Numerical Prediction of Cavitation Damage for the Flow through a Nozzle Using a Multi-Scale Euler-Lagrange Method

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1 Introduction

In maritime flows, cavitation occurs around ship appendages such as rudders and propellers. High flow velocities are accompanied by low static pressures in the liquid that cause vapourisation of cavitation bubbles. In regions of higher pressures, the cavitation bubbles collapse and radiate shock waves of high pressure amplitudes that are able to damage material surfaces. As experimental approaches to predict cavitation erosion for ship appendages involve scale effects, numerical methods to predict erosion have received increasing attention within the last years.

Numerical methods to simulate cavitation are mostly characterised by the approach used to treat the vapour phase. While Euler-Euler methods consider both the liquid and the vapour phase as continua, Euler-Lagrange methods assume only the liquid phase to be continuous and the vapour phase to be an accumulation of spherical Lagrangian bubbles. Despite the details gained by a Lagrangian treatment of the vapour phase, Euler-Lagrange approaches demand a high computational effort. To benefit from the efficiency of the Euler-Euler method and still have insight into details using an Euler-Lagrange method, the approaches are combined in a multi-scale method similar to the ones of Vallier (2013) and Lidtke (2017). Large vapour structures are, therein, treated in an Eulerian framework, while small vapour structures are treated as Lagrangian bubbles. Dynamics and motions of each Lagrangian bubble are calculated and used to identify bubble collapses close to solid surfaces that can possibly cause erosion.

2 Multi-Scale Euler-Lagrange Method

In the multi-scale Euler-Lagrange method, large vapour volumes are captured on the Eulerian grid, while small volumes are treated as spherical Lagrangian bubbles. Vapour volumes are transformed between the two frameworks based on their absolute size – the reference radius of a coherent vapour volume – and their size relative to the numerical grid. Vapour volumes are transformed from the Eulerian grid to Lagrangian bubbles when they decrease below these threshold sizes. Lagrangian bubbles are transformed into Eulerian vapour volumes, when they increase over these threshold sizes or merge into Eulerian vapour volumes.

Cavitating flows are simulated based on a RANS approach using the volume of fluid method to capture the interface between phases. To model turbulence, we apply the k-ω-SST model. For cavitation modelling in regions where the Euler-Euler method is used, we apply the model of Sauer and Schnerr (2000). For each spherical Lagrangian cavitation bubble, its dynamics in terms of growth and collapse are calculated from an equation according to Tomita and Shima (1977). The bubble radius, R, is obtained by solution of the following equation:

$$\frac{p_{r=R} - p_\mathrm{l}}{\rho_\mathrm{l}} + \frac{R\dot{p}_{r=R}}{\rho_\mathrm{l} c_\infty} = R\ddot{R}\left[1 - (1+\epsilon_R)\frac{\dot{R}}{c_\infty}\right] + \frac{3}{2}\dot{R}^2\left(\frac{4-\epsilon_R}{3} - \frac{4}{3}\frac{\dot{R}}{c_\infty}\right) , \tag{1}$$

where $\dot{R}$ and $\ddot{R}$ are the bubble growth rate and the bubble wall acceleration, respectively. p_l is the pressure in the liquid and ρ_l is the density in the liquid. c_∞ is the speed of sound of liquid water in the far field and $\epsilon_R = 1 - \rho_\mathrm{g}/\rho_\mathrm{l}$ with ρ_g as the density of the gas in the bubble. $p_{r=R}$ is the pressure at the inner bubble wall. A first order approximation of it reads:

$$p_{r=R} = p_\mathrm{v} + p_{\mathrm{g},0}\left(\frac{R_0}{R}\right)^{3\gamma_\mathrm{h}} - \frac{2\sigma}{R} - \epsilon_R\frac{4\mu_\mathrm{l}\dot{R}}{R} , \tag{2}$$

and its time derivative is written as:

$$\dot{p}_{r=R} = -3\gamma_\mathrm{h} p_{\mathrm{g},0}\frac{\dot{R}}{R}\left(\frac{R_0}{R}\right)^{3\gamma_\mathrm{h}} + \frac{2\sigma}{R^2}\dot{R} - \epsilon_R 4\mu_\mathrm{l}\frac{\ddot{R}R - \dot{R}^2}{R^2} . \tag{3}$$

p_v is the vapour pressure, μ_l is the viscosity of the liquid, and γ_h is the ratio of specific heats. During bubble growth, the compressibility of the liquid in Eq. (1) is neglected by assuming $\epsilon_R \rightarrow 1$ and $c_\infty \rightarrow \infty$.

To calculate bubble motions, we use the Lagrangian equation of motion which reads:

$$m_\mathrm{b} \frac{\mathrm{d}\vec{u}_\mathrm{b}}{\mathrm{d}t} = \sum_i \vec{F}_i \,. \tag{4}$$

$\vec{u}_\mathrm{b}$ is the velocity of the bubble and $m_\mathrm{b} = \rho_\mathrm{b} V_\mathrm{b}$ is the mass of the bubble. ρ_b is the density of the bubble and the volume of a sphere is $V_\mathrm{b} = \frac{4}{3}\pi R^3$. $\vec{F}_i$ are forces acting on the bubble owing to drag, pressure gradient, virtual mass, buoyancy, volume variation, and lift and depend on the behaviour of the bubble relative to the carrier fluid and the bubble's dynamics.

Interactions between Lagrangian bubbles and the liquid phase are considered in form of a two-way coupling that exchanges momentum from the carrier fluid to the bubbles and vice versa and the calculates mixture properties such as density and viscosity of the fluid based on both phase fractions in a control volume.

3 Results

The flow passing through a vertical nozzle was simulated using the multi-scale Euler-Lagrange method. Franc and Riondet (2006) experimentally investigated this case in terms of cavitation-induced erosion. Flow entered the geometry through a vertical cylinder that was connected to a radial channel via a 1 mm radius. A stagnation type flow developed at the bottom of the domain below the vertical cylinder and caused the flow to diverge radially outwards. At the connecting radius, cavitation structures evolved, travelled further downstream, and collapsed in regions of higher pressure. Fig. 1 shows a time instance of Eulerian vapour structures (dark blue) and Lagrangian bubbles (light blue) travelling through the nozzle. Blue to red colours on the bottom and the side boundaries indicate pressures from 0 to 1 bar. Cavitation structures formed close to the radius at the end of the vertical cylinder and detached in form of cavitation clouds owing to vortices in the flow. In the time instance shown, Lagrangian bubbles appear downstream from the larger vapour structures as a result of detachment from or separation of larger vapour volumes as well as collapse processes. Close to the Lagrangian bubble positions shown, collapses of the bubbles were calculated.

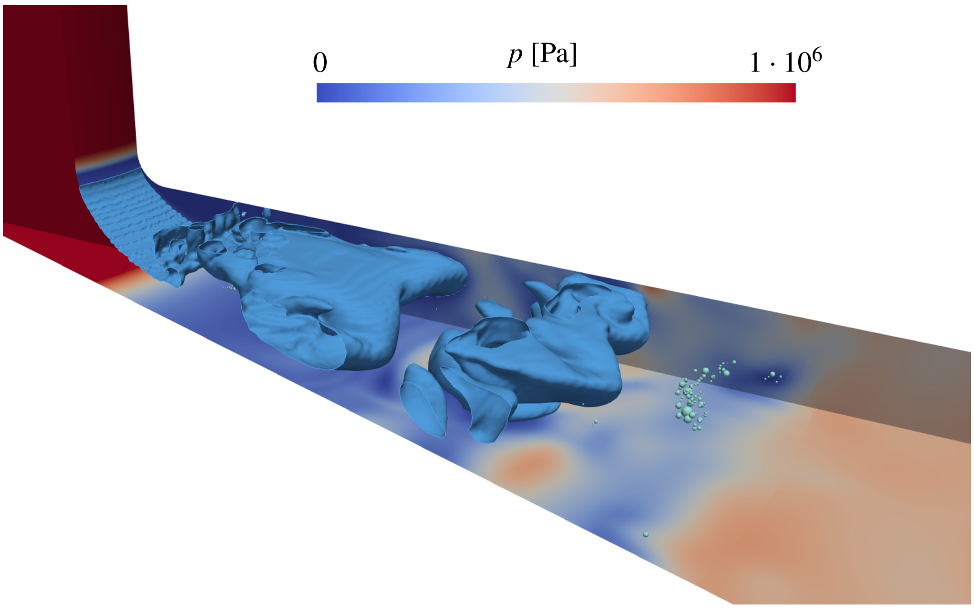

Fig. 1: Perspective side view of Eulerian vapour structures and Lagrangian bubbles in the nozzle

Fig. 2 depicts the predicted areas of erosion on the bottom surface along with positions of Lagrangian bubble collapses. Blue colours on the bottom surface mark regions where the erosion coefficient c_ero and the erosion risk are low; red colours, regions of high erosion risk. c_{ero} was calculated considering the amount of bubble collapses close to the bottom surface and their respective collapse pressures. Spheres indicate collapse positions and are scaled and coloured with their maximum bubble radii, R_{max}, at the beginning of the respective collapse. On these spheres, light grey colours indicate maximum collapse radii of $1 \cdot 10^{-5}$ m; dark grey colours, radii of $1 \cdot 10^{-4}$ m. Within radial distances of 19 to 24 mm from the centre axis of the nozzle, a light green ring marks the area where erosion was measured in experiments of Franc and Riondet (2006). Although, compared to the experimental prediction, bubble collapses were calculated in a larger area, the highest calculated impacts on the bottom surface agreed well with the experimentally measured erosion pattern.

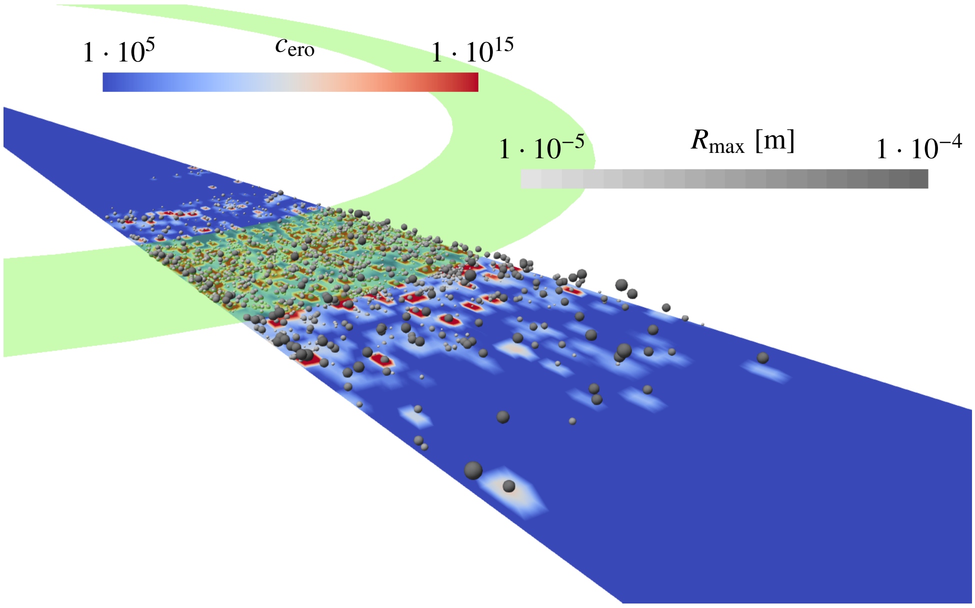

Fig. 2: Erosion prediction on bottom surface and bubble collapses indicated by spheres of maximum radii during respective collapses; green ring marks region where erosion was measured in experiments of Franc and Riondet (2006)

4 Conclusion

The present multi-scale Euler-Lagrange method allows the identification of time instances, positions, bubble radii, and pressures of Lagrangian bubble collapses, while maintaining computational efficiency. Taking into account the behaviour of single cavitation bubbles, more accurate numerical erosion predictions can be performed than for Euler-Euler simulations that neglect single bubble behaviour. The multi-scale method enables to correlate erosion rates with Lagrangian bubble collapse rates and ring-shaped erosion patterns (“pits”) with the radii of Lagrangian bubble collapses.

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