# Workshop on Estimation of Discretization Errors Based on Grid Refinement Studies

## Extrapolation to Cell Size Zero

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The Workshop on estimation of discretization errors based on grid refinement studies includes simulations performed for two geometries with three turbulence models (a,b,c) using the flow solver ReFRESCO [1]:

1. Flow over a flat plate at three Reynolds numbers, (I,II and III), Cases Ia, Ib, Ic, IIa, IIb, IIc, IIIa, IIIb and IIIc.

2. Flow around the NACA 0012 airfoil at three angles of attack (IV, V and VI), Cases IVa, IVb, IVc, Va, Vb, Vc, VIa, VIb and VIc.

These 18 test cases were simulated in sets of geometrically similar grids including 13 grids for the flat plate flow and 9 grids for the flow around the NACA 0012 airfoil. For each test case, the grids of the different sets have the same total number of cells but different sizes of the near-wall cell size (the usually called $y_2^+$).

In the flat plate flow, there is data from simulations performed at Reynolds numbers $Re = 10^7$ (I), $Re = 10^8$ (II) and $Re = 10^9$ (III). On the other hand, all flows around the NACA 0012 airfoil were calculated with $Re = 6 \times 10^6$, but at three different angles of attack $\alpha$: $\alpha = 0^o$ (IV); $\alpha = 4^o$ (V) and $\alpha = 10^o$ (VI). The designation a, b and c identifies the turbulence model used in the (time-averaged) Reynolds-Averaged Navier-Stokes equations for a single-phase incompressible fluid.

a Spalart & Allmaras one-equation eddy-viscosity model [2] (SPAL).

b Shear-Stress transport $k - \omega$ two-equation eddy-viscosity model [3, 4] (SST).

c $k - \sqrt{k}L$ two-equation eddy-viscosity model (KSKL) [5] .

In order to evaluate uncertainty estimates performed for these 18 test cases a reference solution is generated for the quantities of interest selected for each test case using fits to a power series expansion, based on data obtained in grids which are significantly more refined than those used to obtain the Workshop data. Details of the procedure used to obtain these solutions are given in [6]. In this document, we present the reference solutions for all the selected quantities of interest, which include functional (integral), surface and interior quantities[1] . Furthermore, the plots include the fits to the data obtained in extra

[1]Complete list of variables is available at [7]

refined grids and the Workshop data. The observed orders of grid convergence are given in tables together with the reference solutions.

For each test case and selected flow quantity, this document contains the following information:

- The average and maximum $y^+$ (or $d^+$) for all the grid sets proposed for the Workshop.
- Tables with the reference solution and the order of grid convergence used in the extrapolation.
- Plots including the Workshop data, the extra solutions used in the extrapolation and the reference solution. These plots are not presented for the interior locations of the NACA 0012 test cases where the interpolation technique used for the Workshop data is not reliable in any of the extra grids.
- Distributions of the selected flow quantities on the surface obtained in the finest grid available.
- Profiles of velocity components and eddy-viscosity at the selected interior locations obtained in the finest grid available. $V_x$ and $V_y$ are used for the flat plate cases ($V_x = V_t$ and $V_y = V_n$), but tangential $V_t$ and normal $V_n$ components to the surface are used for the NACA 0012 test cases.

There are several remarks that should be made about the data and plots presented in the following sections:

- There are three situations where the reference solution is set equal to the solution of the finest grid ($p = 0$ in the tables):

    1. The difference between the solutions obtained in all the grids of a given test case are smaller than $10^{-8}$.
    2. The standard deviation of the fit obtained from the power series expansion [6] is larger than the data range divided by number of grids minus one.
    3. The extrapolated solution is physically unacceptable.

  The first exception is motivated by the possible influence of the iterative error in the data and the second one is a consequence of noisy data, which can be a consequence of switches/limiters in the turbulence model. The final exception applies only to $\nu_t$ that must be larger than zero.

- The interior locations selected for the Workshop do not coincide with cell centres and so interpolation was required to generate the data for the Cartesian velocity components $V_x$ and $V_y$ and eddy-viscosity $\nu_t$. The bilinear interpolation technique used to generate the Workshop data relies on the determination of the nearest cell centres in each of the four quadrants of a local coordinate system centered at the desired location with $x$ aligned with the incoming flow. Such technique is fully reliable for the flat plate flow, but it may experience difficulties for the selected locations in the near-wall region (linear and buffer layers) of the NACA 0012 airfoil. In order to avoid a new definition of the Workshop data, some of these locations were not included in the determination of the reference solutions and so the corresponding plots are not presented and the data is not included in the tables. Nonetheless, the number of interior locations for test cases IV, V and VI is still larger than 100.

- The vertical velocity component $V_y$ along the line $y = 0$ for the the flow around the NACA 0012 at $0^o$ angle of attack has a known exact solution and so it does not require extrapolation, $V_y = 0$.

- The vertical axis of all the plots was automatically selected by Tecplot [8] using the tool "Fit to Full Size".

- Number of the figures corresponds to the number of the selected local (surface or interior) quantities. Observed order of grid convergence and coordinates of the locations are only given in the tables.

- The distributions of surface quantities ($C_f$ and $C_p$) and the profiles of interior quantities ($V_x$, $V_y$ and $\nu_t$) obtained in the finest grid used for each test case are presented at the begining of the sections in the figures with number 0.

The results for each of the test cases are presented in the following files:

- reference_data_caseI.pdf, flow over a flat plate at $Re = 10^7$.

- reference_data_caseII.pdf, flow over a flat plate at $Re = 10^8$.

- reference_data_caseIII.pdf, flow over a flat plate at $Re = 10^9$.

- reference_data_caseIV.pdf, flow around the NACA 0012 airfoil at $\alpha = 0^o$ and $Re = 6 \times 10^6$.

- reference_data_caseV.pdf, flow around the NACA 0012 airfoil at $\alpha = 4^o$ and $Re = 6 \times 10^6$.

- reference_data_caseVI.pdf, flow around the NACA 0012 airfoil at $\alpha = 10^o$ and $Re = 6 \times 10^6$.

# References


[1] www.refresco.org

[2] Spalart P.R., Allmaras S.R. - *A One-Equation Turbulence Model for Aerodynamic Flows* - AIAA 30$^{\text{th}}$ Aerospace Sciences Meeting, Reno, U.S.A., 1992.

[3] Menter F.R., - *Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications* - AIAA Journal, Vol. 32, No. 8, 1994, pp. 1598–1605.

[4] Menter F.R., Kuntz M., Langtry R. - *Ten Years of Industrial Experience with the SST Turbulence Model* - Turbulence, Heat and Mass Transfer 4, ed: K. Hanjalic, Y. Nagano, and M. Tummers, Begell House, Inc., 2003, pp. 625 - 632.

[5] Menter F.R., Egorov Y., Rusch D. - *Steady and Unsteady Flow Modelling Using the* $k-\sqrt{k}L$ *Model* - Turbulence Heat and Mass Transfer 5, 2006.

[6] Eça L., Vaz G. and Hoekstra M. - *A Contribution for the Assessment of Discretization Error Estimators based on Grid Refinement Studies* - submitted to the ASME Journal of Verification, Validation and Uncertainty Quantification, 2018.

[7] http://web.tecnico.ulisboa.pt/ist12278/Discretization/Workshop_discretization_2017.htm

[8] www.tecplot.com

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# Nomenclature

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| $c$ | Chord of the airfoil. |
| $C_{Df}$ | Friction drag/resistance coefficient. |
| $C_{Dp}$ | Pressure drag/resistance coefficient. |
| $C_f$ | Skin friction coefficient, $\tau_w/(1/2\rho V_\infty^2)$. |
| $C_p$ | Pressure coefficient, $(p - p_\infty)/(1/2\rho V_\infty^2)$. |
| $d$ | Normal distance to the airfoil surface. |
| $d^+$ | Normal distance to the airfoil surface in wall coordinates, $(u_\tau d)/\nu$.. |
| $h_i$ | Typical cell size of grid $i$. |
| $h_1$ | Typical cell size of finest grid. |
| $L$ | Length of the plate. |
| $p$ | Observed order of grid convergence. |
| $p_\infty$ | Maximum $p$ at the inlet boundary. |
| $Re$ | Reynolds number. |
| $u_\tau$ | Friction velocity, $\sqrt{\tau_w/\rho}$. |
| $V_x$ | Mean Cartesian horizontal velocity component. |
| $V_y$ | Mean Cartesian vertical velocity component. |
| $V_\infty$ | Velocity of the incoming uniform flow, aligned with $x$. |
| $x$ | Horizontal coordinate of the Cartesian coordinate system. |
| $x^*$ | Coordinate aligned with the chord of the foil and the origin at the leading edge. |
| $y$ | Vertical coordinate of the Cartesian coordinate system. |
| $y^+$ | Distance to the plate in wall coordinates, $(u_\tau y)/\nu$. |
| $y_2$ | Height of near-wall cells. |
| $y_2^+$ | Dimensionless height of near-wall cells in wall coordinates, $(u_\tau y_2)/\nu$. |
| $\alpha$ | Angle of attack. |
| $\nu$ | Kinematic viscosity of the fluid. |
| $\nu_t$ | Eddy-viscosity. |
| $\rho$ | Density of the fluid. |
| $\tau_w$ | Shear-stress at the wall. |