Turbulence Decay Corrections for Transitional Flow Calculations

Rui Lopes[1], **Luís Eça**[1], **and Guilherme Vaz**[2,3]
[1] Instituto Superior Técnico - University of Lisbon,
[2] Maritime Research Institute Netherlands
[3] WavEC - Offshore Renewables
rui.a.lopes@tecnico.ulisboa.pt

1 Introduction

Transitional flow calculations are becoming increasingly common due to the emergence of applications operating at low Reynolds numbers and the appearance of mathematical models focused on modeling transition. From a physical perspective, transition is a complex phenomenon, non-linear and unsteady, in which flow disturbances from the freestream cause the laminar flow in the boundary layer to become unstable and transition to turbulent flow. As a result, the exact location for the transition region is dependent on the characteristics of the freestream flow and its disturbances. From the numerical standpoint, this sensitivity to the freestream conditions is obtained from inlet boundary conditions for turbulence. In calculations using the Reynolds-Averaged Navier-Stokes (RANS) equations, when transition models are not employed, transition is handled by the underlying turbulence model. This leads to transition occurring at too low Reynolds numbers, which originates turbulent flow close to the leading edge of a body, regardless of the specified turbulence quantities at the inlet.

However, when transition modelling is desired, the inlet turbulence quantities have a strong influence on the transition location as shown by Eça et al. (2016). Despite being physically expected, this influence causes difficulties, since the specification of these values becomes a challenge as little information about turbulence is known in order to determine both variables (in the case of two-equation models). Additionally, common two-equation eddy-viscosity models such as the $k-\epsilon$ and $k-\omega$ turbulence models predict a very strong decay of the turbulence variables in the freestream, which is related to the value of the eddy-viscosity at the inlet according to Spalart and Rumsey (2007). Thus it is common to observe that calculations with transition modelling are accompanied by very high values of $\frac{\nu_t}{\nu}$ in order to maintain a 'reasonable' decay of the turbulence intensity.

This means that for practical applications, one must not only know the correct value for the turbulence intensity, but also has to estimate the eddy-viscosity value that will result in the correct evolution of the turbulence intensity along the flow. This situation severely hinders the predictive capability of transition models (Li et al. (2019)), and often results in awkward values for the eddy-viscosity at the inlet, which may become physically questionable.

In this paper, we explore an alternative technique to control the decay of turbulence kinetic energy in the freestream that modifies dissipation in the k and ω transport equations. This modification is calibrated for the flow around a flat-plate and then subsequently tested on the flow around the NACA 0012 airfoil. The mathematical formulation is described in section 2. The test cases and numerical settings are described in sections 3 and 4 while the results are presented and discussed in section 5. Section 6 summarizes the conclusions of this work.

2 Mathematical Models

This study is concerned with the statistically steady flow of a single-phase incompressible Newtonian fluid, solved by the RANS equations. These are obtained by applying time-averaging to the flow properties and to the continuity and momentum equations. A turbulence closure is necessary in order to determine the Reynolds-stress tensor. In this work, we use the two-equation $k-\omega$ Shear-Stress Transport (SST) turbulence model which resorts to the concept of eddy-viscosity in order to close the system. The γ-$\tilde{Re}_{\theta_t}$ model by Langtry and Menter (2009) is used to account for transition modelling.

$k-\omega$ SST model

The two-equation $k-\omega$ SST turbulence model by Menter et al. (2003) solves transport equations for the turbulence kinetic energy k and for the specific turbulence dissipation ω. The model includes a blending function F_1 that is designed to switch from $k-\omega$ behaviour near the wall to the $k-\epsilon$ equations in the freestream. Model constants and blending functions are given in Menter et al. (2003).

Considering steady, uniform flow aligned with the x axis and neglecting the cross-diffusion term $CD_{k\omega}$, the transport equations of the SST model can be written as:

$$U\frac{dk}{dx} = -\beta^*\omega k \tag{1}$$

$$U\frac{d\omega}{dx} = -\beta\omega^2 \tag{2}$$

where β and β^* are constants. Under these conditions the eddy-viscosity can be obtained from:

$$\nu_t = \frac{k}{\omega} \tag{3}$$

Using dimensionless variables $k^* = k/V_\infty^2$, $\omega^* = \omega * L_{ref}/V_\infty$, $U^* = U/V_\infty$, $x^* = x/L_{ref}$ with $U^* = 1$, the solution to these equations can be written as:

$$k^* = \frac{k^*_{in}}{\left(1+\beta(x^*-x^*_{in})k^*_{in}\left(\frac{\nu}{\nu_t}\right)_{in}Re\right)^{\frac{\beta^*}{\beta}}} \tag{4}$$

$$\omega^* = \frac{\omega^*_{in}}{1+\beta(x^*-x^*_{in})k^*_{in}\left(\frac{\nu}{\nu_t}\right)_{in}Re}. \tag{5}$$

From these equations it can be observed that the decay of ω and k in the freestream along the domain depends on β, β^*, $\left(\frac{\nu}{\nu_t}\right)_{in}$, k^*_{in} and the Reynolds number. These equations explain why high values for the eddy-viscosity ratio are commonly observed in calculations that make use of transition models. The approach presented here follows a different route: instead of controlling the decay through $\left(\frac{\nu}{\nu_t}\right)_{in}$, the constants β and β^* are redefined:

$$\begin{array}{rcl}\beta & = & (1-F_{FS})\beta_o + F_{FS}\beta_{FS} \\ \beta^* & = & (1-F_{FS})\beta^*_o + F_{FS}\beta^*_{FS}\ ,\end{array} \tag{6}$$

where β_0 and β^*_0 are the original constants of the SST model and β_{FS}, β^*_{FS} are the new values which will be active in the freestream and will be responsible for controlling the decay:

$$\begin{array}{rcl}\beta_{FS} & = & \lambda\beta_o \\ \beta^*_{FS} & = & \lambda\beta^*_o\ ,\end{array} \tag{7}$$

λ is defined in the range $0 < \lambda < 1$. Of course, this modification changes the original calibration of the SST model, and therefore it should only be active in the freestream. Hence, we use the function F_{FS}, which is meant to identify whether a given point in the domain lies in the freestream, in a similar approach to that of Lopes et al. (2017), although the transition model was not used in that study. The F_{FS} function is built using the eddy-viscosity ratio and the vorticity magnitude:

$$F_{FS} = \min\left(F_{FS1}, F_{FS2}\right) \tag{8}$$

$$F_{FS1} = \begin{cases} 1 & , \ \frac{\Omega L}{V_\infty} \leq 20 \\ 1-3\xi^2+2\xi^3 & , \ 20 < \frac{\Omega L}{V_\infty} < 50 \\ 0 & , \ \frac{\Omega L}{V_\infty} \geq 50 \end{cases} \quad \text{and} \quad F_{FS2} = \begin{cases} 1 & , \ \frac{\nu_t}{\nu} \leq \left(\frac{\nu_t}{\nu}\right)_{in} \\ 1-3\Psi^2+2\Psi^3 & , \ \left(\frac{\nu_t}{\nu}\right)_{in} < \frac{\nu_t}{\nu} < 1.5\left(\frac{\nu_t}{\nu}\right)_{in} \\ 0 & , \ \frac{\nu_t}{\nu} \geq 1.5\left(\frac{\nu_t}{\nu}\right)_{in} \end{cases}. \tag{9}$$

where $\xi = \frac{\frac{\Omega L}{V_\infty}-20}{30}$ and $\Psi = \frac{\frac{\nu_t}{\nu}-\left(\frac{\nu_t}{\nu}\right)_{in}}{0.5\left(\frac{\nu_t}{\nu}\right)_{in}}$. The F_{FS1} auxiliary function based on the vorticity magnitude is built to identify the laminar boundary layer and the near-wall linear sublayer, while the F_{FS2} function identifies the edge of the turbulent boundary layer. Combined together, these two functions identify the regions of the domain where viscous effects are relevant, i.e., where the turbulence model must be working with its original constants to guarantee that the behaviour of the original model in the laminar and turbulent regions is preserved.

Under the proposed formulation, the decay of turbulence in the freestream is now controlled by the product $\frac{\nu}{\nu_t}\lambda$. Hence, low values of λ can be used to obtain a slow decay instead of decreasing $\frac{\nu}{\nu_t}$ at the inlet.

3 Test Cases

In this work the previous corrections are applied to the two-dimensional steady flows over a flat plate and around the NACA 0012 airfoil. The setup for these cases has been used before by Eça et al. (2016) so it is only briefly mentioned below. The Reynolds number for the flow over the flat plate is $Re_L = 10^7$ (based on the plate length L). The plate extends from $x = 0$ to $x = L$, and the inlet boundary is placed at $x = -0.25L$, while the outlet is located at $x = 1.25L$. The top boundary is $0.25L$ away from the plate. For the flow around the NACA 0012 airfoil the Reynolds number (based on the chord of the airfoil c) is $Re_c = 2.88 \times 10^6$. The top and bottom boundaries are approximately $12c$ away from the airfoil while the inlet and outlet boundaries are placed $12c$ upstream of the leading edge and $23c$ downstream of the trailing edge, respectively.

For both cases the velocity and turbulence variables are specified at the inlet while pressure is extrapolated from the interior. Pressure is fixed at the top boundary for the flat plate flow, and at the outlet for the flow around the airfoil. Zero streamwise derivatives (Neumann conditions) are used for the remaining variables at these boundaries and for the bottom boundary of the airfoil domain as well. Symmetry conditions are applied on the regions upstream and downstream of the plate on the bottom boundary. At the surface of the plate and airfoil, the no-slip condition is enforced and the pressure normal derivative is set to zero. At the wall k is equal to zero, whereas Neumann conditions apply to γ and $\tilde{Re}_{\theta_t}$. ω is specified at the near-wall cell centre according to the near-wall solution (Wilcox (1998)).

4 Numerical Settings

The finite-volume flow solver ReFRESCO (www.refresco.org) is used for all calculations. It uses cell-centered collocated variables and, to ensure mass conservation, a pressure-correction equation based on the SIMPLE algorithm. Second-order schemes are applied to the convective and diffusive terms of all transport equations. Iterative convergence criteria is set so that the L_∞ norm of the normalized residuals of all transport equations must be below 10^{-6} for the flat plate flow and below 10^{-8} for the flow around the NACA 0012 airfoil. The normalized residuals are equivalent to dimensionless variable changes in a simple Jacobi iteration.

The grid used for the flat plate flow has 1024 cells on the surface of the plate and a total of $294,962$ cells while the grid for the flow around the airfoil has 1024 cells on the surface of the airfoil and a total of $391,168$ cells. The maximum dimensionless near-wall cell size, y^+_{max}, is always below 0.5. The grids are illustrated in Figure 1.

5 Results

Flat plate flow

The flat plate test case served as a calibration for the F_{FS} function. At this stage, the goal is to ensure that the sensitivity of the location of the transition region is changed from $\left(\frac{\nu_t}{\nu}\right)_{in}$ to λ. To that end, baseline calculations are performed using a given value for the viscosity ratio at the inlet and setting $\lambda = 1$, which results in the unmodified SST model. Then, a series of calculations using different combinations of $\left(\frac{\nu_t}{\nu}\right)_{in}$ and λ that lead to the same decay (same $\left(\frac{\nu}{\nu_t}\right)_{in}\lambda$) for the freestream turbulence as the baseline calculations are performed. These combinations are shown in Tab. 1: pairs with matching decay type have the same freestream turbulence decay, and should, therefore, lead to a similar location of the transition region.

Figure 2 presents the results obtained for an inlet turbulence intensity of 3%. As expected, combinations of $\frac{\nu_t}{\nu}$ and λ that share the decay type exhibit transition at the same location and matching turbulence

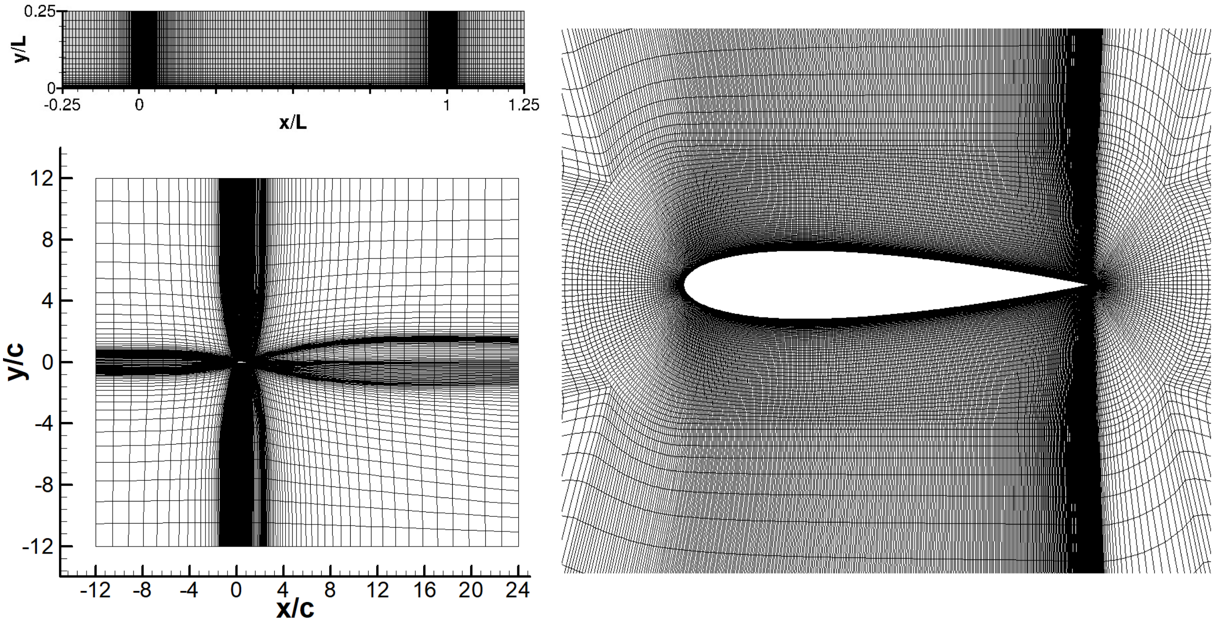

Fig. 1: Illustration of the domain and grids of the two test cases.

Table 1: Combinations used for the flat plate test and corresponding decay type.

$\left(\frac{\nu_t}{\nu}\right)_{in}$	100	10		1			0.1		
λ	1.0	1.0	0.1	1.0	0.1	0.01	0.1	0.01	0.001
Decay type	A	B	A	C	B	A	C	B	A

decay. The only exception occurs for $\frac{\nu_t}{\nu} = 0.1$ and $\lambda = 0.001$, in which a significant deviation for the transition region can be observed. All calculations using the modified dissipation terms show that the solution in both the laminar and turbulent regions was not changed when compared to the baseline calculations, hence the behaviour of the original model is preserved.

A comparison of F_{FS} for two calculations with decay type A is shown in Fig. 3. It is clear that the current formulation for this function makes it dependent on the value of λ, which is not the physically expected behaviour (F_{FS} should be not depend on λ). However, the main challenge of this approach

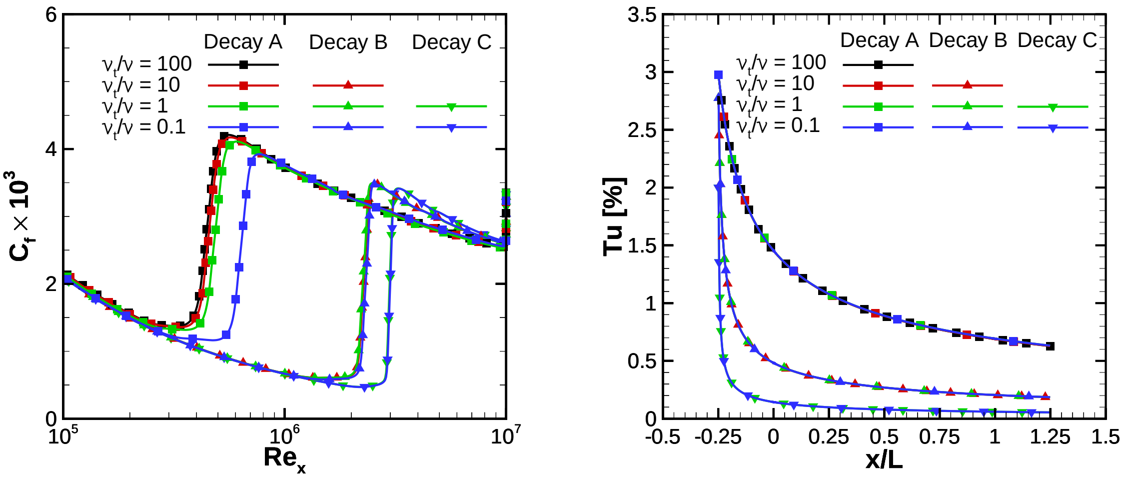

Fig. 2: Local skin friction coefficient C_f along the plate (left) and turbulence intensity Tu along the domain in the freestream (right). Inlet turbulence intensity of 3%.

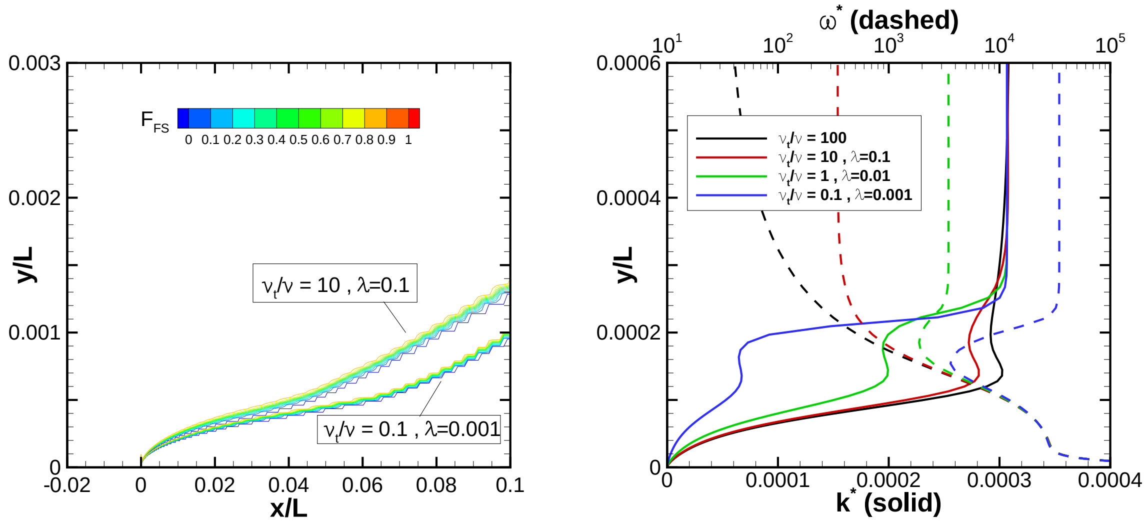

Fig. 3: Comparison of the F_{FS} function in decay type A (left) and k and ω profiles at $x/L = 0.01$ (right).

is illustrated in the right plot of Fig. 3 that depicts the k and ω profiles at $x = 0.01L$, upstream of the critical Reynolds number. The change of λ and $\frac{\nu_t}{\nu}$ leads to a significant difference of the level of ω in the freestream. On the other hand, the four ω profiles are coincident in the boundary-layer region. Therefore, to obtain approximately similar k profiles for different values of λ (the goal of this approach), F_{FS} must start decaying at the same location and decay faster as λ decreases, since freestream ω increases with the decrease of λ. At this stage, the F_{FS} definition is not able to exhibit these properties, especially for $\lambda = 0.001$. Additional calculations were performed for an inlet turbulence intensity of 1%, which led to similar observations. Consequently, they are not shown here.

Flow around the NACA 0012 airfoil

For this case, the combinations for inlet eddy-viscosity and λ are the same as those used for the flat plate case and presented in Tab. 1. However, there is one further addition: the equations of the turbulence model are solved without dissipation terms up to the plane $x/c = -0.5$. This effectively keeps the turbulence variables constant up until the mentioned plane. The goal of this setting is to avoid having the turbulence quantities suffering from a considerable decrease from the inlet up until the leading edge of the airfoil. The results are exhibited in Fig. 4 for an inlet turbulence intensity of 1%.

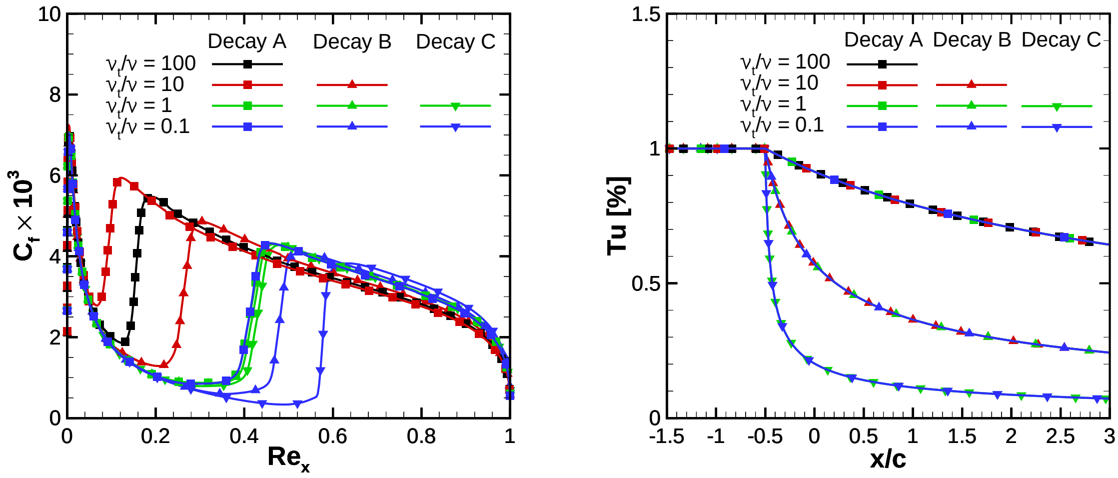

Fig. 4: Local skin friction coefficient C_f on the upper surface of the airfoil (left) and freestream turbulence intensity Tu along part of the domain (right).

Once again the use of the modified dissipation terms causes matching decay of turbulence in the freestream. However, unlike the previous case, this is not translated in the same location for the transition region. For decay type A, the calculation with $\frac{\nu_t}{\nu} = 100$ predicts transition to start at around $x/c = 0.16$, while for the case with $\frac{\nu_t}{\nu} = 10$ it starts before $x/c = 0.08$ and after $x/c = 0.25$ for the remaining combinations. The remaining decay types also exhibit highly varying locations for the transition region.

This case involves a more complex flow than the previous one, thus it is to be expected that it is harder to achieve the desired properties of F_{FS} in this case. The results confirm that the success of this approach requires improvements in the formulation of F_{FS}.

6 Conclusions

A modification to the dissipation terms of the $k - \omega$ SST turbulence model is presented in this paper. Its goal is to reduce the influence of the inlet eddy-viscosity in the decay of the turbulence variables in the freestream, allowing for low values to be used when small decay is desired, which is usually the case for transitional flow simulations. In the present approach, the desired decay of freestream k is achieved through the decrease of the constants of the dissipation terms of the k and ω transport equations.

Calculations with the proposed modification were performed, employing the widely used $\gamma - Re_\theta$ transition model, for the flow over a flat plate and the flow around the NACA 0012 airfoil. Different combinations for the inlet turbulence quantities were tested in order to assess whether the proposed modification achieved the desired purpose. The calculations for the flat plate, which were also used for the calibration of some parameters, exhibited promising results. However, the desired properties were not obtained when the freestream constants of the dissipation terms are less than two orders smaller than the standard values. This was further confirmed in the NACA 0012 test case, in which the shortcomings of the current approach became even more evident.

Therefore, further work is required to obtain a robust formulation which is able to keep the behaviour of the original model in the laminar and turbulent regions, while allowing for control of the freestream turbulence decay such that the same decay leads to the same location of the transition region.

Acknowledgements

The first author acknowledges financial support provided by Fundação para a Ciência e Tecnologia through a research grant under the Bolsas de Doutoramento and Pós-Doutoramento 2016 program. Funding from the Dutch Ministry of Economic Affairs is also acknowledged.

References

Eça, L., Lopes, R., Vaz, G., Baltazar, J., and Rijpkema, D. (2016). Validation Exercises of Mathematical Models for the Prediction of Transitional Flows. In *Proceedings of 31st Symposium on Naval Hydrodynamics, 11th-16th September, Berkeley*.

Langtry, R. B. and Menter, F. R. (2009). Correlation-based Transition Modeling for Unstructured Parallelized Computational Fluid Dynamics Codes. *AIAA journal*, 47(12):2894–2906.

Li, D., P., L., and Werner, S. (2019). Transitional flow on model scale propellers and their likely influence on performance prediction. In *Proceedings of* 6^{th} *International Symposium on Marine Propellers, Rome, Italy*.

Lopes, R., Eça, L., and Vaz, G. (2017). On the Decay of Free-stream Turbulence Predicted by Two-equation Eddy-viscosity Models. In *Proceedings of* 20^{th} *NuTTS, Wageningen, The Netherlands*.

Menter, F. R., Kuntz, M., and Langtry, R. (2003). Ten Years of Industrial Experience with the SST Turbulence Model. *Turbulence, Heat and Mass Transfer*, 4(1):625–632.

Spalart, P. R. and Rumsey, C. L. (2007). Effective Inflow Conditions for Turbulence Models in Aerodynamic Calculations. *AIAA Journal*, 45(10):2544–2553.

Wilcox, D. C. (1998). *Turbulence Modeling for CFD, 2nd ed.* DCW Industries.