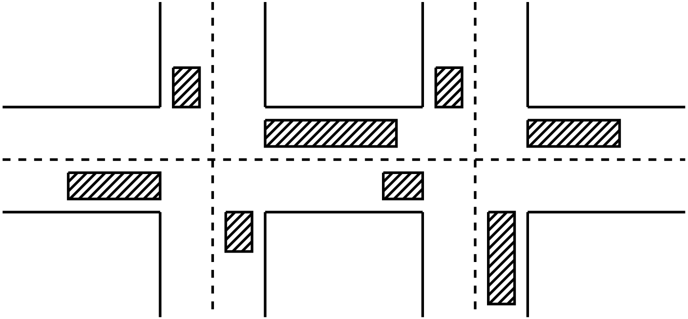
Store-and-forward macroscopic model ([Gazis and Potts, 1963, Aboudolas et al., 2009])

Match the performance of **centralized** TUC ([Diakaki, 1999])

Pedroso, L. and Batista, P. (2021a).
Decentralized store-and-forward based strategies for the signal control problem in large-scale congested urban road networks.
Transportation Research Part C: Emerging Technologies, 132:103412.

Example: Signal control for urban traffic networks

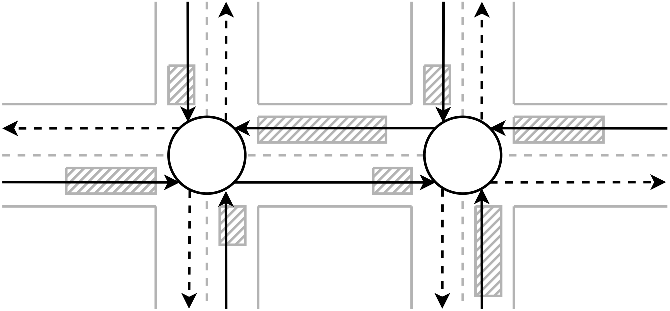
Store-and-forward model



Z links, J signalized junctions

Example: Signal control for urban traffic networks

Store-and-forward model

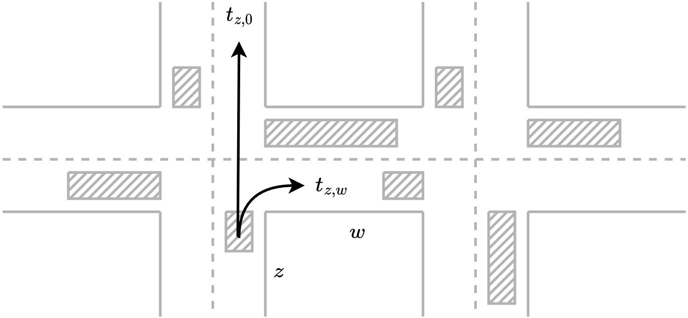


Directed graph ${}^{d}\mathcal{G} := (\mathcal{V}_{{}^{d}\mathcal{G}}, \mathcal{E}_{{}^{d}\mathcal{G}})$:

- Each **junction** is a **vertex**
- Each **link** is an **edge**

Example: Signal control for urban traffic networks

Store-and-forward model

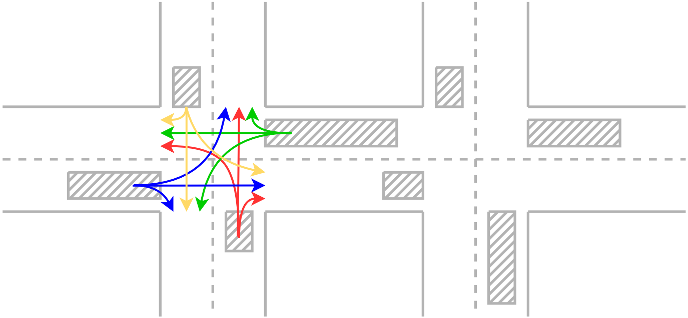


Each link z is characterized by:

- Saturation flow, S_z
- Turning rates, $\mathbf{T}$: $[\mathbf{T}]_{z,w} := t_{w,z}$
- Exit rates, $\mathbf{t_0} := [t_{1,0} \ \ldots \ t_{Z,0}]^T$

Example: Signal control for urban traffic networks

Store-and-forward model



Signal control strategy:

- **Cycle** of duration C
- For each junction j there is a set of **stages** $s \in \mathcal{F}_j$
- For each **stages** there is a set of links that have **right of way**

Example: Signal control for urban traffic networks

Store-and-forward model

Green time g_s of stage s:

- **Minimum** constraint

$$g_s \geq g_{s,\mathrm{min}}\ , s \in \{1, \ldots, S\}$$

- **Cycle** duration constraint

$$\sum_{s \in \mathcal{F}_j} g_s + L_j = C\ , \quad j \in \{1, \ldots, J\}$$

where L_j is the **inter-green** time.

Example: Signal control for urban traffic networks

Store-and-forward model

Store-and-forward Traffic flow approximation

Models **green-red switchings** within a whole cycle as a **continuous flow** of vehicles

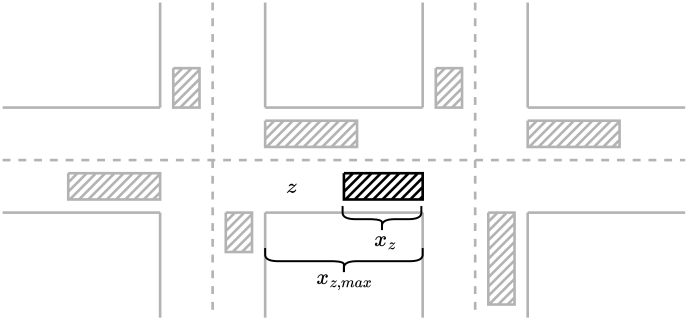
$$u_z(k) = S_z G_z(k)/C \ , \quad z \in \{1, \dots, Z\}$$

$G_z(k)$ is the **total green time** of link z

$$G_z(k) = \sum\nolimits_{s:[\mathbf{S}]_{zs} \neq 0} g_s(k)$$

Example: Signal control for urban traffic networks

Store-and-forward model



$x_z(k)$: number of **vehicles** in link z

$$0 \leq x_z(k) \leq x_{z,\max}$$

Example: Signal control for urban traffic networks

Store-and-forward model

Store-and-forward system (stage green-times $\mathbf{g}(k) \in \mathbb{R}^S$)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_\mathbf{g}\mathbf{g}(k) + C\mathbf{d}(k)$$

- Can be **freely** selected

Store-and-forward system (link green-times $\mathbf{G}(k) \in \mathbb{R}^Z$)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_\mathbf{G}\mathbf{G}(k) + C\mathbf{d}(k)$$

- Distributed among the stages in **post-processing**

Example: Signal control for urban traffic networks

Cost function

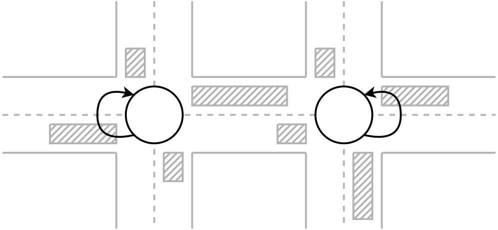
$$J(k) := \mathbf{x}^T(T)\mathbf{Q}\mathbf{x}(T) + \sum_{\tau=k}^{k+T-1} \left(\mathbf{x}^T(\tau)\mathbf{Q}\mathbf{x}(\tau) + (\mathbf{g}(\tau) - \bar{\mathbf{g}}(\tau))^T \mathbf{R} (\mathbf{g}(\tau) - \bar{\mathbf{g}}(\tau)) \right)$$

Penalize **relative occupancy**

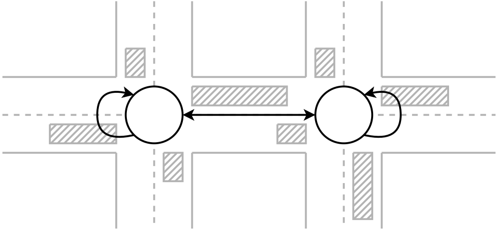
$$\mathbf{Q} = \mathrm{diag}\left(\frac{1}{x_{1,\max}}, \ldots, \frac{1}{x_{Z,\max}} \right)$$

Example: Signal control for urban traffic networks

Decentralized framework



Configuration Ψ



Configuration Φ

Example: Signal control for urban traffic networks
DTUC

Input: stage green-times $\mathbf{g}(k) \in \mathbb{R}^S$

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B_g}\mathbf{g}(k) + C\mathbf{d}(k) \tag{1}$$

Proposition (Controllability)

Consider a feasible traffic network characterized by $(\mathcal{G}, \mathbf{T}, \mathbf{t_0})$ and a minimum complete stage strategy characterized by a stage matrix $\mathbf{S}$. Let $\mathcal{C}$ be the controllability matrix of the store-and-forward LTI system (2). Then, $\mathrm{rank}(\mathcal{C}) = S \leq Z$.

It is not controllable in general!

Example: Signal control for urban traffic networks

DTUC

- **Canonical Structure Theorem**: $\mathbf{z}(k) = \mathbf{W}^{-1}\mathbf{x}(k)$
- **Controllable** component: $\mathbf{z_1}(k+1) = \mathbf{z_1}(k) + \hat{\mathbf{B}}_{\mathbf{g1}}\mathbf{g}(k) + C\hat{\mathbf{d}}_{\mathbf{1}}(k)$
- **Uncontrollable** component: $\mathbf{z_2}(k+1) = \mathbf{z_2}(k) + C\hat{\mathbf{d}}_{\mathbf{2}}(k)$.

? The uncontrollable component **grows unbounded**?

- No!
- Queue length constraint: $0 \leq x_z(k) \leq x_{z,\max}$
- Upstream gating

Example: Signal control for urban traffic networks

DTUC

🎯 Regulate the **controllable** component

$$J(k) := \mathbf{z_1}^T(T)\mathbf{Q_1}\mathbf{z_1}(T) + \sum_{\tau=k}^{k+T-1}\left(\mathbf{z_1}^T(\tau)\mathbf{Q_1}\mathbf{z_1}(\tau) + (\mathbf{g}(\tau) - \bar{\mathbf{g}}(\tau))^T\mathbf{R}(\mathbf{g}(\tau) - \bar{\mathbf{g}}(\tau))\right)$$

🔎 Non-ideal $\mathbf{Q_1}$

$$\mathbf{Q_1} = \begin{bmatrix}\mathbf{I}_S & \mathbf{0}_{S\times(Z-S)}\end{bmatrix}\mathbf{W}^T\mathbf{Q}\mathbf{W}\begin{bmatrix}\mathbf{I}_S \\ \mathbf{0}_{(Z-S)\times S}\end{bmatrix}$$

🔗 Sparsity constraint

$$\mathbf{K_1}(\tau)\begin{bmatrix}\mathbf{I}_S & \mathbf{0}_{S\times(Z-S)}\end{bmatrix}\mathbf{W}^{-1} \in \mathrm{Sparse}(\mathbf{E})$$

Example: Signal control for urban traffic networks

D2TUC

Input: link green-times $\mathbf{G}(k) \in \mathbb{R}^Z$

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B_G}\mathbf{G}(k) + C\mathbf{d}(k) \tag{2}$$

Proposition (Controllability)

Consider a feasible traffic network characterized by $(\mathcal{G}, \mathbf{T}, \mathbf{t_0})$. Then, the store-and-forward LTI system (48) is controllable.

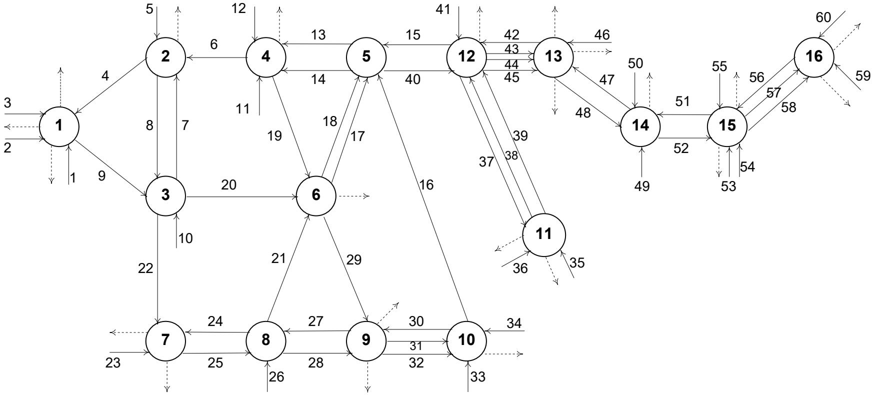
- Apply **one-step method** directly
- Quadratic continuous **knapsack problem** in **each junction**

Example: Signal control for urban traffic networks

Chania urban road network

Chania urban traffic network

- $J = 16$ signalized junctions
- $L = 60$ links



SAFFRON:

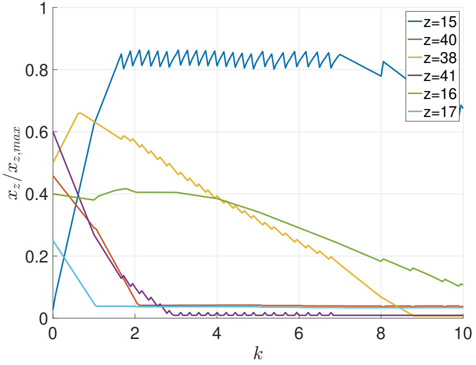
- open-source **tools** for **store-and-forward** models
- traffic network **model** of **Chania**, Greece
- implementation **source-code** of signal control strategies
- nice **international collaboration**
- *github.com/decenter2021/SAFFRON*

Pedroso, L., Batista, P., Papageorgiou, M., and Kosmatopoulos, E. (2022).
Saffron: Store-and-forward model toolbox for urban road network signal control in matlab.
(accepted for presentation at the 25th IEEE Intelligent Transportation Systems Conference (ITSC 2022)).

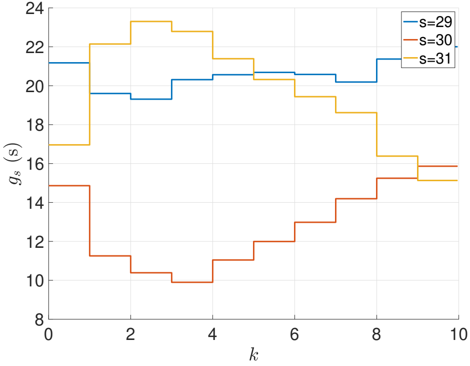
Example: Signal control for urban traffic networks

Chania urban road network

D2TUC configuration Φ



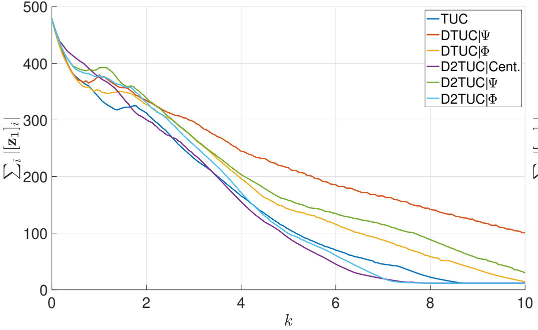
(a) Occupancy



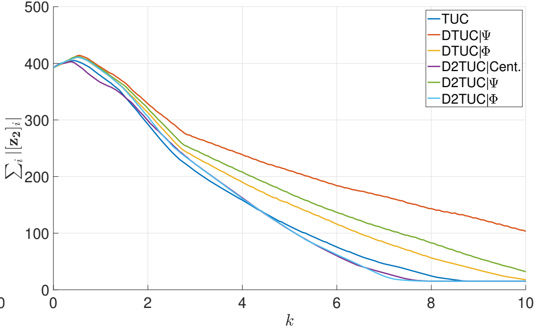
(b) Green-times

Example: Signal control for urban traffic networks

Chania urban road network



(a) Controllable



(b) Uncontrollable

D2TUC matches the perfomance of **TUC**!

Outline

Future work

We achieved:

- Well-performing **convex relaxation**
- **Decentralized** gain synthesis (**local feedback**)
- **Local communication**

But **LTV** gains require **real-time synthesis**:

- **Distributed real-time** synthesis
  - We can **leverage** these results
  - My **MSc thesis**

Thank you!

Outline

References I

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