The use of BEMT and a Transient Timoshenko Beam to Obtain Vibration Properties of Propeller Blades Under Manoeuvring Conditions

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1 Introduction

Propellers are commonly used as propulsors for ships and maritime submarines. Unsteady propeller blade motions can have a significant impact on the fatigue life of the propeller and shaft line and increase the noise generation. If the motions excite a vibration mode of the propeller the effect can be significantly detrimental to the fatigue life of the propeller and drive train and be a source of noise. The motion of the blade also uplifts broadband noise contributions. Excluding cavitation there are two main causes of propeller hydrodynamic noise; swathing and singing. Swathing is the structural response of the propeller blade caused by the unsteady and non-uniform wake inflow. Singing occurs when a trailing edge vortex sheet is shed from the foil and excites one of the vibration modes of the propeller (Sastry 1982). The structural vibration modal frequencies will typically lock-in the vortex frequency rather than that of the shedding frequency for a rigid structure.

The influence of noise uplift is not only a Naval vessel issue but of growing importance because of increasing attention and evidence of its potential impact on the wider marine environment. Typically, a propeller designer mitigates the possibility of blade motion and associated vibrations by designing to reduce blade unsteady forces for a single design condition. This is typically done by careful choice of blade skew and rake distribution. The possibility of blade vibrations is then normally judged based on experience of full scale tests and a limited finite element modal analysis. Full transient simulations are not normally done however CFD is being performed to predict ship loads during manoeuvres and different sea states. Predicting if propeller blade vibration is an issue is a complex task: vibrations can be caused by either or both the unsteady inflow and self-induced vibration from vortex shedding from the trailing edge.

Simulation of this phenomenon using large scale coupled simulations have been developed (Lloyd 2013) , (Domenico 2018) however, typically these are impractical for design purposes due to their high computational cost. For practical design tools, it is desirable to reduce the computational cost as much as possible. This is done by modelling the fluid loading and structural response using well defined models. In this case the fluid loading is modelled using Blade Element Momentum Theory and the structure of the propeller blade is idealized as a Timoshenko beam.

The aim is to present the simulation tools developed to model the fluid structure interaction response of a propeller blade and eventually of use for design assessment of propeller candidates across a range of operating conditions. These include: (1) the model for a time accurate one-dimensional, six degree of freedom Timoshenko beam (2) implement an unsteady wake into a blade element momentum theory code and obtain transient deformation of the propeller blade as it passes through the unsteady wake. This will indicate if the propeller blade natural frequencies are excited by the unsteady wake. For this test case the HMRI propeller was used in the wake of the KVLCC2 hull form (Larsson, 2010).

2 Methodology

2.1 1D Timoshenko Beam Model

To model the structural response of the blade Timoshenko (1921) beam theory was used. It takes into account shear deformation and rotational bending making it suitable for low aspect ratio beams. This was chosen as it is computationally cheap and can accurately model the structure with relatively few elements. The theory has been used to model the structure of horizontal axis wind turbines and helicopter blades (Yardimoglu 2003). Full details of the theory of Timoshenko beams can be found in Andersen et al (2008).

The Timoshenko model is defined as a series of nodes each with six degrees of freedom with the stiffness matrix of the beam model computed using the moment of inertia, the elastic modulus and the
beam geometry. The geometry of the beam is determined by the geometric properties of the propeller blade. The length of the beam is the radius of the propeller subtracting the radius of the hub, the thickness of the beam was determined using the maximum thickness of the blade and the breadth determined by the chord at that section. The code to determine the stiffness matrix is capable of changing the chord and thickness of the beam depending on the chord and thickness distribution along the blade. The beam is modelled as a series of cuboidal sections with the height of the element being 70% of the blade thickness and breadth 70% of blade chord to account for the reduced area of the blade section due to the curvature.

This generates a beam capable of motion in 6 degrees of freedom with similar geometric and physical parameters to the propeller blade. To determine the accuracy of the beam model a 3D FEA model was generated using Ansys v19.2. The Ansys model used one blade of the HMRI with the hub. The Ansys model consists of 20,000 solid 210 elements with care to capture the curvature of the blade using increased refinement at the blade tip, trailing edge and leading edge. Two tests were used to determine the accuracy of the beam model in comparison to the Ansys model, these tests were: comparison of transient response, comparison of modal frequencies.

The damping matrix was determined by determining the damping ratio of the 3D FEA model. Firstly, a transient response is developed in the Ansys model by applying a small velocity to the blade tip. A curve fit was added to the transient response of the 3D Ansys model using damping coefficient $\gamma$. The general shape of the damping was matched using $\gamma = e^{-\gamma t}$ and $\gamma$ is adjusted to determine an accurate damping rate. Once an appropriate damping coefficient was determined, the natural frequencies were used to obtain the mass damping coefficient $\alpha$ and the stiffness matrix damping coefficient $\beta$ as shown in equation 1.

$$\alpha = \frac{\omega_2}{\omega_1}, \quad \beta = \frac{(1 - \alpha)\zeta}{\omega_2 - \alpha\omega_1}$$

(1)

where $\omega_n$ is the modal frequency of mode $n$ and $\zeta = \frac{\gamma}{\omega_1}$. The damping matrix is then determined using $C = \alpha[M] + \beta[K]$ (Capsoni 2013). Once this damping matrix is determined the transient response of the beam model can be compared to the blade model. The modal frequencies are determined through a simple modal analysis where the modal frequencies and shapes are determined by the eigenvalues and eigenvectors. The static analysis is performed by solving the equation $[K]U = [F]$ where $[K]$ is the stiffness matrix $U$ is the deflection vector and $F$ is force matrix.

As the propeller will be operating in water it is important to consider the effects of the surrounding fluid on the response of the structure. The Ansys model was placed in a spherical domain of water, with the domain radius only being slightly larger than that of the propeller. The result of placing the propeller in the domain in water is that is lowered the natural frequencies with dry $\omega_1 = 1144.3$Hz and wet $\omega_1 = 686$Hz . The frequency is changed due to added mass. The added mass is modelled in the beam model by assuming each element to be a ellipse at an angle of attack and using equation 2 as found in Bishop (1979).

$$m_{\text{added}} = \frac{\rho_{\text{fluid}}}{\pi}(t^2 \cos^2(\beta_{\text{beam}}) + c^2 \sin^2(\beta_{\text{beam}}))$$

(2)

Where $\rho_{\text{fluid}}$ is the density of the fluid, $t$ is the thickness of the element, $c$ is the chord and $\beta$ is the element angle of attack relative to the velocity. Now the stiffness $[K]$, mass $[M]$ and damping matrices $[C]$ have been computed the transient response of the beam can be found. To compute the position of all elements at the next time step the $HHT - \alpha$ method has been implemented.

The Hilber-Hughes-Taylor-$\alpha$ ($HHT - \alpha$) is a generalized method of the Newark - $\beta$ numerical integration method which is an unconditionally stable, implicit scheme. The equation of motion of the beam system is $[M]\ddot{x} + [C]\dot{x} + [K]x = f(t)$. Where $x$ is the displacement of the beam elements and $\dot{x}$ is the time derivative and $f(t)$ is the time dependent forces. Full explanation of the $HHT - \alpha$ method can be found in Hughes (1983). For this case the material properties used was that of stainless steel with Youngs modulus $2.1 \times 10^{11}$, Poisson ration $\nu = 0.3$ and density $\rho = 7865$kg/m$^3$.

To further verify the transient response the Fast Fourier Transformation (FFT) of the transient response is found. From this result it can be seen that the vibration response of the beam model and Ansys
model match very well in the bending motion with both having a response frequency of $\sim 1100\, Hz$. However, it can also be seen that the beam model does not capture the torsional response shown at $\sim 2493.3\, Hz$. The results of the modal analysis are shown in Table 1 here it is clear that the beam model fails to capture the torsion mode of the Ansys blade model. This is due to the absence of elements in the chordwise direction as when the mode shape of the 2nd mode of the Ansys model is plotted the blade leading and trailing edge are predominantly excited not the bending axis. To model the torsion it would be advisable to replace the beam model with that of a 2D plate. The model is designed to capture the vibration properties of metallic propellers. The use of composite materials will add further complications due to changes in laminate build up which is not modelled using a Timoshenko beam and therefore require further refinement of the modelling approach.

### 2.2 Blade Element Momentum Theory

Blade element momentum theory is useful for modelling propeller performance. This method combines momentum theory, where the propeller is modelled as an infinitely thin annulus with a momentum change, and blade element theory where the propeller blades are modelled as a 2D lifting surface. This method is computationally cheap and has proven to be reasonably accurate. The method is well established and a comprehensive overview can be found in Molland et al (2017) and Burril (1955).

The BEMT algorithm was adapted to allow for the inclusion of a wake inflow. The disc annulus is split into both radial components and circumferential components. This allows for a more accurate description of the force distribution along the annulus whilst also keeping the computational cost low. For this study the disc annulus is split into 12 radial sections to account for the data given for the HMRI and 36 circumferential sections allowing for $10^\circ$ per section (Badoe 2015).

The BEMT code has been verified using previous codes as benchmark cases. Moreover, a CFD case was run to test the open water results and thrust distribution. The details of the CFD are shown on Table 2.

The BEMT and CFD results had good agreement with thrust shown in Table 2. To accurately verify the thrust distribution along each blade the CFD simulation of the HMRI was split into several radial sections to match with the BEMT sections.

The thrust coefficient against the radial position are compared between the CFD and BEMT thrust results. It can be seen from figure 1a that the thrust from the two methods agree well in parts and are poor in others. The BEMT tends to under-estimate the maximum thrust at position $r/R = 0.6$. Although the overall trend between thrust and radial position is captured well. The overall performance curves using BEMT are compared to the values acheived by experimental data are shown in figure 1b. The curves agree particularly well for efficiency however the BEMT code slightly overestimates the thrust and torque coefficients. This can be improved by appropriately correcting the 2D lift and drag curves using Reynolds number corrections.

### 2.3 Coupling Algorithm

To find a time dependent solution to the structural response due to the fluid loading the Timoshenko beam and Blade Element Momentum theory codes must be coupled. This can be done in two ways: i) One-way coupling is where the fluid loading is applied directly to the structure to obtain a response as

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done in Benra (2011). ii) two-way coupling where the deformation of the structure is iteratively fed back to the fluid solver. The change in geometry will then change the loading on the blades which will then further change the geometry at every timestep.

In this case the one way coupling is achieved by firstly splitting the BEMT output into 36 sections representing 10° per fluid time step, this keeps computational cost low but limits the frequency range that can be captured. Using the rotational rate it is a simple calculation to determine the loading on the beam at each time step. The loading for the BEMT is appropriately spread over the FEA grid as the FEA grid is much finer than the BEMT grid. The transient response of the beam is solved using a time step of 0.001s and the loading is changed at each timestep as the beam rotates through the hull wake. To obtain an accurate response a static analysis is performed first to set the initial conditions of the beam bending and deflection. This is to avoid the propeller blade going from unloaded to fully loaded thus creating an impulse force causing the bending frequency of the blade to be excited.

2.4 Simulation of KVLCC2
The KVLCC2 is a hull form used in several validation cases such as the Gothenburg 2010 workshop and the SIMMAN 2020 workshop. There has been extensive experimental data acquired for this hull form therefore making it a strong validation case (Larsson, 2010). The simulation has been conducted at model scale with the domain shown in figure 2. The model $L_{pp}$ is 6.8927 m with an inlet velocity of 1.702 m/s resulting in $Fr = 0.142$. This was to coincide with available experimental data. It is important to note that the free surface has not been modelled in this simulation to reduce computational cost. Ideally this will be modelled to ensure accuracy and test its requirements.

3 Results
To understand the deflection of the blade the inflow to the propeller has been taken as the wake from the KVLCC2 hull form. The axial velocity is shown in figure 3b as a contour plot with a quiver plot showing the tangential and radial velocity components when the hull form in the straight ahead position. The flow pattern here is relatively clean with the flow slowing at the north position due to the friction of the hull and increasing towards the free stream velocity the further away from the hull the fluid is. The experimental resistance is approximately 34.02N and the numeric resistance gives a value of 36.6N with a mesh size of 30 million cells. Although this is fairly close there is still a slight error. Moreover, the characteristic bilge vortex 'hook' is not well captured in the CFD simulation. This can be due to many issues such as turbulence model or meshing. The $y^+$ value in the simulation was not consistently less than one. The $y^+$ value ranges from 0.6 to 200 so the boundary layer has not been held well throughout the hull form.

The one way coupling technique were used as described previously, however the inflow used was
Fig. 2: KVLCC2 Simulation Domain. Note the two lines from the bottom to the hull is a visualisation issue and not due to refinement. Additional mesh refinement occurs at the hull and into the wake.

(a) FFT of the response of the beam model to the wake from the KVLCC2 hull form using one-way coupling.
(b) Axial Inflow into propeller from KVLCC2 simulation.

Fig. 3: Propeller vibration properties and wake inflow from KVLCC2
that shown in figure 3b. The FFT of the response of the blade tip is shown in figure 3a.

Several peaks exist at the rotation rate of the propeller which is at 9.9 rps and its multiples. This is expected as the wake is steady and clean hence there will be cyclic loading as the propeller rotates. Another peak occurs at the natural wet frequency of the propeller being excited by the axial bending force. With the tangential velocity included there are many secondary peaks in the frequency distribution. The tangential velocity is modelled in the BEMT code by introducing a second angular momentum source $\alpha''$. Depending on the wake angular velocity this can cause the blade section to increase or decrease thus changing the propeller performance.

4 Conclusion

The coupling between a transient timoshenko beam and blade element momentum theory has been achieved. Modelling techniques have been demonstrated to effectively model the structural and performance of the propeller. Further work includes: verification and modification of the Beam theory and BEMT to ensure accurate results. Moreover, simulations of ships in manoeuvres will be conducted to be able to apply unsteady wakes into the coupling technique. The use of unsteady wakes would give a far more interesting dynamic response of the propeller blades and is key to move away from the static analysis.

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