Workshop on Sensing and Control for Autonomous Vehicles
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New Design Techniques for Globally Convergent SLAM

Analysis and Implementation

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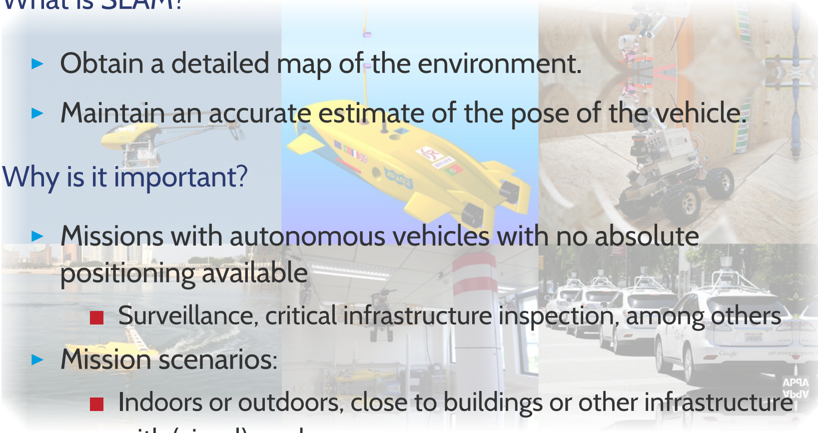
Presentation Outline

INTRODUCTION

- Motivation
- SLAM Formulations
- Sensor suite
- Main challenges

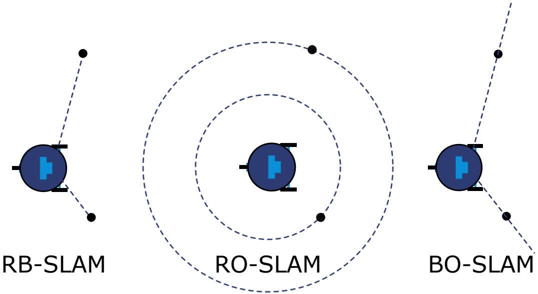
What is SLAM?



- Obtain a detailed map of the environment.
- Maintain an accurate estimate of the pose of the vehicle.

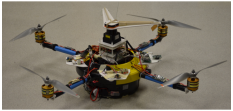
Why is it important?

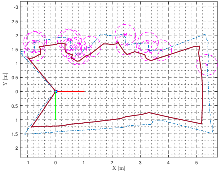
- Missions with autonomous vehicles with no absolute positioning available
  - Surveillance, critical infrastructure inspection, among others
- Mission scenarios:
  - Indoors or outdoors, close to buildings or other infrastructure with (visual) marks

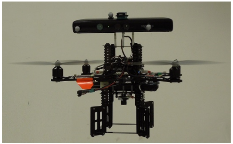


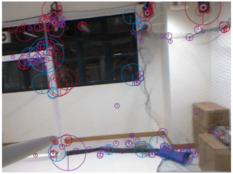
- Measurements with lower dimension than the mapped space:
  - Range-only SLAM
  - Bearing-only SLAM
- Measurements with fully observed space:
  - Range-and-bearing SLAM

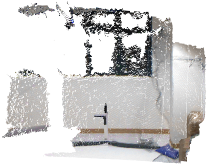
INTRODUCTION > Sensor suite

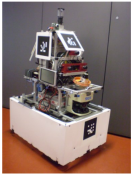


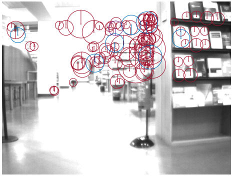












The challenges

- **Undelayed** initialization

The challenges

- **Undelayed** initialization
- Filter with **convergence** guarantees
- Filter with **consistent** estimates

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▷ **How?** ◁

The challenges

- **Undelayed** initialization
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- Filter with **consistent** estimates

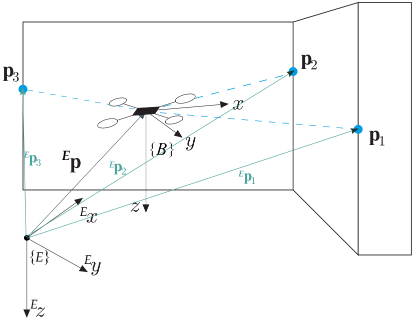
▷ **How?** ◁

Proposed solutions

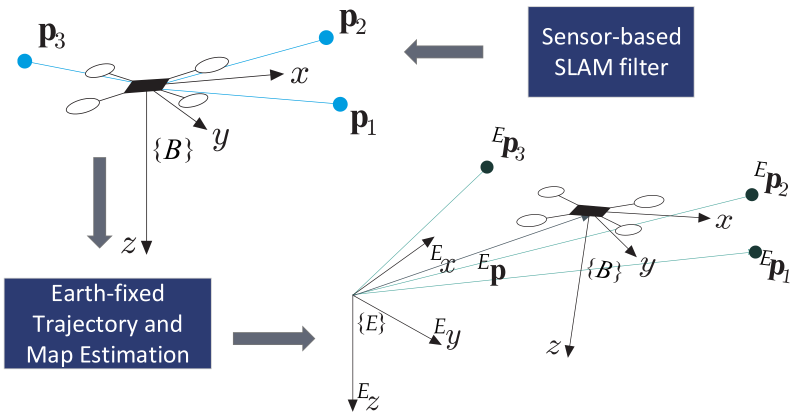
- **Relative** vs absolute filter
- State augmentation & output transformation
- **Avoid** linearizations

Sensor-based SLAM

- Overview
- System design
- Challenges
- Summary



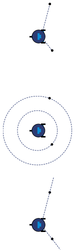
- Make landmark association, loop closing, control, and decision procedures in the sensor-based frame
- Minimize the effects of nonlinearities in the consistency


p3
p2
x
p1
{B}
y
z
Sensor-based
SLAM filter
Earth-fixed
Trajectory and
Map Estimation
Ep3
Ep2
Ep1
Ep
Ex
Ey
Ez
{E}

▷ System dynamics

$$\begin{cases} \dot{\mathbf{p}}_i(t) = -\mathbf{S}\left[\omega(t)\right]\mathbf{p}_i(t) - \mathbf{v}(t) & i \in \mathscr{L} \\ \mathbf{y}_j(t) = \mathbf{f}(\mathbf{p}_j(t)) & j \in \mathscr{L}_O \end{cases}.$$

▷ Available measurements



$$\mathbf{f}(\mathbf{p}_j(t)) = \mathbf{p}_j(t)$$

▷ Range-and-bearing

$$\mathbf{f}(\mathbf{p}_j(t)) = \|\mathbf{p}_j(t)\|$$

▷ Range-only

$$\mathbf{f}(\mathbf{p}_j(t)) = \frac{\mathbf{p}_j(t)}{\|\mathbf{p}_j(t)\|}$$

▷ Bearing-only

- Nonlinear dynamics
- Nonlinear output

- Rewrite dynamics and use measured quantities
- Augment state and/or transform the output depending on specific nonlinearities

	RB-SLAM	RO-SLAM	BO-SLAM
States	▷ map ▷ linear velocity ▷ bias estimation	▷ map ▷ ranges	▷ map ▷ ranges
Inputs/ Outputs	▷ biased angular velocity ▷ landmarks	▷ angular velocity ▷ linear velocity ▷ ranges	▷ angular velocity ▷ linear velocity ▷ bearings
Details	▷ $\mathbf{S}[\omega]\mathbf{p}_i$ ▷ $\omega = \omega_m - \mathbf{b}_\omega$ ▷ $\mathbf{S}[\mathbf{y}_i]\mathbf{b}_\omega$	▷ new state	▷ $\mathbf{b}_i = \frac{\mathbf{p}_i}{\|\mathbf{p}_i\|}$ ▷ $\mathbf{b}_i \|\mathbf{p}_i\| - \mathbf{p}_i = \mathbf{0}$ ▷ new state

▷ States ▷ Measurements

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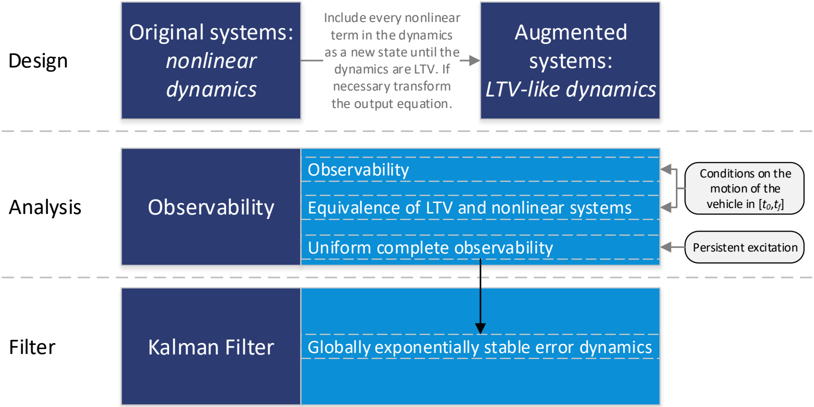
▷ States ▷ Measurements

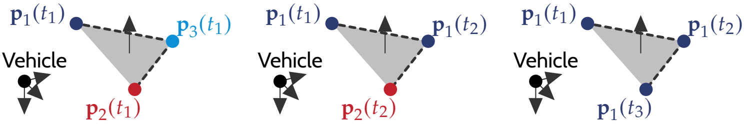
Observability

- Overview
- Results

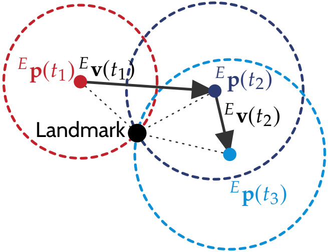

Design
Original systems: *nonlinear dynamics*
Include every nonlinear term in the dynamics as a new state until the dynamics are LTV. If necessary transform the output equation.
Augmented systems: *LTV-like dynamics*
Analysis
Observability
Observability
Equivalence of LTV and nonlinear systems
Uniform complete observability
Conditions on the motion of the vehicle in $[t_0,t_f]$
Persistent excitation
Filter
Kalman Filter
Globally exponentially stable error dynamics

Observability and Equivalence	Two landmarks are visible or one landmark is visible and there is an instant when its derivative is nonzero.
UCO	Two landmarks are visible or one landmark is visible and its derivative is sufficiently away from zero, uniformly in time.

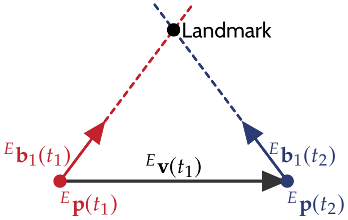
Observability and Equivalence	Three landmarks form a plane in one observation, two observations of two landmarks form a plane or three observations of one landmark form a plane.
UCO	The vectors defined by the three landmarks that form a plane (regardless of the observation moment) are sufficiently away from collinearity, uniformly in time.



Observability and Equivalence	The linear velocity in three observation moments spans $\mathbb{R}^3$.
UCO	The vectors defined by the three velocity measurements are sufficiently away from co-planarity so that the spanned space does not degenerate in time.

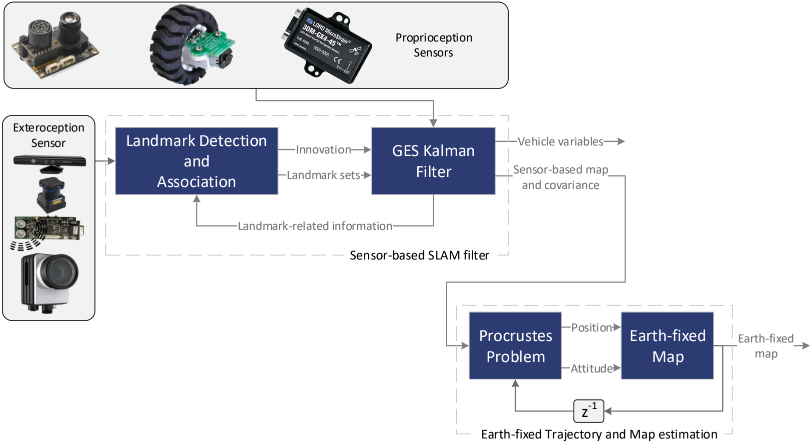


Observability and Equivalence	Two different absolute bearings to one landmark are measured.
UCO	The variation in the bearing measurement is sufficiently away from zero to not degenerate in time.



Earth-fixed Trajectory and Map

- Overview

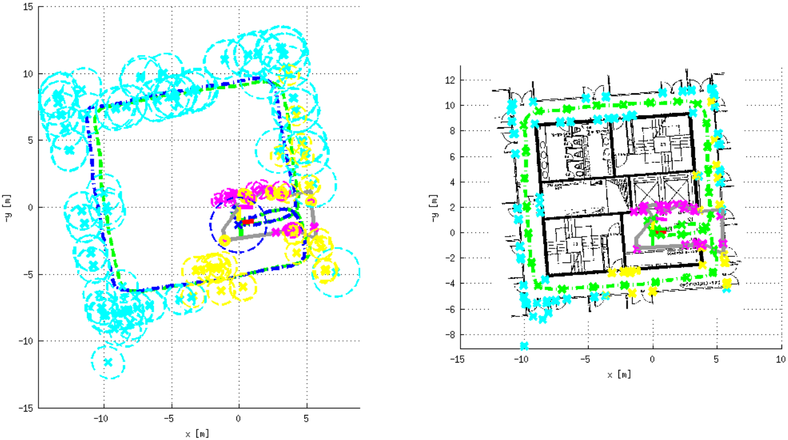

Proprioception Sensors
Exteroception Sensor
Landmark Detection and Association
Innovation
Landmark sets
GES Kalman Filter
Vehicle variables
Sensor-based map and covariance
Landmark-related information
Sensor-based SLAM filter
Procrustes Problem
Position
Attitude
Earth-fixed Map
Earth-fixed map
z-1
Earth-fixed Trajectory and Map estimation

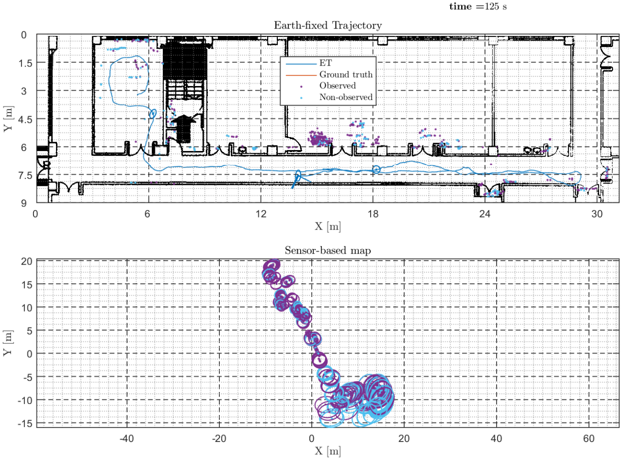
PRACTICAL EXAMPLES

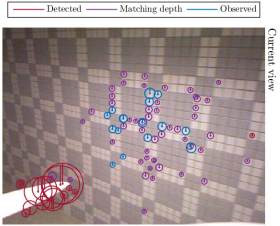
- Overview
- Range-and-bearing SLAM
- Range-only SLAM
- Bearing-only SLAM

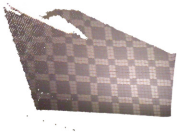
Table: Measurements and their respective sensors

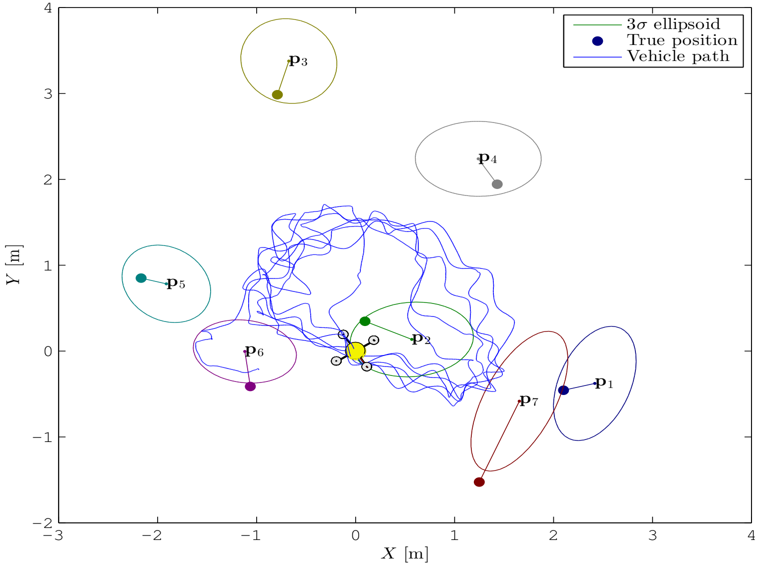
Quantities	Sensors
Landmark position	LiDAR (RB-2D) / RGB-D camera (RB-3D) / Stereo or trinocular camera
Landmark range	Radio/acoustic transceivers (RO)
Landmark bearing	Radio/acoustic transceivers / Single camera (BO)
Linear velocity	Odometry (BO) / Optical flow (RO)
Angular velocity	IMU (RB-2/3D,RO,BO)


15
10
5
0
-5
-10
-15
-y [m]
-10
-5
0
5
x [m]
12
10
8
6
4
2
0
-2
-4
-6
-8
-y [m]
-15
-10
-5
0
5
10
x [m]

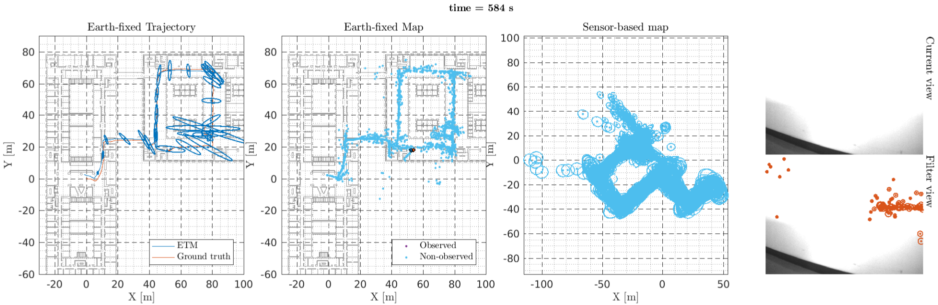

time =125 s
Earth-fixed Trajectory
ET
Ground truth
Observed
Non-observed
Y [m]
X [m]
0
1.5
3
4.5
6
7.5
9
0
6
12
18
24
30
Sensor-based map
Y [m]
X [m]
20
15
10
5
0
-5
-10
-15
-40
-20
0
20
40
60


Detected
Matching depth
Observed
Current view



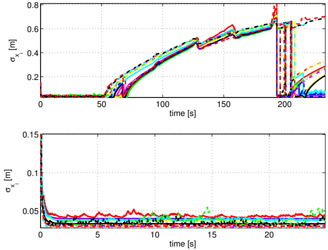


3σ ellipsoid
True position
Vehicle path
p1
p2
p3
p4
p5
p6
p7
Y [m]
X [m]
4
3
2
1
0
−1
−2
−3

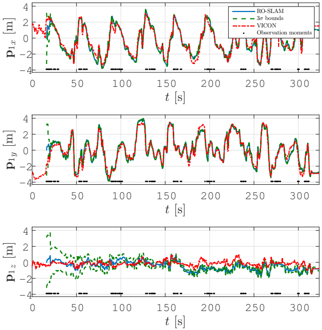


time = 584 s
Earth-fixed Trajectory
Earth-fixed Map
Sensor-based map
ETM
Ground truth
Observed
Non-observed
X [m]
Y [m]
Current view
Filter view

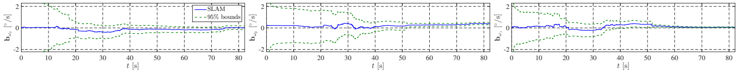
Landmark uncertainty (RB-2D)



Landmark position (RO)



Angular rate bias (RB-3D)



- Tools to tackle the nonlinearities of the main SLAM formulations were presented.
- A class of sensor-based simultaneous localization and mapping filters with global convergence guarantees was introduced.
- Constructive and physically intuitive necessary and sufficient conditions for observability, and thus convergence were provided.
- Experimental examples of practical implementations were illustrated.